Last time:

EW Lagrangian
$e, \mu, \tau, \nu_e, \nu_\mu, \nu_\tau$
$W^\pm, Z, A_\mu$, Higgs

$\leftarrow$ cross talk? $\rightarrow$
$W^\pm, Z, A_\mu$
Higgs

QCD Lagrangian
quarks, q^{if}
gluons, A_μ^a

(I) Couple quarks to $W_\mu^\pm, Z_\mu, A_\mu$. Define doublets under weak isospin: $L_u = \begin{pmatrix} u \\ d' \end{pmatrix}_L$, $L_c = \begin{pmatrix} c \\ s' \end{pmatrix}_L$, $L_t = \begin{pmatrix} t \\ b' \end{pmatrix}_L$

& right-handed singlets: $u_R, d_R, s_R, c_R, b_R, t_R$

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \underbrace{\begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}}_{\text{CKM matrix}} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

$\Rightarrow$ the Lagrangian is:

$$\boxed{\begin{aligned}\mathcal{L}_{\text{quarks+gauge}} &= \bar{L}_u\, i\gamma^\mu\left(\partial_\mu - i\frac{g'}{6}B_\mu - ig\frac{\vec{\tau}}{2}\cdot\vec{W}_\mu\right)L_u + \bar{R}_u\, i\gamma^\mu\left(\partial_\mu - i\frac{2}{3}g' B_\mu\right)R_u \\ &+ \bar{R}_d\, i\gamma^\mu\left(\partial_\mu + i\frac{g'}{3}B_\mu\right)R_d + (\text{2 more generations})\end{aligned}}$$

(II) Coupling quarks to Higgs: terms like $\bar{L}_u \phi R_u$ won't work

$\Rightarrow$ Def. $\boxed{\tilde{\phi} \equiv i\tau^2\phi^*}$ and the most general $\mathcal{L}$ has terms like $\bar{L}_u \tilde{\phi} R_u$ and $\bar{L}_u \phi R_d$.

When the dust settled we got (for 2 generations)

$$\mathcal{L}^{2\,\mathrm{gen}}_{\mathrm{quarks+Higgs}} = -G_1\left[\bar{L}_u \tilde{\phi} R_u + \bar{R}_u \tilde{\phi}^+ L_u\right] - G_2\left[\bar{L}_u \phi R_d + \bar{R}_d \phi^+ L_u\right] - G_3\left[\bar{L}_u \phi R_s + \bar{R}_s \phi^+ L_u\right] - G_4\left[\bar{L}_c \tilde{\phi} R_c + \bar{R}_c \tilde{\phi}^+ L_c\right] - G_5\left[\bar{L}_c \phi R_d + \bar{R}_d \phi^+ L_c\right] - G_6\left[\bar{L}_c \phi R_s + \bar{R}_s \phi^+ L_c\right]$$

$$\langle 0|\phi|0\rangle = \begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix}, \qquad \langle 0|\tilde{\phi}|0\rangle = \begin{pmatrix} v/\sqrt{2} \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} d' \\ s' \end{pmatrix} = \begin{pmatrix} \cos\theta_c & \sin\theta_c \\ -\sin\theta_c & \cos\theta_c \end{pmatrix} \begin{pmatrix} d \\ s \end{pmatrix}$$ with $\theta_c \simeq 13^\circ$ Cabibbo angle

$$\Rightarrow m_u = \frac{G_1 v}{\sqrt{2}}, \quad m_d = \frac{G_2 v}{\sqrt{2}\cos\theta_c}, \quad m_c = \frac{G_4 v}{\sqrt{2}}, \quad m_s = \frac{G_3 v}{\sqrt{2}\cos\theta_c}$$

$$\Rightarrow \mathcal{L}^{d,s\ \text{part}}_{\text{quark-Higgs}} = -\frac{v}{\sqrt{2}}\Big\{ G_2\big[\bar{d}d\cos\theta_C + (\bar{s}_L d_R + \bar{d}_R s_L)\cdot$$

$$\sin\theta_C\big] + G_3\big[\bar{s}s\sin\theta_C + (\bar{d}_L s_R + \bar{s}_R d_L)\cos\theta_C\big] +$$

$$+ G_5\big[-\bar{d}d\sin\theta_C + (\bar{s}_L d_R + \bar{d}_R s_L)\cos\theta_C\big] +$$

$$+ G_6\big[\bar{s}s\cos\theta_C - (\bar{d}_L s_R + \bar{s}_R d_L)\sin\theta_C\big]\Big\} =$$

$$= -\bar{d}d\,\frac{v}{\sqrt{2}}\big[G_2\cos\theta_C - G_5\sin\theta_C\big] - \bar{s}s\,\frac{v}{\sqrt{2}}\big[G_3\sin\theta_C +$$

$$+ G_6\cos\theta_C\big] - \frac{v}{\sqrt{2}}(\bar{s}_L d_R + \bar{d}_R s_L)\underbrace{\big[G_2\sin\theta_C + G_5\cos\theta_C\big]}_{=0}$$

$$- \frac{v}{\sqrt{2}}(\bar{d}_L s_R + \bar{s}_R d_L)\underbrace{\big[G_3\cos\theta_C - G_6\sin\theta_C\big]}_{=0}$$

$\Rightarrow$ don't want $d \leftrightarrow s$ $\Rightarrow$ $G_5 = -G_2\tan\theta_C$

$G_6 = G_3\cot\theta_C$

$$\Rightarrow m_d = \frac{v}{\sqrt{2}}\big[G_2\cos\theta_C - G_5\sin\theta_C\big] = \boxed{\frac{v}{\sqrt{2}}\,\frac{G_2}{\cos\theta_C} = m_d}$$

$$m_s = \frac{v}{\sqrt{2}}\big[G_3\sin\theta_C + G_6\cos\theta_C\big] = \boxed{\frac{v}{\sqrt{2}}\,\frac{G_3}{\sin\theta_C} = m_s}$$

$\Rightarrow$ instead of unknown m_u, m_d, m_s, m_c have constants G_1, G_2, G_3, G_4 also unknown...

CKM matrix (absolute values)

$$\begin{pmatrix} |V_{ud}| & |V_{us}| & |V_{ub}| \\ |V_{cd}| & |V_{cs}| & |V_{cb}| \\ |V_{td}| & |V_{ts}| & |V_{tb}| \end{pmatrix} \simeq \begin{pmatrix} 0.974 & 0.226 & 0.004 \\ 0.226 & 0.973 & 0.042 \\ 0.009 & 0.041 & 0.999 \end{pmatrix}$$

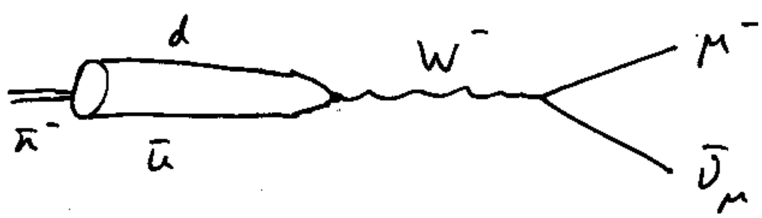
~ "almost" diagonal.

Why do we need d', s', b'? Look at $\mathcal{L}$:

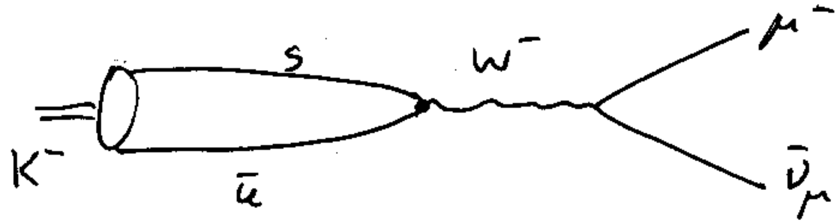
$$g(\bar u_L \ \bar d'_L)\, i\gamma^\mu \underbrace{\frac{\vec\tau}{2}\cdot\vec W_\mu}_{W_\mu} \begin{pmatrix} u_L \\ d'_L \end{pmatrix} \Rightarrow \text{has}$$

$$g\, \bar u_L \gamma\cdot W_\mu d'_L + g\, \bar d'_L \gamma\cdot W^+_\mu u_L$$

Experimentally one has the following decays:



$\pi^- \to \mu^- \bar\nu_\mu$



$K^- \to \mu^- \bar\nu_\mu$

$\Rightarrow$ if $d' = d$ $\Rightarrow$ then $K^- \to \mu^- \bar{\nu}_\mu$ process would have been prohibited $\Rightarrow$ but it exists

$\Rightarrow$ in 1963 Cabibbo postulated his mixing

$\Rightarrow$ as $d' = d\cos\theta_c + s\sin\theta_c$ $\Rightarrow$ get $s\bar{u}$ coupling!

$$\left.\begin{aligned} &\Rightarrow M_{\pi^- \to \mu^- \bar{\nu}_\mu} \propto \cos\theta_c \\ &M_{K^- \to \mu^- \bar{\nu}_\mu} \propto \sin\theta_c \end{aligned}\right\} \Rightarrow \frac{\sigma_{\pi^- \to \mu^- \bar{\nu}_\mu}}{\sigma_{K^- \to \mu^- \bar{\nu}_\mu}} \approx \frac{\cos^2\theta_c}{\sin^2\theta_c} =$$

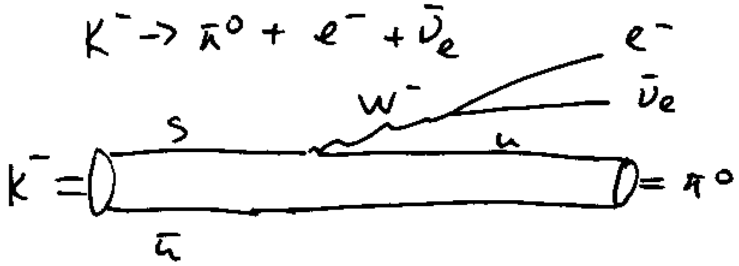
$= \cot^2\theta_c \simeq 18.8$ (experiment $\simeq 13.2$)

$\pi^- \to \mu^- \bar{\nu}_\mu$ is Cabibbo-favored

$K^- \to \mu^- \bar{\nu}_\mu$ is Cabibbo-surpressed.

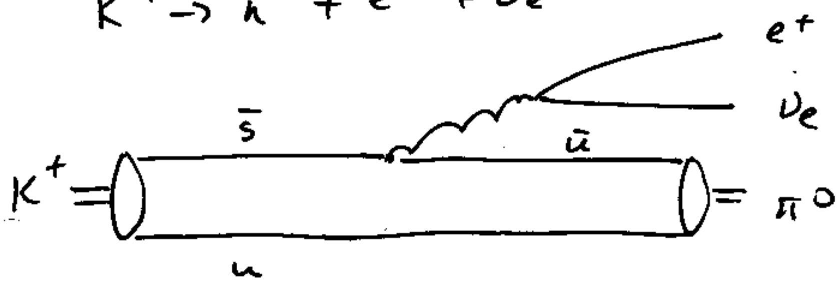
Other relevant processes:

$K^- \to \pi^0 + e^- + \bar{\nu}_e$



$\Rightarrow M_{K^- \to \pi^0 + e^- + \bar{\nu}_e} \propto \sin\theta_c \sim V_{us}$

$K^+ \to \pi^0 + e^+ + \nu_e$



$\Rightarrow M_{K^+ \to \pi^0 + e^+ + \nu_e} \propto \sin\theta_c \sim V_{us}$ too.

<u>Semi-leptonic decays.</u>

cf. $K^+ \to \mu^+ + \nu_\mu$ ~ leptonic decay
(all leptons in final state)

$K^+ \to \pi^0 \pi^-$ hadronic (non-leptonic) decay.

Interactions of W's and Z's with Quarks

$$\mathcal{L}_{quarks+W,Z} = \bar{L}_u i\gamma^\mu \left(\partial_\mu - i\frac{g'}{6}B_\mu - ig\frac{\vec{\tau}}{2}\cdot\vec{W}_\mu\right)L_u +$$

$$+ \bar{R}_u i\gamma^\mu\left(\partial_\mu - i\frac{2}{3}g'B_\mu\right)R_u + \bar{R}_d i\gamma^\mu\left(\partial_\mu + i\frac{1}{3}g'B_\mu\right)R_d + (c,t)$$

$$= \bar{L}_u i\gamma^\mu\left(\partial_\mu - i\frac{g'}{2}Y_{L_u}B_\mu - ig\frac{\vec{\tau}}{2}\cdot\vec{W}_\mu\right)L_u + \bar{R}_u i\gamma^\mu\left(\partial_\mu - i\frac{g'}{2}Y_{R_u}B_\mu\right)$$

$$\cdot R_u + \bar{R}_d\, i\gamma^\mu\left(\partial_\mu - i\frac{g'}{2}Y_{R_d}B_\mu\right)R_d + (c,t).$$

Define $\quad U_L = \begin{pmatrix} u \\ c \\ t \end{pmatrix}_L, \quad D_L = \begin{pmatrix} d \\ s \\ b \end{pmatrix}_L, \quad U_R, D_R$ ~ similarly

with $\psi = \begin{pmatrix} U_L \\ D_L \end{pmatrix}, \quad M = \begin{pmatrix} \mathbb{1} & 0 \\ 0 & V \end{pmatrix},$ V is a 3×3 CKM Matrix,

M is 6×6, $M^+M = \mathbb{1}$

$$\Rightarrow \boxed{\begin{aligned} \mathcal{L} = \bar{\psi} M^+ i\gamma^\mu\left(\partial_\mu - i\frac{g'}{2}\overset{=1/3}{Y}B_\mu - ig\frac{\vec{\tau}}{2}\cdot\vec{W}_\mu\right)M\psi \,+ \\ + \sum_f \bar{R}_f i\gamma^\mu\left(\partial_\mu - i\frac{g'}{2}Y_f B_\mu\right)R_f \end{aligned}}$$

(more compact notation)

Write $\begin{cases} B_\mu = A_\mu \cos\theta_w - Z_\mu \sin\theta_w \\ W_\mu^3 = A_\mu \sin\theta_w + Z_\mu \cos\theta_w \end{cases}$

$$\Rightarrow \boxed{\begin{aligned} \mathcal{L} &= \bar{\Psi}\, i\gamma^\mu \Big[\partial_\mu - i\frac{g'}{6}(A_\mu \cos\theta_w - Z_\mu \sin\theta_w) - i g \frac{\tau^3}{2} \cdot \\ &\cdot (A_\mu \sin\theta_w + Z_\mu \cos\theta_w) - i\frac{g}{\sqrt{2}} m^+ (\tau^+ W_\mu + \tau^- W_\mu^+) m\Big]\Psi \\ &+ \sum_f \bar{R}_f\, i\gamma^\mu \Big(\partial_\mu - i\frac{g'}{2} Y_f (A_\mu \cos\theta_w - Z_\mu \sin\theta_w)\Big) R_f . \end{aligned}}$$

We have used: $m^+ \tau^3 m = \begin{pmatrix} 1 & 0 \\ 0 & V^+ \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 0 & V \end{pmatrix} =$

$$= \begin{pmatrix} 1 & 0 \\ 0 & V^+ \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 0 & -V \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & -V^+V \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} = \tau^3 .$$

<u>We defined</u>: $\tau^\pm = \dfrac{\tau_1 \pm i\tau_2}{2} \Rightarrow \tau^+ = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}, \tau^- = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}$

$$\Rightarrow \frac{1}{2}\left(\tau^1 W_\mu^1 + \tau^2 W_\mu^2\right) = \left| \begin{aligned} W_\mu^1 &= \frac{W_\mu + W_\mu^+}{\sqrt{2}} \\ W_\mu^2 &= (-i)\frac{W_\mu^+ - W_\mu}{\sqrt{2}} \end{aligned} \right. =$$

$$= \frac{1}{2}\frac{1}{\sqrt{2}}\left[\tau^1 (W_\mu + W_\mu^+) - i\tau^2 (W_\mu^+ - W_\mu)\right] = \frac{1}{\sqrt{2}}\Big[W_\mu \frac{\tau^1 + i\tau^2}{2} +$$

$$+ W_\mu^+ \frac{\tau^1 - i\tau^2}{2}\Big] = \frac{1}{\sqrt{2}}\left(\tau^+ W_\mu + \tau^- W_\mu^+\right) \text{ as desired.}$$

(i) Charged current (coupling of $W^{\pm}$ bosons)

$$m^{+}\tau^{+}m = \begin{pmatrix} 1 & 0 \\ 0 & V^{+} \end{pmatrix}\begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 0 & V \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & V^{+} \end{pmatrix}\begin{pmatrix} 0 & V \\ 0 & 0 \end{pmatrix} =$$

$$= \begin{pmatrix} 0 & V \\ 0 & 0 \end{pmatrix}; \quad m^{+}\tau^{-}m = \begin{pmatrix} 1 & 0 \\ 0 & V^{+} \end{pmatrix}\begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 0 & V \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & V^{+} \end{pmatrix}\begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}$$

$$= \begin{pmatrix} 0 & 0 \\ V^{+} & 0 \end{pmatrix}$$ => the charged current part of the Lagrangian is:

$$\mathcal{L}_{c.c.} = \frac{g}{\sqrt{2}}\,\bar{\Psi}\,\gamma^{\mu}\left[\begin{pmatrix} 0 & V \\ 0 & 0 \end{pmatrix} W_{\mu} + \begin{pmatrix} 0 & 0 \\ V^{+} & 0 \end{pmatrix} W_{\mu}^{+}\right]\Psi =$$

$$= \frac{g}{\sqrt{2}}\,(\bar{U}_L \;\; \bar{D}_L)\,\gamma^{\mu}\begin{pmatrix} 0 & V W_{\mu} \\ V^{+}W_{\mu}^{+} & 0 \end{pmatrix}\begin{pmatrix} U_L \\ D_L \end{pmatrix} = \frac{g}{\sqrt{2}}\Big[\bar{U}_L\gamma^{\mu} V W_{\mu} D_L$$

$$+ \bar{D}_L\gamma^{\mu}V^{+}W_{\mu}^{+}U_L\Big] = \frac{g}{2\sqrt{2}}\Big\{\bar{u}\,\gamma\cdot W\,(1-\gamma_5)\left[V_{ud}\cdot d + V_{us}\,s + V_{ub}\,b\right] + \bar{c}\,\gamma\cdot W(1-\gamma_5)\left[V_{cd}\,d + V_{cs}\,s + V_{cb}\,b\right] + \bar{t}\,\gamma\cdot W\,(1-\gamma_5)\left[V_{td}\,d + V_{ts}\,s + V_{tb}\,b\right] + h.c.\Big\} = \mathcal{L}_{cc}$$

=> we dropped L subscripts => got $(1-\gamma_5)$'s

=> spelled out VD_L => got CKM matrix elements.

(ii) Neutral current (coupling of Z bosons, photons)

a) Photons => A_μ terms

$$\mathcal{L}^{\text{photons}} = \frac{g'}{6}\cos\theta_W\ \bar{\Psi}\,\gamma\cdot A\,\Psi + \frac{g}{2}\sin\theta_W\ \bar{\Psi}\,\gamma\cdot A\,\tau^3\Psi$$

$$+ \sum_f \frac{g'}{2} Y_f \cos\theta_W\ \bar{R}_f\,\gamma\cdot A\,R_f$$

$\Rightarrow$ remember $e = g'\cos\theta_W = g\sin\theta_W$

$$\mathcal{L}^{\text{photons}} = \bar{\Psi}\,\gamma\cdot A\left(\frac{e}{6} + \frac{e}{2}\tau^3\right)\Psi + \sum_f \frac{e}{2} Y_f\ \bar{R}_f\,\gamma\cdot A\,R_f$$

$$\Rightarrow e\left(\frac{1}{6} + \frac{\tau^3}{2}\right) = e\left(\frac{Y}{2} + \frac{\tau^3}{2}\right) = e\left(\frac{Y}{2} + I_3\right) = Q_{LHQ} = e_f$$

$\Rightarrow$ Gell-Mann-Nishijima formula

(check: $\frac{Y}{2} + \frac{\tau^3}{2} = \frac{1}{6} + \frac{1}{2}\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix} = \begin{pmatrix}\frac{2}{3} & 0\\ 0 & -\frac{1}{3}\end{pmatrix}$ ← charge of u, c, t; ← charge of d, s, b)

$$\frac{e}{2}\cdot Y_f = e\cdot\frac{1}{2}\cdot\begin{cases} 4/3 & \text{for } u,c,t \\ -2/3 & \text{for } d,s,b\end{cases} = e\cdot\begin{cases} 2/3 & \text{for } u,c,t \\ -1/3 & \text{for } d,s,b\end{cases}$$

$\Rightarrow$ get $Q_{RHQ} = e_f$ again

$$\boxed{e_f = e\cdot\begin{cases} 2/3 & \text{for } u,c,t \\ -1/3 & \text{for } d,s,b\end{cases}}$$

$$\Rightarrow \boxed{\mathcal{L}^{\text{photons}} = \sum_f e_f\ \bar{q}_f\,\gamma\cdot A\,q_f}$$

Regular QED term as expected!

b) Z-bosons: $\mathcal{L}^Z = \bar{\Psi}\gamma^\mu\left[-\frac{g'}{6}\sin\theta_W + g\frac{\tau^3}{2}\cdot\right.$

$\left.\cdot\cos\theta_W\right] Z_\mu \Psi - \sum_f \bar{R}_f \gamma^\mu Z_\mu \frac{g'}{2} Y_f \sin\theta_W R_f = \left| \begin{array}{l} g'\sin\theta_W = \\ = g\frac{\sin^2\theta_W}{\cos\theta_W}\end{array}\right.$

$$= (\bar{U}_L \ \bar{D}_L)\,\gamma\cdot Z \begin{pmatrix} \frac{g}{2}\cos\theta_W - \frac{g}{6}\frac{\sin^2\theta_W}{\cos\theta_W} & 0 \\ 0 & -\frac{g}{2}\cos\theta_W - \frac{g}{6}\frac{\sin^2\theta_W}{\cos\theta_W}\end{pmatrix}\begin{pmatrix} U_L \\ D_L\end{pmatrix}$$

$$- \frac{g\sin^2\theta_W}{2\cos\theta_W}\cdot\sum_f \bar{R}_f\gamma\cdot Z\, Y_f R_f = \frac{g}{2\cos\theta_W}\Big[\bar{u}_L\gamma\cdot Z\, u_L\cdot$$

$$\cdot\left(\cos^2\theta_W - \frac{1}{3}\sin^2\theta_W\right) - \bar{D}_L\gamma\cdot Z\, D_L\left(\cos^2\theta_W + \frac{1}{3}\sin^2\theta_W\right) -$$

$$- \frac{g\sin^2\theta_W}{2\cos\theta_W}\left(\bar{u}_R\gamma\cdot Z\, u_R\cdot\frac{4}{3} + \bar{D}_R\gamma\cdot Z\, D_R\left(-\frac{2}{3}\right)\right)\Big]$$

$$\Rightarrow \mathcal{L}^Z = \frac{g}{4\cos\theta_W}\Big\{\bar{u}\,\gamma\cdot Z\left[(1-\gamma_5)\left(1-\frac{4}{3}\sin^2\theta_W\right) - (1+\gamma_5)\frac{4}{3}\sin^2\theta_W\right]\cdot$$

$$\cdot u - \bar{D}\,\gamma\cdot Z\left[(1-\gamma_5)\left(1-\frac{2}{3}\sin^2\theta_W\right) - (1+\gamma_5)\frac{2}{3}\sin^2\theta_W\right] D$$

Putting photons & Z-bosons together get

$$\mathcal{L}_{n.c.} = \frac{g}{4\cos\theta_W}\Big\{\bar{u}\,\gamma\cdot Z\left[(1-\gamma_5)\left(1-\frac{4}{3}\sin^2\theta_W\right) - (1+\gamma_5)\frac{4}{3}\sin^2\theta_W\right] u -$$

$$- \bar{D}\,\gamma\cdot Z\left[(1-\gamma_5)\left(1-\frac{2}{3}\sin^2\theta_W\right) - (1+\gamma_5)\frac{2}{3}\sin^2\theta_W\right] D + \sum_f e_f\,\bar{q}_f\,\gamma\cdot A\, q_f$$

(neutral current $\mathcal{L}$)