Last time: Finished talking about local non-Abelian symmetries: showed that the gauged Lagrangian

$$\mathcal{L} = \bar{\psi}\left[i\gamma^\mu D_\mu - m\right]\psi - \frac{1}{2}\,\mathrm{tr}\left[F_{\mu\nu}F^{\mu\nu}\right], \quad D_\mu = \partial_\mu - igA_\mu$$

is gauge invariant. $\quad F_{\mu\nu} = \frac{i}{g}\left[D_\mu, D_\nu\right]$

Under gauge transform: $\psi \to \psi' = S(x)\psi$

$\Rightarrow \quad D_\mu \to S(x)\,D_\mu\,S^{-1}(x), \quad F_{\mu\nu} \to S\,F_{\mu\nu}\,S^{-1}$

$\Rightarrow$ all is invariant

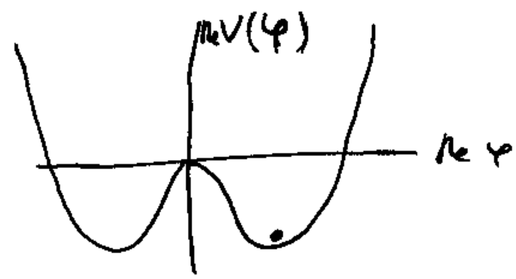
$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu - ig\left[A_\mu, A_\nu\right]$$

~includes self-int. of gauge fields(!)

The Higgs Mechanism (U(1) model)

$$\mathcal{L} = (D_\mu\varphi)^*(D^\mu\varphi) - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \mu^2\varphi^*\varphi - \lambda(\varphi^*\varphi)^2.$$

U(1) gauge symmetry



pick SSB VEV:

$$\langle 0|\varphi|0\rangle = \frac{v}{\sqrt{2}} = \frac{\mu}{\sqrt{2\lambda}}.$$

First define ρ', θ (real fields) by $\varphi = \frac{\rho'(x)}{\sqrt{2}}e^{i\theta(x)}$

$\Rightarrow \mathcal{L}$ becomes

$$\mathcal{L} = \frac{1}{2}\left[(\partial_\mu + igB_\mu)\rho'\right]\left[(\partial_\mu - igB_\mu)\rho'\right] - \frac{1}{4}G_{\mu\nu}G^{\mu\nu} + \frac{\mu^2}{2}\rho'^2 - \frac{\lambda}{4}\rho'^4$$

with $B_\mu = A_\mu - \frac{1}{g}\partial_\mu\theta$, $G_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu = F_{\mu\nu}$.

The VEV is $\langle 0|\rho'|0\rangle = v \Rightarrow$ write $\rho' = v + \rho$ to get

$$\mathcal{L} = \frac{1}{2}\partial_\mu\rho\,\partial^\mu\rho - \mu^2\rho^2 - \frac{1}{4}G_{\mu\nu}G^{\mu\nu} + \frac{1}{2}g^2v^2 B_\mu B^\mu + \frac{1}{2}g^2 B_\mu B^\mu(2v\rho + \rho^2) - \lambda v\rho^3 - \frac{\lambda}{4}\rho^4$$

have field ρ with mass $\sqrt{2}\mu$

field B_μ — " — $gv = m_B$

---

massive gauge fields: $\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{m^2}{2}A_\mu A^\mu$

$\Rightarrow$ EOM (HW2): $\partial_\nu F^{\mu\nu} = m^2 A^\mu \Rightarrow \underbrace{\partial_\mu\partial_\nu F^{\mu\nu}}_{=0} = m^2\partial_\mu A^\mu$

$\Rightarrow$ $\partial_\mu A^\mu = 0$ always Lorentz gauge

<u>Recall massless ($m=0$) fields</u>: $\partial_\mu A^\mu = 0$ gauge $\Rightarrow k_\mu\varepsilon^\mu = 0$

$\Rightarrow$ take $k^\mu = \overset{t\ x\ y\ z}{(k, 0, 0, k)} \Rightarrow \varepsilon^\pm_\mu = \frac{1}{\sqrt{2}}(0, 1, \pm i, 0)$ (transverse)

$\varepsilon_L = (1, 0, 0, 1) \Rightarrow \varepsilon_L^2 = 0 \Rightarrow$ zero probability $\Rightarrow$ 2 d.o.f.

<u>Now, for $m \neq 0$</u>: $\partial_\mu A^\mu = 0$ gauge always $\Rightarrow k_\mu\varepsilon^\mu = 0$

$\Rightarrow$ can now take $k^\mu = (m, \vec{0})$ in the particle's rest frame

$\Rightarrow \varepsilon^{(1)}_\mu = (0, 1, 0, 0)$, $\varepsilon^{(2)}_\mu = (0, 0, 1, 0)$, $\varepsilon^{(3)}_\mu = (0, 0, 0, 1)$

with $(\varepsilon^{(i)})^2 = -1 \Rightarrow$ non-zero prob. for all $\Rightarrow$ 3 d.o.f.

potential: $\frac{\mu^2}{2}\left(2\rho\cdot v+\rho^2\right)-\frac{\lambda}{4}\left(\rho^4+4\rho^3 v+6\rho^2 v^2\right.$

$\left.+4\rho v^3+v^4\right)=\rho\,\cancel{\left(\mu^2 v-\lambda v^3\right)}\to 0+\rho^2\cdot\Big(\frac{\mu^2}{2}-$

$-\frac{3}{2}\underbrace{\lambda v^2}_{\mu^2}\Big)-\lambda\rho^3 v-\frac{\lambda}{4}\rho^4=-\mu^2\rho^2-\lambda\rho^3 v-\frac{\lambda}{4}\rho^4$

$$\mathcal{L}=\frac{1}{2}\partial_\mu\rho\,\partial^\mu\rho-\mu^2\rho^2-\frac{1}{4}G_{\mu\nu}G^{\mu\nu}+\frac{1}{2}g^2v^2B_\mu B^\mu$$
$$+\frac{1}{2}g^2B_\mu B^\mu\left(2\rho v+\rho^2\right)-\lambda\rho^3 v-\frac{\lambda}{4}\rho^4$$

particle content:

~ a scalar ρ with mass $\mu\sqrt{2}$.

~ a massive gauge field B_μ with mass

$m_B = g v$ (see HW #2, problem on Proca Lagrangian: $\mathcal{L}=-\frac{1}{4}F_{\mu\nu}F^{\mu\nu}+\frac{m^2}{2}A_\mu A^\mu$.)

~ field θ got "eaten up" by B_μ, as massless gauge field A_μ had 2 d.o.f., now $\oplus$ 1 (θ)

$\Rightarrow$ B_μ has 3 degrees of freedom.

θ ~ "would-be" Goldstone boson

~ if we had not absorbed θ into B_μ would have gotten terms like $A_\mu\partial^\mu\theta$ ~ not clear how to interpret. (related to negative normproblem) (~~~~---) ?

$\Rightarrow$ SSB of gauge symmetry ~ no Goldstone bosons

~ but get massive vector fields!

(e.g. Meissner effect in superconductivity when photon gets a "mass" and is screened in superconductor ~ P.W. Anderson, '58)

$\Rightarrow$ in particle physics this is known as the <u>Higgs phenomenon</u>. (P. Higgs, 1964).

<u>$SU(2) \otimes U(1)$ Electroweak Theory.</u>

history: Pauli postulated neutrino

Fermi ('34) : to explain β-decay $n \to p e \bar{\nu}$ suggested an interaction term

$$\mathcal{L}_F = -\frac{G_F}{\sqrt{2}}\left[\bar{p}\,\gamma_\mu n\right]\left[\bar{e}\,\gamma^\mu \nu\right] + h.c.$$

with $G_F = \frac{10^{-5}}{m_p^2}$. $\Rightarrow$ but as $[G_F] = \frac{1}{M^2}$ $\Rightarrow$ not renormalizable

$\Rightarrow$ as theory has W, Z vector bosons $\Rightarrow$ Glashow, Salam proposed a gauge theory ('61, '64)

$\Rightarrow$ problem with massive gauge fields:

the propagator is $-i \dfrac{g_{\mu\nu} - \frac{k_\mu k_\nu}{M^2}}{k^2 - M^2 + i\epsilon}$ $\Rightarrow$ also non-renormalizable, as $\to$ const as $k^\mu \to \infty$ $\Rightarrow$

$\Rightarrow$ loops badly diverge...

$\Rightarrow$ Weinberg ('67) suggested using SSB to cure the problem

$\Rightarrow$ Glashow – Weinberg – Salam model

$\Rightarrow$ define fermion fields of leptons:

$e, \mu, \tau, \nu_e, \nu_\mu, \nu_\tau$

$\Rightarrow$ define left & right handed ones $\psi_{L,R} = \frac{1 \mp \gamma_5}{2} \psi$

$\Rightarrow$ group leptons in left-handed weak-isospin doublets:

$$L_e = \begin{pmatrix} \nu_e \\ e \end{pmatrix}_L , \quad L_\mu = \begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix}_L , \quad L_\tau = \begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix}.$$

and in right-handed isospin singlets:

$$R_e = e_R , \quad R_\mu = \mu_R , \quad R_\tau = \tau_R$$

$\Rightarrow$ write the Lagrangian for the 3 generations (aka families) of leptons:

$$\boxed{\mathcal{L}_{free} = \bar{R}_e \, i\gamma\cdot\partial R_e + \bar{L}_e \, i\gamma\cdot\partial L_e + (\mu \,\&\, \tau\text{-terms})}$$

$\Rightarrow$ quantum #'s: $\vec{I}$ ~ weak isospin $\Rightarrow$

$\Rightarrow$ doublets have $I = \frac{1}{2}$, singlet has $I = 0$.

$\Rightarrow$ neutrinos have zero electric charge: $Q_{electric} = 0$

$\Rightarrow$ if we want to have $\boxed{Q = I_3 + \frac{Y}{2}}$ $\Rightarrow$
(Gell-Mann–Nishijima-type)

$\Rightarrow$ define weak hypercharge Y: neutrinos have $Q = 0$, $I_3 = +\frac{1}{2}$ $\Rightarrow$ $Y = -1$ $\Rightarrow$ all doublets L_e, L_μ, L_τ have $Y = -1$.

(check: electron has $Q = -1 \Rightarrow -1 = -\frac{1}{2} - \frac{1}{2}$, ok)

~ the singlet: electron $Q = -1 = \underset{\overset{\|}{0}}{I_3} + \frac{Y}{2} \Rightarrow Y = -2$

$\Rightarrow$ iso-singlets have weak hypercharge $Y = -2$.
R_e, R_μ, R_τ

$\Rightarrow$ back to $\mathcal{L}_{free}$: it clearly has the following global symmetries:

$U(1)$: $L_e \to e^{-i\alpha_Y} L_e$, $R_e \to e^{-2i\alpha_Y} R_e$ (required by anomaly cancellation to be the same α_Y)

$SU(2)$: $L_e \to e^{i\vec{\alpha}\cdot\frac{\vec{\tau}}{2}} L_e$, $\vec{\tau}$ ~ Pauli matrices

$\Rightarrow$ Gauge $U(1)$ symmetry first: introduce an abelian vector field $B_\mu(x)$ with field strength $f_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu$ & coupling to leptons of $g'/2$: $\quad Y=-2 \Rightarrow \left(\partial_\mu - i \frac{g'}{2} Y B_\mu\right)$

$$\mathcal{L} = \bar{R}_e \, i\gamma^\mu \left(\partial_\mu \underbrace{+2}_{Y=-2} i \left(\frac{g'}{2}\right) B_\mu\right) R_e + \bar{L}_e \, i\gamma^\mu \cdot$$

$$\cdot \left(\partial_\mu \underbrace{+1}_{Y=-1} \cdot i \left(\frac{g'}{2}\right) B_\mu\right) L_e - \frac{1}{4} f_{\mu\nu} f^{\mu\nu} + (\mu, \tau).$$

$\Rightarrow$ Now let us gauge the $SU(2)$ symmetry: introduce a gauge field $\vec{W}_\mu = (W^1_\mu, W^2_\mu, W^3_\mu)$ with the field strength $F_{\mu\nu} = \partial_\mu W_\nu - \partial_\nu W_\mu - ig[W_\mu, W_\nu]$

$W_\mu = \vec{W}_\mu \cdot \frac{\vec{\tau}}{2}$, $\;g$ is the coupling of W_μ to itself & to the leptons:

$$\boxed{\begin{aligned}\mathcal{L} = \bar{R}_e \, i\gamma^\mu (\partial_\mu + ig' B_\mu) R_e + \bar{L}_e \, i\gamma^\mu \Big(\partial_\mu + i\frac{g'}{2} B_\mu - \\ - ig \frac{\vec{\tau}}{2} \cdot \vec{W}_\mu\Big) L_e - \frac{1}{4} f_{\mu\nu} f^{\mu\nu} - \frac{1}{4} \vec{F}_{\mu\nu} \cdot \vec{F}^{\mu\nu} + (\mu, \tau).\end{aligned}}$$

now we have a Lagrangian for the leptons & 4 gauge fields $(B_\mu, \vec{W}_\mu)$, but so far everything is massless ($\Rightarrow$ bad.).

$\Rightarrow$ to give particles (especially $\vec{W}_\mu$'s) a mass need SSB mechanism

$\Rightarrow$ so far the Lagrangian is $SU(2) \otimes U(1)$ invariant

$\Rightarrow$ to break this symmetry introduce Higgs field: a weak isospin doublet

$$\phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} \begin{matrix} \Rightarrow Q = +1 \\ \Rightarrow Q = 0 \end{matrix} \Rightarrow Q = I_3 + \frac{Y}{2}, \quad I_3 = \pm\frac{1}{2}$$

$\Rightarrow$ weak hypercharge is $Y = 1$.

$$\phi^+ = (\phi^-, \phi^{0+})$$

$\Rightarrow$ add Higgs field to the Lagrangian:

$$\boxed{\mathcal{L}_{Higgs} = \left(\left[\partial_\mu - i\frac{g'}{2}B_\mu - ig\frac{\vec{\tau}}{2}\cdot\vec{W}_\mu\right]\phi\right)^+ \left[\partial_\mu - i\frac{g'}{2}B_\mu - ig\frac{\vec{\tau}}{2}\cdot\vec{W}_\mu\right]\phi + \mu^2\phi^+\phi - \lambda\left(\phi^+\phi\right)^2}$$

Note that Higgs has $Y = +1 \Rightarrow \partial_\mu - i\frac{g'}{2}Y B_\mu = \partial_\mu - i\frac{g'}{2}B_\mu$

$\Rightarrow$ Higgs also couples to fermions (Yukawa coupling)

$$\boxed{\mathcal{L}_{Higgs-leptons} = -G_e\left[\bar{L}_e\,\phi\,R_e + R_e^+\,\phi^+ L_e\right] + (\mu, \tau\text{-terms})}$$

as $L_e = \begin{pmatrix} \nu_e \\ e \end{pmatrix}_L \Rightarrow \bar{L}_e \phi R_e = (\bar{\nu}_e \;\; \bar{e}_L) \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} e_R$

$= \bar{\nu}_e \phi^+ e_R + \bar{e}_L \phi^0 e_R$ (matrices in isospin space).

$\Rightarrow$ the full Lagrangian is:

$$\boxed{\begin{aligned} \mathcal{L} &= \bar{R}_e \, i\gamma^\mu (\partial_\mu + i g' B_\mu) R_e + \bar{L}_e \, i\gamma^\mu \left(\partial_\mu + i\frac{g'}{2} B_\mu - i g \frac{\vec{\tau}}{2}\cdot\vec{W}_\mu\right) L_e \\ &+ (\mu,\tau\text{-terms}) - \frac{1}{4} f_{\mu\nu} f^{\mu\nu} - \frac{1}{4} \vec{F}_{\mu\nu}\cdot\vec{F}^{\mu\nu} + \\ &+ \left[\left(\partial_\mu - i\frac{g'}{2} B_\mu - i g \frac{\vec{\tau}}{2}\cdot\vec{W}_\mu\right)\phi\right]^+ \left[\left(\partial_\mu - i\frac{g'}{2} B_\mu - i g \frac{\vec{\tau}}{2}\cdot\vec{W}_\mu\right)\phi\right] \\ &+ \mu^2 \phi^+\phi - \lambda (\phi^+\phi)^2 - G_e \left[\bar{L}_e \phi R_e + R_e^+ \phi^+ L_e\right] \end{aligned}}$$

$SU(2)_L \otimes U(1)_Y$ electroweak theory.

$\Rightarrow$ use the Higgs field to break the symmetry:

$$SU(2)_L \otimes U(1)_Y \rightarrow U(1)_{EM}$$

$\Rightarrow \phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}$ ~ has electric charge +1 $\Rightarrow$ if in vacuum have $\langle 0 | Q | 0 \rangle \neq 0$ (SSB)
$\Rightarrow$ no charge symmetry $\Rightarrow$ no electric charge conservation $\Rightarrow$ don't want this

$\Rightarrow$ to conserve the electric charge require the vacuum of Higgs field to be at

$$\langle 0 | \phi | 0 \rangle = \begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix}.$$

$\Rightarrow \quad V(v) = -\mu^2 \frac{v^2}{2} + \frac{\lambda}{4} v^4 \Rightarrow$ minimize to get

$v = \mu/\sqrt{\lambda}$ $\Rightarrow$ write
$$\phi(x) = e^{-i \frac{\vec{\tau}}{2} \cdot \vec{\theta}(x)} \begin{pmatrix} 0 \\ \frac{v+\eta(x)}{\sqrt{2}} \end{pmatrix}$$

with $\vec{\theta}$, η real fields.

Just like in the abelian $U(1)$ case can absorb $\vec{\theta}$ field into $\vec{W}_\mu$ by performing gauge rotation:

if $S(x) = e^{i \frac{\vec{\tau}}{2} \cdot \vec{\theta}(x)} \Rightarrow \phi \to \phi' = S\phi$, $L_{e,\mu,\tau} \to L'_{e,\mu,\tau} = S L_{e,\mu,\tau}$

and $W_\mu \to W_\mu' = S W_\mu S^{-1} - \frac{i}{g}(\partial_\mu S) S^{-1}$.

$\Rightarrow$ can drop primes to write (keep electrons only)

$$\begin{aligned} \mathcal{L} = {} & \bar{R}_e i\gamma^\mu (\partial_\mu + i g' B_\mu) R_e + \bar{L}_e i\gamma^\mu \left(\partial_\mu + i\frac{g'}{2} B_\mu - i g \frac{\vec{\tau}}{2} \cdot \vec{W}_\mu\right) L_e \\ & - \frac{1}{4} f_{\mu\nu} f^{\mu\nu} - \frac{1}{4} \vec{F}_{\mu\nu} \cdot \vec{F}^{\mu\nu} + \frac{1}{2} \left[\left(\partial_\mu - i\frac{g'}{2} B_\mu - i g \frac{\vec{\tau}}{2} \cdot \vec{W}_\mu\right) \begin{pmatrix} 0 \\ v+\eta \end{pmatrix} \right]^\dagger \\ & \left[\left(\partial_\mu - i\frac{g'}{2} B_\mu - i g \frac{\vec{\tau}}{2} \cdot \vec{W}_\mu\right) \begin{pmatrix} 0 \\ v+\eta \end{pmatrix} \right] + \frac{\mu^2}{2} (v+\eta)^2 - \frac{\lambda}{4} (v+\eta)^4 \\ & - G_e \frac{1}{\sqrt{2}} \left[\begin{pmatrix} \bar{\nu}_e & \bar{e}_L \end{pmatrix} \begin{pmatrix} 0 \\ v+\eta \end{pmatrix} e_R + h.c. \right]. \end{aligned}$$

Start with η-particle: linear terms in η in the potential cancel as usual, as we are expanding around a minimum in η.