Homework Set No. 2, Physics 880.02

Deadline – Thursday, May 7, 2009

1. (20 pts) Consider a theory of massive fermions ψ coupled to massive (real) scalar bosons φ with the Lagrangian

$$\mathcal{L} = \bar{\psi}\,(i\gamma^\mu \partial_\mu - M)\,\psi + \frac{1}{2}\,\partial_\mu\varphi\,\partial^\mu\varphi - \frac{m^2}{2}\,\varphi^2 - g\,\varphi\,\bar{\psi}\,\psi.$$

Assume that the fermions are much heavier than the bosons, $M \gg m$. Calculate the static potential between a fermion and anti-fermion due to an exchange of a single boson field, similar to how we calculated the heavy quark potential in class. The fermion-boson vertex gives a factor of $-ig$. (This is Yukawa theory of nuclear interactions, with the fermions being protons and neutrons and the scalar particle being the pion. The potential you should obtain is called the Yukawa potential.)

2. (10 pts) Now consider QED with *massive* photons:

$$\mathcal{L} = \bar{\psi}\,(i\gamma^\mu D_\mu - M)\,\psi - \frac{1}{4}\,F_{\mu\nu}\,F^{\mu\nu} + \frac{m^2}{2}\,A_\mu\,A^\mu.$$

Assuming again that fermions (electrons) are much heavier than the "photons", $M \gg m$, calculate the static electron-positron potential due to an exchange of a single "photon". (Hint: the calculation should be similar to that of problem 1.) Note the difference between your result and the Coulomb potential.