Last time: SU(2) ⊗ U(1) Electroweak Theory (cont'd).

defined left-handed doublets: $L_e = \begin{pmatrix} \nu_e \\ e \end{pmatrix}_L$, $L_\mu = \begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix}_L$, $L_\tau = \begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix}_L$

and right-handed singlets: $R_e = e_R$, $R_\mu = \mu_R$, $R_\tau = \tau_R$

under weak isospin.

$$\boxed{\mathcal{L}_{free} = \bar{R}_e \, i\gamma\cdot\partial R_e + \bar{L}_e \, i\gamma\cdot\partial L_e + (\mu,\tau\text{-terms})}$$

~ global $SU(2)_L \otimes U(1)_Y$ symmetry ⇒ gauge it!

⇒ introduced a non-Abelian field $\vec{W}_\mu = (W_\mu^1, W_\mu^2, W_\mu^3)$ with $\vec{F}_{\mu\nu}$,

and an Abelian field B_μ with $f_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu$.

$$\boxed{\mathcal{L}_{leptons+gauge} = \bar{R}_e \, i\gamma^\mu(\partial_\mu + ig' B_\mu) R_e + \bar{L}_e \, i\gamma^\mu\left(\partial_\mu + i\frac{g'}{2}B_\mu - ig\frac{\vec{\tau}}{2}\cdot\vec{W}_\mu\right) \cdot L_e - \frac{1}{4} f_{\mu\nu}f^{\mu\nu} - \frac{1}{4}\vec{F}_{\mu\nu}\cdot\vec{F}^{\mu\nu} + (\mu,\tau)}$$

⇒ all fields are massless ⇒ to make them massive need

SSB of the $SU(2)_L \otimes U(1)_Y$ gauge symmetry: introduced

Higgs field: $\phi = \begin{pmatrix} \phi^{(+)} \\ \phi^{(0)} \end{pmatrix}$, $\phi^+ = \begin{pmatrix} \phi^{(-)} & \phi^{(0)*} \end{pmatrix}$

⇒
$$\boxed{\begin{aligned}\mathcal{L}_{leptons+gauge+Higgs} &= \bar{R}_e \, i\gamma^\mu(\partial_\mu + ig' B_\mu) R_e + \bar{L}_e \, i\gamma^\mu\Big(\partial_\mu + i\frac{g'}{2}B_\mu - \\ &- ig\frac{\vec{\tau}}{2}\cdot\vec{W}_\mu\Big)L_e + (\mu,\tau) - \frac{1}{4} f_{\mu\nu}f^{\mu\nu} - \frac{1}{4}\vec{F}_{\mu\nu}\cdot\vec{F}^{\mu\nu} + \\ &+ \left[\left(\partial_\mu - i\frac{g'}{2}B_\mu - ig\frac{\vec{\tau}}{2}\cdot\vec{W}_\mu\right)\phi\right]^+ \left[\left(\partial_\mu - i\frac{g'}{2}B_\mu - ig\frac{\vec{\tau}}{2}\cdot\vec{W}_\mu\right)\phi\right] \\ &+ \mu^2\phi^+\phi - \lambda(\phi^+\phi)^2 - G_e\left[\bar{L}_e\phi R_e + R_e^+\phi^+ L_e\right]\end{aligned}}$$

$\Rightarrow$ pick a VEV : $\langle 0|\phi|0\rangle = \begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix}$ ~ no charged vacuum

, $\boxed{v = \mu/\sqrt{\lambda}}$

$\Rightarrow$ write $\phi(x) = e^{-i\frac{\vec{\tau}}{2}\cdot\vec{\theta}(x)} \begin{pmatrix} 0 \\ \frac{v+\eta(x)}{\sqrt{2}} \end{pmatrix}$, $\vec{\theta} = (\theta^1, \theta^2, \theta^3)$ and η are real fields.

$\Rightarrow$ the VEV breaks the $SU(2)_L \otimes U(1)_Y$ symmetry down to $U(1)_{EM}$!

$\Rightarrow \quad V(v) = -\mu^2 \frac{v^2}{2} + \frac{\lambda}{4} v^4 \Rightarrow$ minimize to get

$v = \mu/\sqrt{\lambda}$ $\Rightarrow$ write $\qquad \phi(x) = e^{-i\frac{\vec{\tau}}{2}\cdot\vec{\theta}(x)} \begin{pmatrix} 0 \\ \frac{v+\eta(x)}{\sqrt{2}} \end{pmatrix}$

with $\vec{\theta}, \eta$ ~ real fields.

Just like in the abelian $U(1)$ case can absorb $\vec{\theta}$ field into $\vec{W}_\mu$ by performing gauge rotation:

if $S_{(x)} = e^{i\frac{\vec{\tau}}{2}\cdot\vec{\theta}(x)} \Rightarrow \phi \to \phi' = S\phi, \; L_{\substack{e\\ \mu\\ \tau}} \to L'_{\substack{e\\ \mu\\ \tau}} = S L_{\substack{e\\ \mu\\ \tau}}$

and $W_\mu \to W_\mu' = S W_\mu S^{-1} - \frac{i}{g}(\partial_\mu S) S^{-1}$. ($\vec{\theta}$ ~ would-be-Goldstone boson)

$\Rightarrow$ can drop primes to write (keep electrons only)

$$\boxed{\begin{aligned}\mathcal{L} &= \bar{R}_e i\gamma^\mu(\partial_\mu + ig' B_\mu) R_e + \bar{L}_e i\gamma^\mu\left(\partial_\mu + i\frac{g'}{2}B_\mu - ig\frac{\vec{\tau}}{2}\cdot\vec{W}_\mu\right)L_e \\ &- \frac{1}{4} f_{\mu\nu} f^{\mu\nu} - \frac{1}{4}\vec{F}_{\mu\nu}\cdot\vec{F}^{\mu\nu} + \frac{1}{2}\left[\left(\partial_\mu - i\frac{g'}{2}B_\mu - ig\frac{\vec{\tau}}{2}\cdot\vec{W}_\mu\right)\begin{pmatrix}0\\ v+\eta\end{pmatrix}\right]^\dagger \\ &\left[\left(\partial_\mu - i\frac{g'}{2}B_\mu - ig\frac{\vec{\tau}}{2}\cdot\vec{W}_\mu\right)\begin{pmatrix}0\\ v+\eta\end{pmatrix}\right] + \frac{\mu^2}{2}(v+\eta)^2 - \frac{\lambda}{4}(v+\eta)^4 \\ &- G_e\frac{1}{\sqrt{2}}\left[\left(\bar{\nu}_e \;\; \bar{e}_L\right)\begin{pmatrix}0\\ v+\eta\end{pmatrix} e_R + h.c.\right].\end{aligned}}$$

Start with η-particle: linear terms in η in the potential cancel as usual, as we are expanding around a minimum in η.

$O(\eta^2)$: $\frac{1}{2}\partial_\mu\eta\,\partial^\mu\eta + \eta^2\left(\frac{\mu^2}{2} - \frac{6}{4}\lambda v^2\right) = \frac{1}{2}\partial_\mu\eta\,\partial^\mu\eta - \mu^2\eta^2$

$\Rightarrow$ η is a massive particle with mass $\sqrt{2}\,\mu$.

"the Higgs particle"

Now, look at leptons: e, ν: get a term

$$-G_e \frac{v}{\sqrt{2}}\left[\bar{e}_L e_R + \bar{e}_R e_L\right] = -\frac{G_e v}{\sqrt{2}}\bar{e}e$$

$\Rightarrow$ electron has a mass $m_e = \frac{G_e v}{\sqrt{2}}$

(ibid for μ, τ : $m_\ell = \frac{G_\ell v}{\sqrt{2}}$, $\ell = e, \mu, \tau$)

note that $m_\nu = 0$ in the Standard Model

(not true in nature, more on this later)

$\Rightarrow$ Gauge bosons also get a mass: get a term

$$+\frac{1}{2}\left[\left[i\frac{g'}{2}B_\mu + ig\frac{\vec{\tau}}{2}\cdot\vec{W}_\mu\right]\begin{pmatrix}0\\v\end{pmatrix}\right]^\dagger\left[\left(-i\frac{g'}{2}B_\mu - ig\frac{\vec{\tau}}{2}\cdot\vec{W}_\mu\right)\begin{pmatrix}0\\v\end{pmatrix}\right]$$

$$\left(\frac{g'}{2}B_\mu + \frac{g}{2}\,\vec{\tau}\cdot\vec{W}_\mu\right)\begin{pmatrix}0\\v\end{pmatrix} = \begin{pmatrix}\frac{g'}{2}B_\mu + \frac{g}{2}W_\mu^3 & \frac{g}{2}(W_\mu^1 - iW_\mu^2)\\ \frac{g}{2}(W_\mu^1 + iW_\mu^2) & \frac{g'}{2}B_\mu - \frac{g}{2}W_\mu^3\end{pmatrix}\begin{pmatrix}0\\v\end{pmatrix}$$

Define $W_\mu^{(\pm)} \equiv \frac{1}{\sqrt{2}}\left(W_\mu^1 \mp i W_\mu^{(2)}\right)$ ($W^\pm$ bosons)

(also $W_\mu^{(+)} = W_\mu$, $W_\mu^{(-)} = W_\mu^+$)

$$Z_\mu \equiv \frac{-g' B_\mu + g W_\mu^3}{\sqrt{g^2+g'^2}} \qquad (Z\text{-boson})$$

$$A_\mu \equiv \frac{g B_\mu + g' W_\mu^3}{\sqrt{g^2+g'^2}} \qquad (\text{the photon})$$

Equivalently: $\begin{cases} Z_\mu = -B_\mu \sin\theta_W + W_\mu^3 \cos\theta_W \\ A_\mu = B_\mu \cos\theta_W + W_\mu^3 \sin\theta_W \end{cases}$

with $\tan\theta_W = g'/g$ (Weinberg angle)

$\Rightarrow \begin{cases} B_\mu = A_\mu \cos\theta_W - Z_\mu \sin\theta_W \\ W_\mu^3 = A_\mu \sin\theta_W + Z_\mu \cos\theta_W \end{cases}$

$\Rightarrow$ the term in the Lagrangian: $+\frac{v^2}{2}\left|\frac{g}{\sqrt{2}} W_\mu^{(+)}\right|^2 +$

$$+\frac{v^2}{2}\cdot\frac{1}{4}(g^2+g'^2)\,|Z_\mu|^2 = +\frac{g^2 v^2}{4}\underbrace{W_\mu^{(+)} W^{\mu(-)}}_{W_\mu^+ W^\mu} + \frac{v^2}{8}(g^2+g'^2) Z_\mu Z^\mu$$

$W_\mu^+ W^\mu$ – complex vector field

$\underbrace{\text{vector field}}_{\text{"}\frac{1}{2}\text{"}(W_\mu^1 W^{1\mu} + W_\mu^2 W^{2\mu})}$

$\Rightarrow$ $M_W = \frac{g v}{2}$ $M_Z = \frac{\sqrt{g^2+g'^2}}{2} v$.

$\Rightarrow$ W, Z bosons get a mass

$\Rightarrow$ A_μ (photon) remains massless!

The rest of the Lagrangian:

$$\bar{R}_e\, i\gamma^\mu (\partial_\mu + i g' B_\mu) R_e = \bar{e}_R\, i\gamma\cdot\partial\, e_R - g'\, \bar{e}_R \gamma^\mu e_R \cdot$$

$$\cdot (A_\mu \cos\theta_W - Z_\mu \sin\theta_W) = \bar{e}_R\, i\gamma\cdot\partial\, e_R - \underbrace{g' \cos\theta_W}_{\text{QED coupling}} \cdot$$

$$\cdot\, \bar{e}_R \gamma\cdot A\, e_R + \underbrace{g' \sin\theta_W}_{= g \frac{\sin^2\theta_W}{\cos\theta_W}}\, \bar{e}_R \gamma\cdot Z\, e_R$$

QED coupling $= e\ (>0)$

$$\boxed{e = g'\cos\theta_W = g \sin\theta_W}$$

$$\bar{L}_e\, i\gamma^\mu \left(\partial_\mu + i\frac{g'}{2} B_\mu - i g \frac{\vec{\tau}}{2}\cdot\vec{W}_\mu\right) L_e = \bar{\nu}_e\, i\gamma\cdot\partial\, \nu_e +$$

$$+ \bar{e}_L\, i\gamma\cdot\partial\, e_L + \frac{1}{2}\begin{pmatrix}\bar{\nu}_e & \bar{e}_L\end{pmatrix}\cdot \begin{pmatrix} -g'B_\mu + g W_\mu^3 & g(W_\mu^1 - i W_\mu^2) \\ g(W_\mu^1 + i W_\mu^2) & -g' B_\mu - g W_\mu^3 \end{pmatrix}\begin{pmatrix}\nu_e \\ e_L\end{pmatrix}$$

$$= \bar{\nu}_e\, i\gamma\cdot\partial\, \nu_e + \bar{e}_L\, i\gamma\cdot\partial\, e_L + \frac{1}{2}\begin{pmatrix}\bar{\nu}_e & \bar{e}_L\end{pmatrix}\cdot$$

$$\cdot \begin{pmatrix} -g\tan\theta_W (\cancel{A_\mu\cos\theta_W} - Z_\mu \sin\theta_W) + g(\cancel{A_\mu \sin\theta_W} + Z_\mu\cos\theta_W) & \sqrt{2} g W_\mu \\ \sqrt{2}\, g\, W_\mu^+ & -g\tan\theta_W (A_\mu\cos\theta_W - Z_\mu\sin\theta_W) - g(A_\mu\sin\theta_W + Z_\mu\cos\theta_W) \end{pmatrix}$$

$$\cdot \begin{pmatrix}\nu_e \\ e_L\end{pmatrix} = \bar{\nu}_e\, i\gamma\cdot\partial\, \nu_e + \bar{e}_L\, i\gamma\cdot\partial\, e_L + \frac{1}{2}\begin{pmatrix}\bar{\nu}_e & \bar{e}_L\end{pmatrix}\cdot$$

$$\cdot \begin{pmatrix} g Z_\mu \frac{1}{\cos\theta_W} & \sqrt{2} g W_\mu \\ \sqrt{2} g W_\mu^+ & -2g\sin\theta_W A_\mu + g Z_\mu \frac{2\sin^2\theta_W - 1}{\cos\theta_W} \end{pmatrix}\begin{pmatrix}\nu_e \\ e_L\end{pmatrix}$$

$$= \bar{\nu}_e\, i\gamma\cdot\partial\, \nu_e + \bar{e}_L\, i\gamma\cdot\partial\, e_L + \frac{g}{\sqrt{2}}\left[\bar{\nu}_e\, \gamma\cdot W e_L + \bar{e}_L \gamma\cdot W^{+} \nu_e\right]$$ (103)

$$+ \frac{g}{2\cos\theta_W}\, \bar{\nu}_e\, \gamma\cdot Z\, \nu_e - \underbrace{g\sin\theta_W}_{e}\, \bar{e}_L\, \gamma\cdot A\, e_L + \frac{g}{2\cos\theta_W}\,\cdot$$

$$\cdot\left(2\sin^2\theta_W - 1\right)\, \bar{e}_L\, \gamma\cdot Z\, e_L$$

$\Rightarrow$ note that the photon does not couple to neutrino

$\Rightarrow$ it is indeed charge-neutral! (electric charge)

Putting these two terms together get:

$$\bar{R}_e\, i\gamma^\mu\left(\partial_\mu + ig' B_\mu\right) R_e + \bar{L}_e\, i\gamma^\mu\left(\partial_\mu + i\frac{g'}{2} B_\mu - ig\frac{\vec{\tau}}{2}\cdot\vec{W}_\mu\right) L_e$$

$$= \bar{e}\, i\gamma\cdot\partial e + \bar{\nu}_e\, i\gamma\cdot\partial\, \nu_e - e\,\bar{e}\,\gamma\cdot A e + g\frac{\sin^2\theta_W}{\cos\theta_W}\, \bar{e}_R \gamma\cdot Z\, e_R$$

$$+ \frac{g}{2\cos\theta_W}\, \bar{\nu}_e\, \gamma\cdot Z\, \nu_e + \frac{g}{2\cos\theta_W}\left(2\sin^2\theta_W - 1\right)\bar{e}_L\, \gamma\cdot Z\, e_L +$$

$$+ \frac{g}{\sqrt{2}}\left[\bar{\nu}_e\, \gamma\cdot W e_L + \bar{e}_L\, \gamma\cdot W^{+} \nu_e\right].$$

$\Rightarrow$ in principle need to re-write $-\frac{1}{4} f_{\mu\nu} f^{\mu\nu} - \frac{1}{4}\vec{F}_{\mu\nu}\cdot\vec{F}^{\mu\nu}$

in terms of fields A_μ, Z_μ, W_μ ... let's assume we did

$$\Rightarrow \frac{1}{2}\left[\left(\partial_\mu - i\frac{g'}{2}B_\mu - ig\frac{\vec{\tau}}{2}\cdot\vec{W}_\mu\right)\begin{pmatrix}0\\ v+\eta(x)\end{pmatrix}\right]^{+}\left[\left(\partial_\mu - i\frac{g'}{2}B_\mu - \right.\right.$$

$$\left.\left. - ig\frac{\vec{\tau}}{2}\cdot\vec{W}_\mu\right)\begin{pmatrix}0\\ v+\eta(x)\end{pmatrix}\right] = \frac{1}{2}\begin{pmatrix}0 & v+\eta\end{pmatrix}\left(\overleftarrow{\partial}_\mu + i\frac{g'}{2}B_\mu + ig\frac{\vec{\tau}}{2}\cdot\vec{W}_\mu\right).$$

$$\bullet \left(\partial_\mu - i\,\frac{g'}{2} B_\mu - i g\,\frac{\vec{\tau}}{2}\cdot\vec{W}_\mu\right)\begin{pmatrix} 0 \\ v+\eta \end{pmatrix} = \frac{1}{2}\,\partial_\mu \eta\,\partial^\mu \eta$$

$$+ \frac{g^2}{4}(v+\eta)^2\, W_\mu^+ W^\mu + \underbrace{\frac{g^2+g'^2}{8}}_{\frac{1}{8} g^2\left(1+\frac{\sin^2\theta_W}{\cos^2\theta_W}\right) = \frac{g^2}{8\cos^2\theta_W}}(v+\eta)^2\, Z_\mu Z^\mu$$

Note that terms linear in η cancel!

$\Rightarrow$ Combining everything we write:

$$\boxed{\begin{aligned}
\mathcal{L}_{EW} &= \bar{e}\, i\gamma\cdot\partial e + \bar{\nu}_e\, i\gamma\cdot\partial \nu_e - \frac{G_e}{\sqrt{2}}(v+\eta)\,\bar{e} e - \\
&- \frac{1}{4} f_{\mu\nu} f^{\mu\nu} - \frac{1}{4}\vec{F}_{\mu\nu}\cdot\vec{F}^{\mu\nu} + \frac{1}{2}\partial_\mu\eta\,\partial^\mu\eta - \mu^2\eta^2 - \lambda v \eta^3 \\
&- \frac{\lambda}{4}\eta^4 + \frac{g^2}{4}(v+\eta)^2 W_\mu^+ W^\mu + \frac{g^2}{8\cos^2\theta_W}(v+\eta)^2 Z_\mu Z^\mu \\
&+ \frac{g}{2\cos\theta_W}\left[2\sin^2\theta_W\,\bar{e}_R\gamma\cdot Z e_R + (2\sin^2\theta_W - 1)\,\bar{e}_L\gamma\cdot Z e_L\right] \\
&- e\,\bar{e}\gamma\cdot A e + \frac{g}{2\cos\theta_W}\bar{\nu}_e\gamma\cdot Z \nu_e - \frac{g}{\sqrt{2}}\left[\bar{\nu}_e\gamma\cdot W e_L + \right. \\
&\left. + \bar{e}_L\gamma\cdot W^+ \nu_e\right]
\end{aligned}}$$

$+$ (μ,τ terms)

. The Electroweak lagrangian.

$M_W = 80.398 \pm 0.025$ GeV

$M_Z = 91.1876 \pm 0.0021$ GeV