The Electroweak Theory

the theory of <u>leptons</u> (spin - 1/2): $\begin{matrix} e & \mu & \tau \\ \nu_e & \nu_\mu & \nu_\tau \end{matrix}$

<u>gauge bosons</u> (spin - 1): γ, $\underbrace{W^+, W^-, Z}_{\text{massive}}$.

& <u>Higgs boson</u>: (spin - 0): φ or H (not yet observed)

Local Gauge Symmetries

Start with quantum electrodynamics (QED).

Take electrons only: $\mathcal{L} = \bar{\psi}\,[\,i\gamma\cdot\partial - m\,]\,\psi$

$\psi \sim$ Dirac field for electrons.

$\mathcal{L}$ is invariant under $\psi \to \psi' = e^{i\alpha}\psi$

$\alpha \sim$ real number (a constant).

$\Rightarrow$ this is a <u>global</u> $U(1)$ symmetry!

<u>Global symmetry</u>: indep. of x^μ.

$\Rightarrow$ Say we want to have $\alpha(x)$: $\psi \to \psi' = e^{i\alpha(x)}\psi(x)$.

Want to have $\mathcal{L}$ invariant under this <u>local $U(1)$ symmetry</u> (<u>local</u> = $\alpha = \alpha(x)$).

$$\bar\psi\,[i\gamma\cdot\partial - m]\,\psi \to \bar\psi\, e^{-i\alpha(x)}\,[i\gamma\cdot\partial - m]\, e^{i\alpha(x)}\psi$$

$$= \bar\psi\,[\,i\gamma\cdot\partial + i\gamma\cdot\partial(i\alpha) - m\,]\psi = \bar\psi\,[i\gamma\cdot\partial - m]\psi$$

$$- \bar\psi\,\gamma^\mu(\partial_\mu\alpha)\psi$$

$\Rightarrow$ $\mathcal{L}$ is not invariant under local $U(1)$ symmetry!

$\Rightarrow$ Fix it by introducing local gauge field $A_\mu(x)$ (gauge the lagrangian): $\left(\begin{matrix} g=-e \\ \text{electron charge}\end{matrix}\right)$

$$\mathcal{L} = \bar\psi\,[i\gamma\cdot\partial - m]\,\psi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} + g\bar\psi\gamma^\mu A_\mu\psi$$

$\Rightarrow$ require that: $\boxed{\begin{array}{l}\psi \to e^{i\alpha(x)}\psi \\ A_\mu \to A_\mu + \frac{1}{g}\partial_\mu\alpha(x)\end{array}}$

$$\Rightarrow \mathcal{L} \to \mathcal{L}' = \bar\psi\,[i\gamma\cdot\partial - m]\psi - \cancel{\bar\psi\gamma^\mu(\partial_\mu\alpha)\psi} - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}$$

$$+ g\bar\psi\gamma^\mu A_\mu\psi + \cancel{\bar\psi\gamma^\mu(\partial_\mu\alpha)\psi} = \mathcal{L}$$

$\Rightarrow$ now it is invariant!

$\Rightarrow$ Def. Covariant derivative $D_\mu \equiv \partial_\mu - igA_\mu$

$$\Rightarrow \mathcal{L}_{QED} = \bar\psi\,[i\gamma^\mu D_\mu - m]\,\psi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}$$

as usual $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$, $F_{\mu\nu} = [D_\mu D_\nu - D_\nu D_\mu]\frac{i}{g}$ $= \frac{i}{g}[D_\mu, D_\nu]$

Now imagine a theory with a non-abelian symmetry, like SU(2): $\psi = \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix}$, $\psi_1, \psi_2 \sim$ spinors

ψ_1 & ψ_2 are different by some quantum #
(e.g. color, weak isospin)

$\mathcal{L} = \bar{\psi}\left[i\gamma^\mu \partial_\mu - m \right]\psi$ with $m = \begin{pmatrix} m & 0 \\ 0 & m \end{pmatrix}$ is invariant under $\psi \to \psi' = e^{i\vec{\alpha}\cdot\frac{\vec{\tau}}{2}}\psi$

$\vec{\tau}$ are Pauli matrices in $\begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix}$ space.

$\Rightarrow$ global SU(2) symmetry.

$\Rightarrow$ let's make it local (gauge it): $\vec{\alpha} = \vec{\alpha}(x)$

$$\Rightarrow \psi \to \psi' = e^{i\vec{\alpha}(x)\cdot\frac{\vec{\tau}}{2}}\psi(x) \equiv S(x)\,\psi(x)$$

with $S^+ S = S S^+ = \mathbb{1}$.

$$\Rightarrow \mathcal{L} \to \mathcal{L}' = \bar{\psi} S^+ \left[i\gamma^\mu \partial_\mu - m \right] S \psi = \bar{\psi}\left[i\gamma\cdot\partial - m \right]\psi$$

$$+ \bar{\psi}\, i\gamma^\mu \left(S^+ \partial_\mu S \right)\psi \quad \Rightarrow \text{not invariant}$$

$\Rightarrow$ add a gauge field A_μ^a, $a = 1, 2, 3$:

$$\boxed{\mathcal{L} = \bar{\psi}\left[i\gamma\cdot\partial - m \right]\psi - \frac{1}{4} F_{\mu\nu}^a F^{a\mu\nu} + g\bar{\psi}\gamma^\mu A_\mu^a \frac{\tau^a}{2}\psi}$$

$$\mathcal{L} \to \mathcal{L}' = \bar{\psi}\left[i\gamma^\mu \partial_\mu - m\right]\psi + \bar{\psi}\, i\gamma^\mu \left(S^+ \partial_\mu S\right)\psi +$$

$$+ g\,\bar{\psi}\gamma^\mu S^+ A'_\mu S\psi - \frac{1}{4} F'^{a}_{\mu\nu} F'^{a\mu\nu}$$

where $A_\mu = A_\mu^a \frac{\tau^a}{2}$ is a matrix.

Collect ψ-terms: $g\bar{\psi}\gamma^\mu \underbrace{\left[S^+ A'_\mu S + \frac{i}{g} S^+ \partial_\mu S\right]}_{\text{require } = A_\mu}\psi$

$\Rightarrow A_\mu = S^+ A'_\mu S + \frac{i}{g} S^+ \partial_\mu S \Rightarrow S A_\mu S^+ = A'_\mu +$

$+ \frac{i}{g}(\partial_\mu S) S^+ \Rightarrow$ $A'_\mu = S A_\mu S^+ - \frac{i}{g}(\partial_\mu S) S^+$

$\psi' = S\psi$

non-abelian gauge transformation!

Def. Covariant derivative $D_\mu = \partial_\mu - igA_\mu$

(note: now it's a matrix!)

$$\Rightarrow \boxed{\mathcal{L} = \bar{\psi}\left[i\gamma^\mu D_\mu - m\right]\psi - \frac{1}{4} F^a_{\mu\nu} F^{a\mu\nu}}$$

<u>But</u>: we never checked the invariance of $F^a_{\mu\nu} F^{a\mu\nu}$ term. What is $F^a_{\mu\nu}$ anyway? Using abelian analogy write $F_{\mu\nu} = \frac{i}{g}\left[D_\mu, D_\nu\right]$

where $F_{\mu\nu} = F_{\mu\nu}^a \frac{\tau^a}{2}$.

$$F_{\mu\nu} = \frac{i}{g}\left[D_\mu, D_\nu\right] = \frac{i}{g}\left[\partial_\mu - igA_\mu, \partial_\nu - igA_\nu\right] =$$

$$= \frac{i}{g}\left\{-ig\left[\partial_\mu, A_\nu\right] - ig\left[A_\mu, \partial_\nu\right] - g^2\left[A_\mu, A_\nu\right]\right\}$$

$$= \frac{i}{g}\left\{-ig\left(\partial_\mu A_\nu - \partial_\nu A_\mu\right) - g^2\left[A_\mu, A_\nu\right]\right\} =$$

$$= \partial_\mu A_\nu - \partial_\nu A_\mu - ig\left[A_\mu, A_\nu\right]$$

$$\Rightarrow \boxed{F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu - ig\left[A_\mu, A_\nu\right]}$$

$$F_{\mu\nu}^a \frac{\tau^a}{2} = \left(\partial_\mu A_\nu^a - \partial_\nu A_\mu^a\right)\frac{\tau^a}{2} - ig A_\mu^b A_\nu^c \underbrace{\left[\frac{\tau^b}{2}, \frac{\tau^c}{2}\right]}_{i\varepsilon^{bca}\frac{\tau^a}{2}} \leftarrow SU(2)$$

$$= \frac{\tau^a}{2}\left[\partial_\mu A_\nu^a - \partial_\nu A_\mu^a + g\,\varepsilon^{abc} A_\mu^b A_\nu^c\right]$$

$$\Rightarrow \boxed{F_{\mu\nu}^a = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a + g\,\varepsilon^{abc} A_\mu^b A_\nu^c}$$

~ true for $SU(2)$

~ other groups have different group structure constants:

$$F_{\mu\nu}^a = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a + g f^{abc} A_\mu^b A_\nu^c.$$