Last time: finished talking about EW lagrangian (leptons only). Talked about it's parameters:

$$\begin{cases} M_W = \frac{gv}{2} = 80.398 \pm 0.025 \text{ GeV} \\ M_Z = \frac{gv}{2\cos\theta_W} = 91.1876 \pm 0.0021 \text{ GeV} \\ m_\gamma = 0 \quad m_\nu = 0 \quad m_e = \frac{G_e v}{\sqrt{2}},\ m_\mu = \frac{G_\mu v}{\sqrt{2}},\ m_\tau = \frac{G_\tau v}{\sqrt{2}}. \end{cases}$$

(masses)

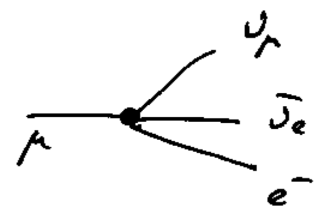
Weinberg angle $\sin^2\theta_W \simeq 0.23120 \pm 0.00015 \approx \frac{1}{4} \Rightarrow$

$$\Rightarrow \theta_W \simeq 30°.$$

$\frac{g^2}{4\pi} \simeq \frac{1}{30}$ ~ small

$\mu \to e \bar{\nu}_e \nu_\mu$

at low energy $\Rightarrow$ M_W is the largest scale $\Rightarrow$ W prop. is $\sim \frac{1}{M_W^2}$ $\Rightarrow$ W^- moves very little in space-time $\sim \frac{1}{M_W}$.
$\Leftrightarrow$ effective vertex $\Rightarrow$



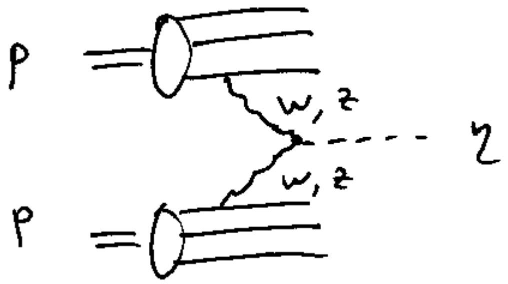
$$\Rightarrow \boxed{\frac{G_F}{\sqrt{2}} = \frac{g^2}{8M_W^2}} \simeq 10^{-5} \text{ GeV}^{-2}.$$

$\boxed{v \simeq 289 \text{ GeV}}$ Higgs VEV

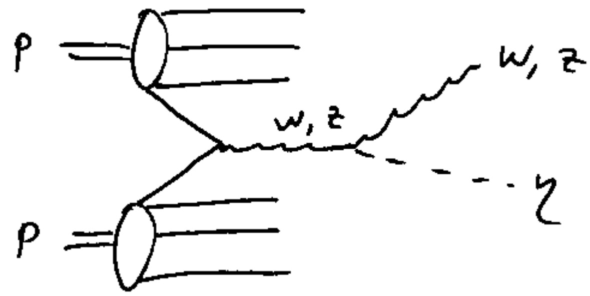
$M_{Higgs} = 129^{+74}_{-49}$ GeV current expectation

Higgs boson will be searched for at LHC.

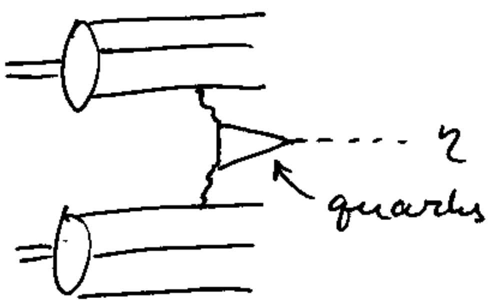
Possible discovery processes are:



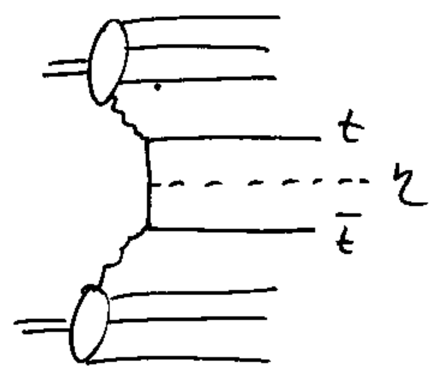
Weak boson fusion



associated Drell-Yan process



gluon fusion



associated top pair ...

Quarks in the Electroweak Theory.

Quarks also form left-handed doublets under weak isospin:

$$L_u = \begin{pmatrix} u \\ d' \end{pmatrix}_L \qquad L_c = \begin{pmatrix} c \\ s' \end{pmatrix}_L \qquad L_t = \begin{pmatrix} t \\ b' \end{pmatrix}_L$$

$$R_u = u_R \qquad R_c = c_R \qquad R_t = t_R$$

$$R_d = d_R \qquad R_s = s_R \qquad R_b = b_R$$

$L_u = \begin{pmatrix} u \\ d' \end{pmatrix}_L$ ~ doublet $\Rightarrow I_3 = \frac{\pm 1}{2} \Rightarrow Q = I_3 + \frac{Y}{2}$

$\Rightarrow Y = 2(Q - I_3) \Rightarrow$ for u have $Q = +\frac{2}{3},\ I_3 = +\frac{1}{2} \Rightarrow$

$\Rightarrow Y = 2\left(\frac{2}{3} - \frac{1}{2}\right) = \frac{1}{3}$; for d' have $Q = -\frac{1}{3},\ I_3 = -\frac{1}{2} \Rightarrow$

$\Rightarrow Y = 2\left(-\frac{1}{3} + \frac{1}{2}\right) = \frac{1}{3} \Rightarrow Y = \frac{1}{3}$ for the doublet!

Singlets: $R_u = u_R$ has $Q = +\frac{2}{3},\ I_3 = 0 \Rightarrow Y = \frac{4}{3}$

$R_d = u_d$ has $Q = -1/3,\ I_3 = 0 \Rightarrow Y = -2/3$

(same for other quark generations / families)

$\Rightarrow$ We have <u>defined</u> the quark <u>weak</u> eigenstates d', s', b' by:

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \underbrace{\begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}}_{\text{CKM matrix}} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

weak eigenstates   CKM matrix   quarks in QCD (mass eigenstates)

$$\text{CKM} = \underbrace{\overset{1963}{\text{Cabibbo}}}_{\text{? No prize?}}\text{-}\underbrace{\overset{1973}{\text{Kobayashi-Maskawa}}}_{\text{Nobel Prize '08}} \text{ matrix}$$

CKM matrix is unitary: $V^\dagger V = V V^\dagger = \mathbb{1}$.

(Logic: our mass matrix for quarks is diagonal, but there is no reason for EW interaction one to be diagonal too.)

Let's write down the Lagrangian:

$$\mathcal{L}_{\text{quarks + gauge}} = \bar{L}_u \, i\gamma^\mu \Big(\partial_\mu - i \frac{g'}{2} \underset{\overset{\shortparallel}{1/3}}{Y} B_\mu - i g \frac{\vec{\tau}}{2} \cdot \vec{W}_\mu \Big) L_u$$

$$+ \bar{R}_u \, i\gamma^\mu \Big(\partial_\mu - i \frac{g'}{2} \underset{\overset{\shortparallel}{\frac{4}{3}}}{Y} B_\mu \Big) R_u + \bar{R}_d \, i\gamma^\mu \Big(\partial_\mu - i \frac{g'}{2} B_\mu \underset{\overset{\shortparallel}{-2/3}}{Y} \Big) R_d$$

+ other 2 generations.

$$\Rightarrow \boxed{\begin{aligned} \mathcal{L}_{\text{quarks + gauge}} &= \bar{L}_u \, i\gamma^\mu \Big(\partial_\mu - i \frac{g'}{6} B_\mu - i g \frac{\vec{\tau}}{2} \cdot \vec{W}_\mu \Big) L_u \\ &+ \bar{R}_u \, i\gamma^\mu \Big(\partial_\mu - i \frac{2}{3} g' B_\mu \Big) R_u + \bar{R}_d \, i\gamma^\mu \Big(\partial_\mu + i \frac{1}{3} g' B_\mu \Big) R_d \\ &+ 2 \text{ more generations.} \end{aligned}}$$

Need to couple quarks to the Higgs: (don't have to, but it would be nice)

$$\phi = \begin{pmatrix} \phi^{(+)} \\ \phi^{(0)} \end{pmatrix}.$$

If we write a term like $\bar{L}_u \phi R_u$ and $\bar{L}_u \phi R_d$.

However the VEV is $\langle 0 | \phi | 0 \rangle = \begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix} \Rightarrow$

$\Rightarrow$ near the Higgs VEV get

$$\bar{L}_u \phi R_u = (\bar{u}_L \; \bar{d}'_L) \begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix} u_R = \bar{d}'_L u_R \frac{v}{\sqrt{2}}$$ ~ no mass for u ...

$\bar{L}_u$: $Y=-1/3$, ϕ: $Y=+1$, R_u: $Y=+4/3$ $\Rightarrow$ not $U(1)_Y$ invariant too...

~ like neutrinos, u would not get a mass... ?

(same for c, t quarks).

$\Rightarrow$ to give quarks mass define $\tilde{\phi}(x) \equiv i\tau^2 \phi^*$

for the VEV: $$\langle 0|\tilde{\phi}|0\rangle = i\underbrace{\tau^2}_{\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}} \cdot \begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix} =$$

$$= \begin{pmatrix} v/\sqrt{2} \\ 0 \end{pmatrix}.$$ ~ have the VEV $\neq 0$ on top now

Under $SU(2)_L$ gauge transform: $\phi \to e^{i\frac{\vec{\alpha}\cdot\vec{\tau}}{2}} \phi$

$$\Rightarrow \tilde{\phi} \to i\tau^2 \left(e^{i\frac{\vec{\alpha}\cdot\vec{\tau}}{2}} \phi \right)^* = i\tau^2 e^{-i\frac{\vec{\alpha}\cdot\vec{\tau}^*}{2}} \phi^* =$$

$(\tau^2)^2 = 1$

$$= \underbrace{\tau^2 e^{-i\frac{\vec{\alpha}\cdot\vec{\tau}^*}{2}} \tau_2}_{e^{i\frac{\vec{\alpha}\cdot\vec{\tau}}{2}}} \tilde{\phi}$$

this is true because: $\tau^2 (-\tau^{1*}) \tau^2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \cdot$

$$\cdot \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix} \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \begin{pmatrix} -i & 0 \\ 0 & i \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} = \tau^1$$

Similarly $\tau^2 (-\tau^{2*}) \tau^2 = \tau^2$ (obvious) and

$\tau^2 (-\tau^{3*}) \tau^3 = \tau^3$ $\Rightarrow$ eqn is true $\left(\tau^2(-\vec{\tau}^*)\tau^2 = \vec{\tau}, (\tau^2)^2 = 1 \Rightarrow \text{sandwich } (\tau^2)^2\text{'s in} \right)$

$\Rightarrow$ under $SU(2)_L$ have $\tilde{\phi} \to e^{i\frac{\vec{\alpha}\cdot\vec{\tau}}{2}}\tilde{\phi}$

$\Rightarrow$ transforms just like ϕ!

$\Rightarrow$ can write $\bar{L}_u \tilde{\phi} R_u$ ~ $SU(2)_L$ invariant!

near VEV: $\bar{L}_u \tilde{\phi} R_u = (\bar{u}_L \ \bar{d}'_L)\begin{pmatrix} v/\sqrt{2} \\ 0 \end{pmatrix} u_R = \frac{v}{\sqrt{2}}\bar{u}_L u_R$

$\langle 0|\tilde{\phi}|0\rangle = \begin{pmatrix} v/\sqrt{2} \\ 0 \end{pmatrix}$ $\Rightarrow$ may give u-quark mass!

terms like $\bar{L}_u \phi R_d = (\bar{u}_L \ \bar{d}'_L)\begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix} d_R = \frac{v}{\sqrt{2}}\bar{d}'_L d_R$

can give d-quark mass (and s, b quarks too).

$\Rightarrow$ also need to check weak hypercharge:

ϕ has $Y=+1$ $\Rightarrow$ $\tilde{\phi}$ has $Y=-1$ $\Rightarrow$ $\underset{Y=-1/3}{\bar{L}_u}\ \underset{Y=-1}{\tilde{\phi}}\ \underset{Y=4/3}{R_u}$ $\Rightarrow$ net $Y=0$

$\underbrace{\underset{Y=-1/3}{\bar{L}_u}\ \underset{Y=+1}{\phi}\ R_d}_{\Rightarrow Y=-2/3}$ $\Rightarrow$ net $Y=0$ ~ both work!

To write quarks + Higgs couplings let's limit ourselves to 2 generations: $L_u, L_c, R_u, R_d, R_c, R_s$.

First write all possible terms:

$$\mathcal{L}_{\text{quarks-Higgs}} = -G_1[\bar{L}_u\tilde{\phi}R_u + \bar{R}_u\tilde{\phi}^+L_u] - G_2[\bar{L}_u\phi R_d + \bar{R}_d\phi^+L_u] - G_3[\bar{L}_u\phi R_s + \bar{R}_s\phi^+L_u] - G_4[\bar{L}_c\tilde{\phi}R_c + \bar{R}_c\tilde{\phi}^+L_c]$$

$$-G_5\left[\bar{L}_c\phi R_d + \bar{R}_d\phi^+ L_c\right] - G_6\left[\bar{L}_c\phi R_s + \bar{R}_s\phi^+ L_c\right]$$

$$-G_7\left[\bar{L}_u\tilde{\Phi} R_c + \bar{R}_c\tilde{\Phi}^+ L_u\right] - G_8\left[\bar{L}_c\tilde{\Phi} R_u + \bar{R}_u\tilde{\Phi}^+ L_c\right].$$

Plug in $\langle 0|\Phi|0\rangle = \begin{pmatrix} 0 \\ v/\sqrt{2}\end{pmatrix}$, $\langle 0|\tilde{\Phi}|0\rangle = \begin{pmatrix} v/\sqrt{2} \\ 0\end{pmatrix}$:

$$\mathcal{L}^{\text{2 generations}}_{\text{quark-Higgs}} = -\frac{v}{\sqrt{2}}\Big\{G_1\,\bar{u}u + G_2(\bar{d}'_L d_R + \bar{d}_R d'_L) + G_3(\bar{d}'_L s_R +$$

$$+\bar{s}_R d'_L) + G_4\,\bar{c}c + G_5(\bar{s}'_L d_R + \bar{d}_R s'_L) + G_6(\bar{s}'_L s_R + \bar{s}_R s'_L)$$

$$+ G_7(\bar{u}_L c_R + \bar{c}_R u_L) + G_8(\bar{c}_L u_R + \bar{u}_R c_L)\Big\}$$

$\Rightarrow$ first of all we see $m_u = G_1\frac{v}{\sqrt{2}}$ $m_c = \frac{G_4 v}{\sqrt{2}}$.

$\Rightarrow$ can't have $u \to c$ & vice versa $\Rightarrow \boxed{G_7 = G_8 = 0}$.

$\Rightarrow$ Left with d, s quarks: for those write:

$$\begin{pmatrix} d' \\ s' \end{pmatrix} = \begin{pmatrix} \cos\theta_c & \sin\theta_c \\ -\sin\theta_c & \cos\theta_c \end{pmatrix}\begin{pmatrix} d \\ s \end{pmatrix}$$

$\theta_c \sim$ Cabibbo angle, in CKM matrix $V_{ud} \simeq \cos\theta_c \simeq V_{cs}$,
$V_{us} \simeq \sin\theta_c \simeq -V_{cd}$

$\theta_c \simeq 13^\circ$ small mixing.

$\Rightarrow d' = d\cos\theta_c + s\sin\theta_c$

$s' = -d\sin\theta_c + s\cos\theta_c$

$$\Rightarrow \mathcal{L}^{d,s\ \text{part}}_{\text{quark-Higgs}} = -\frac{v}{\sqrt{2}}\Big\{ G_2\Big[\underline{\bar{d}d\cos\theta_c} + (\bar{s}_L d_R + \bar{d}_R s_L)\cdot$$

$$\cdot\sin\theta_c\Big] + G_3\Big[\underline{\bar{s}s\sin\theta_c} + (\bar{d}_L s_R + \bar{s}_R d_L)\cos\theta_c\Big] +$$

$$+ G_5\Big[\underline{-\bar{d}d\sin\theta_c} + (\bar{s}_L d_R + \bar{d}_R s_L)\cos\theta_c\Big] +$$

$$+ G_6\Big[\underline{\bar{s}s\cos\theta_c} - (\bar{d}_L s_R + \bar{s}_R d_L)\sin\theta_c\Big]\Big\} =$$

$$= -\bar{d}d\,\frac{v}{\sqrt{2}}\Big[G_2\cos\theta_c - G_5\sin\theta_c\Big] - \bar{s}s\,\frac{v}{\sqrt{2}}\Big[G_3\sin\theta_c +$$

$$+ G_6\cos\theta_c\Big] - \frac{v}{\sqrt{2}}(\bar{s}_L d_R + \bar{d}_R s_L)\underbrace{\Big[G_2\sin\theta_c + G_5\cos\theta_c\Big]}_{=0}$$

$$-\frac{v}{\sqrt{2}}(\bar{d}_L s_R + \bar{s}_R d_L)\underbrace{\Big[G_3\cos\theta_c - G_6\sin\theta_c\Big]}_{=0}$$

$\Rightarrow$ don't want $d \leftrightarrow s$ $\Rightarrow$ $G_5 = -G_2\tan\theta_c$

$$G_6 = G_3\cot\theta_c$$

$$\Rightarrow m_d = \frac{v}{\sqrt{2}}\Big[G_2\cos\theta_c - G_5\sin\theta_c\Big] = \boxed{\frac{v}{\sqrt{2}}\,\frac{G_2}{\cos\theta_c} = m_d}$$

$$m_s = \frac{v}{\sqrt{2}}\Big[G_3\sin\theta_c + G_6\cos\theta_c\Big] = \boxed{\frac{v}{\sqrt{2}}\,\frac{G_3}{\sin\theta_c} = m_s}$$

$\Rightarrow$ instead of unknown m_u, m_d, m_s, m_c have constants G_1, G_2, G_3, G_4 also unknown...

CKM matrix (absolute values)

$$\begin{pmatrix} |V_{ud}| & |V_{us}| & |V_{ub}| \\ |V_{cd}| & |V_{cs}| & |V_{cb}| \\ |V_{td}| & |V_{ts}| & |V_{tb}| \end{pmatrix} \simeq \begin{pmatrix} 0.974 & 0.226 & 0.004 \\ 0.226 & 0.973 & 0.042 \\ 0.009 & 0.041 & 0.999 \end{pmatrix}$$

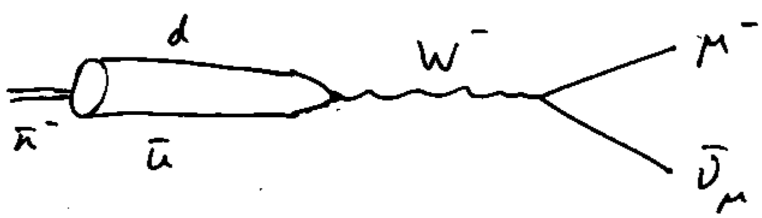
~ "almost" diagonal.

Why do we need d', s', b'? Look at $\mathcal{L}$:

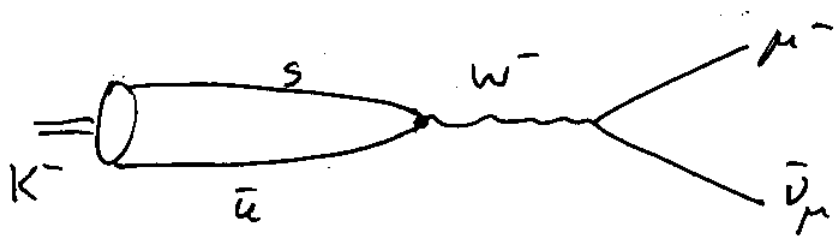
$$g(\bar{u}_L \ \bar{d}'_L) \, i \gamma^\mu \underbrace{\frac{\vec{\tau}}{2} \cdot \vec{W}_\mu}_{W_\mu} \begin{pmatrix} u_L \\ d'_L \end{pmatrix} \Rightarrow \text{has}$$

$$g \, \bar{u}_L \gamma \cdot W_\mu \, d'_L + g \, \bar{d}'_L \gamma \cdot W^+_\mu u_L$$

Experimentally one has the following decays:



$\pi^- \to \mu^- \bar{\nu}_\mu$



$K^- \to \mu^- \bar{\nu}_\mu$

$\Rightarrow$ if $d' = d$ $\Rightarrow$ then $K^- \to \mu^- \bar{\nu}_\mu$ process would have been prohibited $\Rightarrow$ but it exists

$\Rightarrow$ in 1963 Cabibbo postulated his mixing

$\Rightarrow$ as $d' = d\cos\theta_c + s\sin\theta_c$ $\Rightarrow$ get $s\bar{u}$ coupling!

$$\left.\begin{array}{l} \Rightarrow M_{\pi^- \to \mu^- \bar{\nu}_\mu} \propto \cos\theta_c \\ M_{K^- \to \mu^- \bar{\nu}_\mu} \propto \sin\theta_c \end{array}\right\} \Rightarrow \frac{\sigma_{\pi^- \to \mu^- \bar{\nu}_\mu}}{\sigma_{K^- \to \mu^- \bar{\nu}_\mu}} \approx \frac{\cos^2\theta_c}{\sin^2\theta_c} =$$

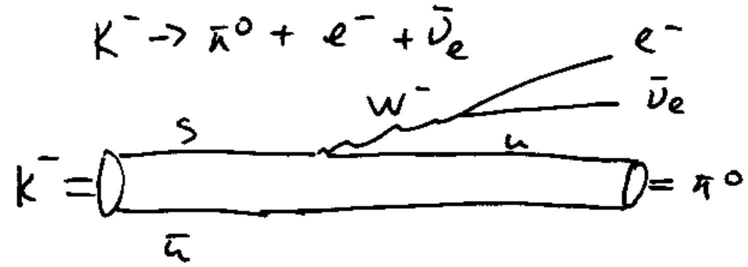
$= \cot^2\theta_c \simeq 18.8$ (experiment $\simeq 13.2$)

$\pi^- \to \mu^- \bar{\nu}_\mu$ is Cabibbo-favored

$K^- \to \mu^- \bar{\nu}_\mu$ is Cabibbo-surpressed.

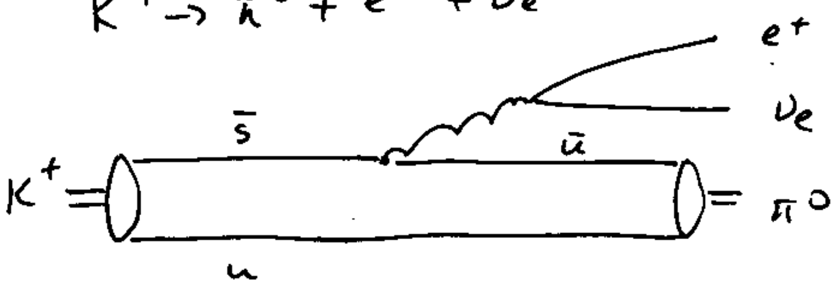
Other relevant processes:

$K^- \to \pi^0 + e^- + \bar{\nu}_e$



$\Rightarrow M_{K^- \to \pi^0 + e^- + \bar{\nu}_e} \propto \sin\theta_c \sim V_{us}$

$K^+ \to \pi^0 + e^+ + \nu_e$



$\Rightarrow M_{K^+ \to \pi^0 + e^+ + \nu_e} \propto \sin\theta_c \sim V_{us}$ too.

<u>Semi-leptonic decays.</u>