Homework Set No. 2, Physics 880.02

Deadline – Thursday, February 5, 2009

1. If a photon was a *massive* particle of mass m its Lagrangian density (in the absence of sources) would have been given by the so-called Proca Lagrangian

$$\mathcal{L}_{\text{Proca}} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{m^2}{2} A_\mu A^\mu \tag{1}$$

where A_μ is the 4-vector photon field.

(a) (5 pts) Find the equations of motion for the Proca Lagrangian (known as the Proca equations).

(b) (5 pts) Take a 4-divergence of the Proca equations obtained in (a) to show that if $m \neq 0$ Proca equations require Lorenz gauge condition $\partial_\mu A^\mu = 0$ to always be valid. (Hence Proca Lagrangian in Eq. (1) is not gauge-invariant!) Rewrite Proca equations imposing Lorenz gauge condition.

2. Consider generators of some Lie group obeying Lie algebra commutation relations

$$[X_a, X_b] = i f_{abc} X_c \tag{2}$$

with anti-symmetric structure constants f_{abc}.

(a) (5 pts) Prove the Jacobi identity

$$[X_a, [X_b, X_c]] + [X_b, [X_c, X_a]] + [X_c, [X_a, X_b]] = 0$$

by expanding out the commutators.

(b) (5 pts) Use the commutation relation (2) for X_a's in the Jacobi identity to show that

$$f_{bcd} f_{ade} + f_{abd} f_{cde} + f_{cad} f_{bde} = 0,$$

which is also often referred to as the Jacobi identity.

3. (5 pts) Using Gell-Mann matrices (and their commutators) find the structure constants f^{156} and f^{678} of the group $SU(3)$.

4. Using Young tableaux method decompose the following product representations of the group $SU(3)$ into sums of irreducible representations:

(a) (5 pts) $\mathbf{8} \otimes \mathbf{8}$,

(b) (5 pts) $\mathbf{8} \otimes \mathbf{3}$.