

Brief Review of Part I.

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~ I started by reviewing quantization of free scalar, Dirac & gauge fields (not going to repeat now)

Quark Model and Group Theory

~ defined isospin operators $\vec{I} \Rightarrow \vec{I}^2, I_z$ have eigenvalues $I(I+1)$ and $-I, \dots, I$ correspondingly (p, n or $\bar{n}^+, \bar{n}^0, \bar{n}^-$, etc.)

~ defined baryon # : # of Baryons (B)

~ strangeness: K^+, K^0, \bar{K}^0, K^-
 $\underbrace{\hspace{1.5cm}}_{S=+1} \quad \underbrace{\hspace{1.5cm}}_{S=-1}$

$$Q = I_3 + \frac{Y}{2}$$

~ hyper charge:

$$Y \equiv B + S$$

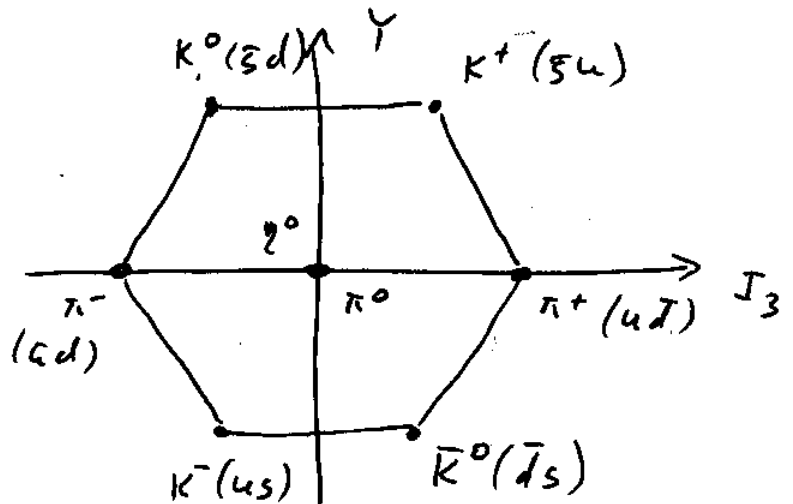
electric charge Gell-Mann-Nishijima

"Eight fold Way":

$$\bar{n}^0 = \frac{\bar{u}u - \bar{d}d}{\sqrt{2}}$$

$$\eta^0 = \frac{1}{\sqrt{6}} (\bar{u}u + \bar{d}d - 2\bar{s}s)$$

0^- mesons (pseudoscalar mesons)



Gell-Mann & Ne'eman ('61)

Quarks: u, d, s, c, b, t

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have $Q = +\frac{2}{3}$ or $-\frac{1}{3}$, $B = +\frac{1}{3}$, u, d have $I = \frac{1}{2}$, $I_3 = \pm\frac{1}{2}$

s -quark has $S = -1$.

\sim another quantum #: color $i=1, 2, 3$

$\Rightarrow u_i(x) \sim 3$ colors of up quark.

mesons, baryons \sim always color-neutral!

\Rightarrow quarks interact with each other by exchanging gluons \sim spin-1 non-abelian

gauge fields: A_μ^a , $a=1, \dots, 8$ \sim gluon color

quark fields: \bar{q}^{if} $\left\{ \begin{array}{l} \leftarrow \text{color} \\ \leftarrow \text{flavor} \end{array} \right.$

$$\mathcal{L}_{QCD} = \bar{q}^{if} (i\gamma \cdot \partial - m_f) q^{if} - \frac{1}{4} F_{\mu\nu}^a F^{a\mu\nu} + g \bar{q}^{if} \gamma^\mu A_\mu^a (T^a)_{ij} q^{if}$$

with T^a the generators of group $SU(3)$ in the fundamental representation: $T^a = \frac{\lambda^a}{2}$

$\lambda^a \sim$ Gell-Mann matrices

$$[T^a, T^b] = i f^{abc} T^c, \quad f^{abc} \sim SU(3) \text{ structure constants.}$$

gluon field strength:

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$$F_{\mu\nu}^a = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a + g f^{abc} A_\mu^b A_\nu^c$$

defining $A_\mu = \sum_{a=1}^8 A_\mu^a T^a$ & defining the

covariant derivative $D_\mu = \partial_\mu - ig A_\mu$ get

$$\mathcal{L}_{QCD} = \bar{q}^f (i \gamma \cdot D - m_f) q^f - \frac{1}{4} F_{\mu\nu}^a F^{a\mu\nu}$$

~ We studied group theory. In particular for SU(3) we showed that the following is true:

$$3 \otimes \bar{3} = 1 \oplus 8$$

$$3 \otimes 3 \otimes 3 = 1 \oplus 8 \oplus 8 \oplus 10$$

\Rightarrow for mesons made out of u, d, s quarks due to $3 \otimes \bar{3} = 1 \oplus 8$ get a flavor-octet.

$\eta' \sim$ singlet.

Baryons: $3 \otimes 3 \otimes 3 = 1 \oplus 8 \oplus 8 \oplus 10 \Rightarrow$ have an octet and a decuplet ~ all agrees.

~ also works for colors.

Quark-only Lagrangian: $N_f = 3$

$$\mathcal{L} = \bar{q} (i\gamma \cdot \partial - m) q, \quad q = \begin{pmatrix} u \\ d \\ s \end{pmatrix}, \quad m = \begin{pmatrix} m_u & 0 & 0 \\ 0 & m_d & 0 \\ 0 & 0 & m_s \end{pmatrix}$$

$$\Rightarrow \text{defined } q_L = \frac{1 - \gamma_5}{2} q, \quad q_R = \frac{1 + \gamma_5}{2} q$$

$$\Rightarrow \mathcal{L}_{m=0} = \bar{q}_L i\gamma \cdot \partial q_L + \bar{q}_R i\gamma \cdot \partial q_R$$

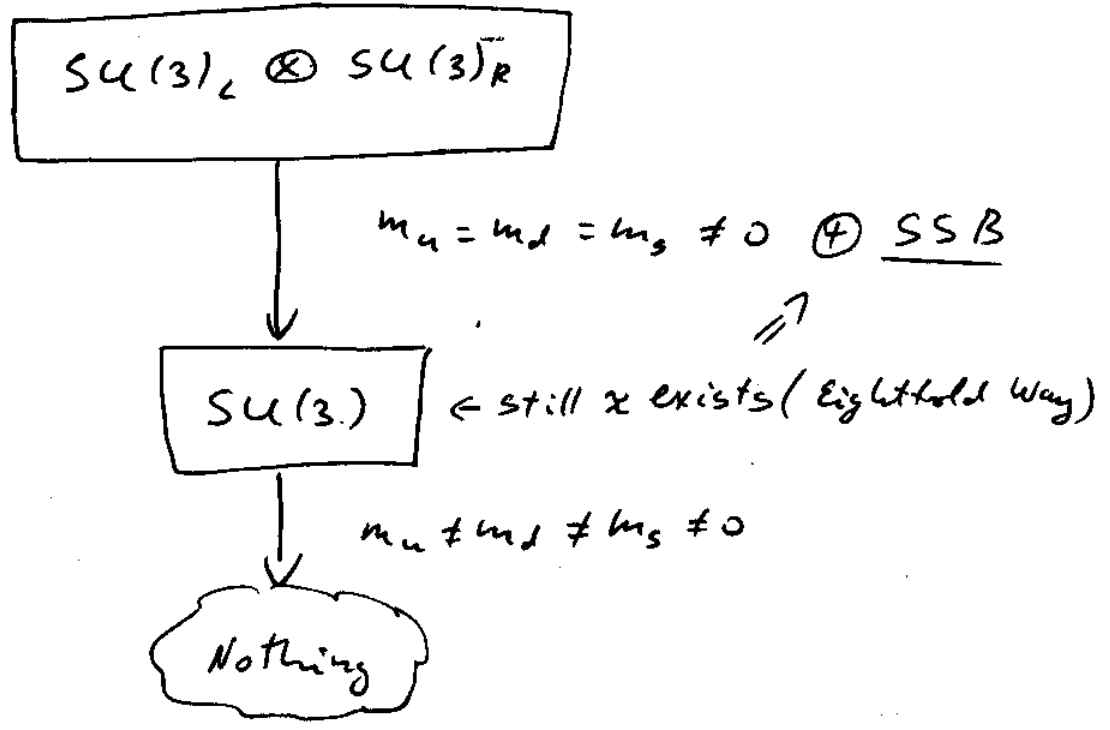
\Rightarrow $SU(3)_L \otimes SU(3)_R$ invariant! $q_L \rightarrow e^{i\vec{L} \cdot \vec{T}} q_L$
chiral symmetry

add mass but with $m_u = m_d = m_s \Rightarrow$

$$\text{get } \mathcal{L} = \bar{q}_L i\gamma \cdot \partial q_L + \bar{q}_R i\gamma \cdot \partial q_R - m [\bar{q}_L q_R + \bar{q}_R q_L]$$

$\Rightarrow SU(3)_L \otimes SU(3)_R$ is broken down to $SU(3)$.

if $m_u \neq m_d \neq m_s$, $SU(3)$ is also broken:



Spontaneous Symmetry Breaking (SSB) (5)

~ symmetry manifest in \mathcal{L} , but not respected by ground state.

~ Nambu - Goldstone theorem: spontaneous breakdown of a continuous symm. \Rightarrow massless spinless particles (Nambu-Goldstone bosons).

Example: Abelian σ -Model

$$\mathcal{L} = \frac{1}{2} \partial_\mu \sigma \partial^\mu \sigma + \frac{1}{2} \partial_\mu \pi \partial^\mu \pi + \frac{\mu^2}{2} (\sigma^2 + \pi^2) - \frac{\lambda}{4} (\sigma^2 + \pi^2)^2$$

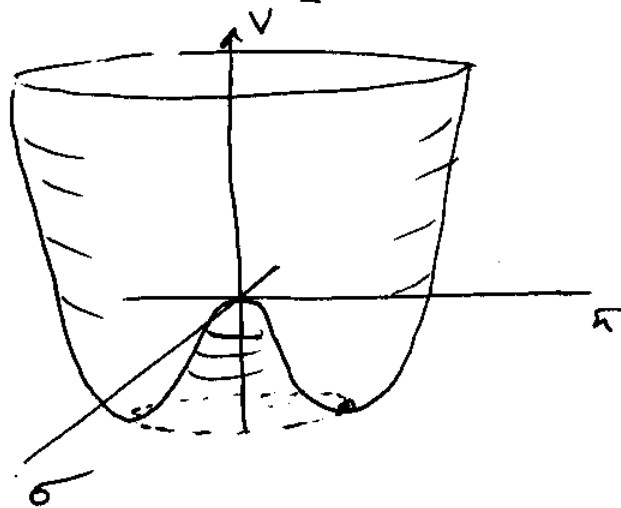
at the minimum

$$\sigma^2 + \pi^2 = v^2 = \frac{\mu^2}{\lambda}$$

\Downarrow
pick vacuum at

$$\langle 0 | \sigma | 0 \rangle = v = \frac{\mu}{\sqrt{\lambda}}$$

$$\langle 0 | \pi | 0 \rangle = 0$$



\Rightarrow expand near the vacuum: $\sigma = v + \sigma'$, π :

$$\mathcal{L} = \frac{1}{2} \partial_\mu \sigma' \partial^\mu \sigma' + \frac{1}{2} \partial_\mu \pi \partial^\mu \pi - \mu^2 \sigma'^2 - \lambda v \sigma' (\sigma'^2 + \pi^2) - \frac{\lambda}{4} (\sigma'^2 + \pi^2)^2$$

$\Rightarrow \pi$ is massless: Goldstone boson

σ' has mass $\sqrt{2}\mu$.

$U(1)$ symmetry broken spontaneously

Example: non-abelian σ -model:

$$\vec{n} \rightarrow \vec{\pi} = (\pi^1, \pi^2, \pi^3) \sim \text{pion field.}$$

$$f^N = \begin{pmatrix} P \\ n \end{pmatrix} \sim \text{fermions.}$$

$SU(2)_L \otimes SU(2)_R$ symmetric

after SSB get: σ' has mass $\mu\sqrt{2}$

f^N have mass $g\sigma$

$\vec{\pi}$ have mass 0 (Goldstone bosons)

$SU(2)_L \otimes SU(2)_R$ spont. broken to $SU(2)$.

In QCD: pions (π^+, π^-, π^0) are Goldstone bosons of chiral SSB, $m_\pi = 0$ (as $m_u \neq m_d \neq 0$ not exact)

$$v = \langle 0 | \bar{\psi} \psi | 0 \rangle = -(230 \text{ MeV})^3.$$

The Electroweak Theory.

local vs global gauge symmetries:

$$\mathcal{L}_{QED} = \bar{\psi} [i\gamma^\mu D_\mu - m] \psi - \frac{1}{4} F_{\mu\nu} F^{\mu\nu}, \quad \begin{matrix} F_{\mu\nu} = \partial_\mu A_\nu - \\ - \partial_\nu A_\mu \end{matrix}$$

$$D_\mu = \partial_\mu - ig A_\mu \Rightarrow \begin{cases} \psi \rightarrow e^{i\alpha(x)} \psi \\ A_\mu \rightarrow A_\mu + \frac{1}{g} \partial_\mu \alpha \end{cases} \quad \begin{matrix} \text{local } U(1) \\ \text{Symmetry} \\ \text{(Abelian)} \end{matrix}$$

non-Abelian:

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$$\mathcal{L} = \bar{\psi} [i \gamma^\mu D_\mu - m] \psi - \frac{1}{4} F_{\mu\nu}^a F^{a\mu\nu}$$

$$D_\mu = \partial_\mu - ig A_\mu, \quad A_\mu = \sum_a T^a A_\mu^a, \quad F_{\mu\nu} = \frac{i}{g} [D_\mu, D_\nu] = \partial_\mu A_\nu - \partial_\nu A_\mu - ig [A_\mu, A_\nu].$$

$$\begin{cases} \psi \rightarrow S(x) \psi \\ A_\mu \rightarrow S(x) A_\mu S^{-1}(x) - \frac{i}{g} (\partial_\mu S) S^{-1} \end{cases}$$

$S(x)$ ~ unitary $N \times N$ matrix \Rightarrow $SU(N)$ local symm.

Higgs mechanism: $\mathcal{L} = (D_\mu \varphi)^* (D^\mu \varphi) - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \mu^2 \varphi^* \varphi - \lambda (\varphi^* \varphi)^2$

$U(1)$ model

$\Rightarrow U(1)$ gauge symm. \Rightarrow pick a VEV: $\langle 0 | \varphi | 0 \rangle = \frac{v}{\sqrt{2}} = \frac{\mu}{\sqrt{2\lambda}}$

\Rightarrow write $\varphi = \frac{\rho'(x)}{\sqrt{2}} e^{i\theta(x)}$, $B_\mu(x) = A_\mu - \frac{1}{g} \partial_\mu \theta(x)$

& expand $\rho' = v + \rho$ around the VEV. One gets:

$$\mathcal{L} = \frac{1}{2} \partial_\mu \rho \partial^\mu \rho - \mu^2 \rho^2 - \frac{1}{4} G_{\mu\nu} G^{\mu\nu} + \frac{1}{2} g^2 v^2 B_\mu B^\mu + \frac{1}{2} g^2 B_\mu B^\mu (2v\rho + \rho^2) - \lambda v \rho^3 - \frac{\lambda}{4} \rho^4$$

ρ has mass $\sqrt{2}\mu$

B_μ ~ massive gauge field $m_B = g v$

(no Goldstone bosons) θ - would-be Goldstone b.

SU(2) ⊗ U(1) Electroweak theory:

Leptons

$$Y=-1 \quad L_e = \begin{pmatrix} \nu_e \\ e \end{pmatrix}_L, \quad L_\mu = \begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix}_L, \quad L_\tau = \begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix}_L, \quad \boxed{Q = I_3 + \frac{Y}{2}}$$

$Y=-2 \quad R_e = e_R, \quad R_\mu = \mu_R, \quad R_\tau = \tau_R.$

↑ weak isospin ↑ weak hypercharge

gauge field: $\vec{W}_\mu = (W_\mu^1, W_\mu^2, W_\mu^3), \quad \vec{F}_{\mu\nu}$
 B_μ with $f_{\mu\nu}$

Higgs field: $\phi = \begin{pmatrix} \phi^{(+)} \\ \phi^{(0)} \end{pmatrix}, \quad Y = +1, \quad \phi^\dagger = (\phi^{(-)}, \phi^{(0)\dagger})$

$$\begin{aligned} \mathcal{L}_{\text{leptons + gauge + Higgs}} = & \bar{R}_e i \gamma^\mu (\partial_\mu + i g' B_\mu) R_e + \bar{L}_e i \gamma^\mu (\partial_\mu + i \frac{g'}{2} B_\mu - \\ & - i g \frac{\vec{\sigma}}{2} \cdot \vec{W}_\mu) L_e + (\mu, \tau) - \frac{1}{4} f_{\mu\nu} f^{\mu\nu} - \frac{1}{4} \vec{F}_{\mu\nu} \cdot \vec{F}^{\mu\nu} + \\ & + \left[(\partial_\mu - i \frac{g'}{2} B_\mu - i g \frac{\vec{\sigma}}{2} \cdot \vec{W}_\mu) \phi \right]^\dagger \left[(\partial_\mu - i \frac{g'}{2} B_\mu - i g \frac{\vec{\sigma}}{2} \cdot \vec{W}_\mu) \phi \right] \\ & + m^2 \phi^\dagger \phi - \lambda (\phi^\dagger \phi)^2 - G_e [\bar{L}_e \phi R_e + \text{c.c.}] - (\mu, \tau) \end{aligned}$$

$Y = +1, +1, -2 = 0$

VEV $\langle 0 | \phi | 0 \rangle = \begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix}, \quad \phi(x) = e^{-i \frac{\vec{\tau}}{2} \cdot \vec{\theta}(x)} \begin{pmatrix} 0 \\ \frac{v + \eta(x)}{\sqrt{2}} \end{pmatrix}$

Def.

$$\begin{cases} W_\mu = \frac{1}{\sqrt{2}} (W_\mu^1 - i W_\mu^2) \\ W_\mu^\dagger = \frac{1}{\sqrt{2}} (W_\mu^1 + i W_\mu^2) \\ Z_\mu = -B_\mu \sin \theta_w + W_\mu^3 \cos \theta_w \\ A_\mu = B_\mu \cos \theta_w + W_\mu^3 \sin \theta_w \end{cases}$$

$\tan \theta_w = g'/g$
Weinberg angle

get $\mathcal{L} = \bar{e} i \gamma \cdot \partial e + \bar{\nu}_e i \gamma \cdot \partial \nu_e - \frac{g_e}{\sqrt{2}} (v + \eta) \bar{e} e -$
 $-\frac{1}{4} f_{\mu\nu} f^{\mu\nu} - \frac{1}{4} \vec{F}_{\mu\nu} \cdot \vec{F}^{\mu\nu} + \frac{1}{2} \partial_\mu \eta \partial^\mu \eta - \mu^2 \eta^2 - \lambda v \eta^3 - \frac{\lambda}{4} \eta^4$
 $+ \frac{g^2}{4} (v + \eta)^2 W_\mu^\dagger W^\mu + \frac{g^2}{8 \cos^2 \theta_w} (v + \eta)^2 Z_\mu Z^\mu +$
 $+ \frac{g}{2 \cos \theta_w} [2 \sin^2 \theta_w \bar{e}_R \gamma \cdot Z e_R + (2 \sin^2 \theta_w - 1) \bar{e}_L \gamma \cdot Z e_L]$
 $- e \bar{e} \gamma \cdot A e + \frac{g}{2 \cos \theta_w} \bar{\nu}_e \gamma \cdot Z \nu_e - \frac{g}{\sqrt{2}} [\bar{\nu}_e \gamma \cdot W e_L + c.c.] +$
 $+ (M, \epsilon)$

$M_W = \frac{g v}{2} \approx 80.4 \text{ GeV}$
 $M_Z = \frac{g v}{2 \cos \theta_w} \approx 91.2 \text{ GeV}$
 $m_\gamma = 0$
 $m_e = \frac{G_e v}{\sqrt{2}}, \quad m_\mu = \frac{G_\mu v}{\sqrt{2}}, \quad m_\tau = \frac{G_\tau v}{\sqrt{2}}$
 $m_{\nu_e} = m_{\nu_\mu} = m_{\nu_\tau} = 0 \quad (\gg .04 \text{ eV in reality})$

$\theta_w \approx 30^\circ, \quad \frac{g^2}{4\pi} \approx \frac{1}{30} \sim \text{small}$

$M_H = \mu \sqrt{2} = v \sqrt{2\lambda}, \quad v \approx 289 \text{ GeV}$

$M_H = 129 \begin{matrix} +74 \\ -49 \end{matrix} \text{ GeV}$

Quarks in EW theory:

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$$\mathcal{L}_{\text{quarks} + \text{gauge}} = \bar{L}_u i \gamma^\mu (\partial_\mu - i \frac{g'}{6} B_\mu - i g \frac{\tau^a}{2} W_\mu^a) L_u$$

$$+ \bar{R}_u i \gamma^\mu (\partial_\mu - i \frac{2}{3} g' B_\mu) R_u + \bar{R}_d i \gamma^\mu (\partial_\mu + i \frac{g'}{3} B_\mu) R_d$$

+ 2 more generations.

$$L_u = \begin{pmatrix} u \\ d' \end{pmatrix}_L, \quad L_c = \begin{pmatrix} c \\ s' \end{pmatrix}_L, \quad L_t = \begin{pmatrix} t \\ b' \end{pmatrix}_L$$

$$R_u = u_R \quad R_c = c_R \quad R_t = t_R$$

$$R_d = d_R \quad R_s = s_R \quad R_b = b_R$$

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \underbrace{\begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}}_{\text{CKM matrix}} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

↑ weak eigenstates

CKM matrix (Cabibbo-Kobayashi-Maskawa, unitary)

↖ mass eigenstates

quarks-Higgs couplings: $\tilde{\phi} = i \tau^2 \phi^*$, $Y = -1$

⇒ write $\mathcal{L} = -G_1 [\bar{L}_u \tilde{\phi} R_u + \bar{R}_u \tilde{\phi}^\dagger L_u] -$

$$- G_2 [\bar{L}_u \phi R_d + \bar{R}_d \phi^\dagger L_u] + \dots$$

(all terms with $Y=0$ ($U(1)$ inv.) & $SU(2)$ inv.)