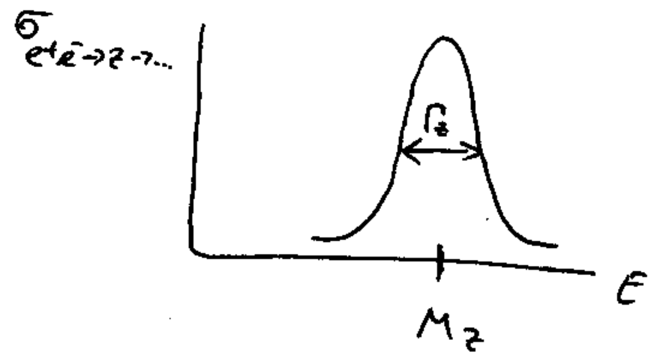
$\Rightarrow$ get $\boxed{N_\nu = 2.984 \pm 0.008}$

(look at $e^+e^- \to Z \to \ldots$ at LEP, SLAC ...)

$\Rightarrow$ there are 3 neutrino generations!



Neutrino Masses and Oscillations.

$\Rightarrow$ imagine that neutrinos have a mass (we know they do) & lepton # is not conserved ($\nu_e \to \nu_\mu, \ldots$)

$\Rightarrow$ again mass eigenstate $\neq$ weak eigenstates
(ν_1, ν_2, ν_3) $(\nu_e, \nu_\mu, \nu_\tau)$

just like quarks

$\Rightarrow$ consider 2 generations for simplicity:

mixing angle

$$\begin{pmatrix} \nu_\mu \\ \nu_e \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \end{pmatrix}$$

Maki et al '62
Pontecorvo '58, '6?
Gribov & Pontecorvo '6?

$\Rightarrow$ in EW process one produces ν_e or ν_μ.

propagation: $\nu_1(t) = \nu_1(0)\, e^{-iE_1 t}$, $\nu_2(t) = \nu_2(0)\, e^{-iE_2 t}$

$\Rightarrow$ write (at $t=0$) that we have $\nu_\mu(0)=1$, $\nu_e(0)=0$

~ a pure μ-neutrino:

$$\Rightarrow \begin{cases} 1 = \nu_1(0)\cos\theta + \nu_2(0)\sin\theta \\ 0 = -\nu_1(0)\sin\theta + \nu_2(0)\cos\theta \end{cases} \Rightarrow \begin{array}{l} \nu_1(0) = \cos\theta \\ \nu_2(0) = \sin\theta \end{array}$$

$$\Rightarrow \nu_\mu(t) = \underbrace{e^{-iE_1 t}\nu_1(0)}_{\nu_1(t)}\cos\theta + \underbrace{e^{-iE_2 t}\nu_2(0)}_{\nu_2(t)}\sin\theta =$$

$$= e^{-iE_1 t}\cos^2\theta + e^{-iE_2 t}\sin^2\theta$$

$$\Rightarrow |\nu_\mu(t)|^2 = \left| e^{-iE_1 t}\cos^2\theta + e^{-iE_2 t}\sin^2\theta \right|^2 =$$

$$= \cos^4\theta + \sin^4\theta + \cos^2\theta\cdot\sin^2\theta\cdot 2\cos\left((E_1 - E_2)t\right)$$

$$= 1 + 2\sin^2\theta\cos^2\theta\left[\cos\left((E_1-E_2)t\right) - 1\right] =$$

$$= 1 - \sin^2(2\theta)\cdot\sin^2\left(\frac{E_1-E_2}{2}t\right).$$

For small masses: $E = \sqrt{m^2+p^2} \simeq p + \dfrac{m^2}{2p}$

$$\Rightarrow E_1 - E_2 \simeq \frac{m_1^2}{2E_1} - \frac{m_2^2}{2E_2} \simeq \frac{m_1^2 - m_2^2}{2E}$$ as $E_1 \simeq E_2$ at this order

$$\Rightarrow \boxed{P(\nu_\mu \to \nu_\mu) = 1 - \sin^2(2\theta)\cdot\sin^2\left(\frac{1{,}27\,\Delta m^2 L}{E}\right)}$$

with $\Delta m^2 \equiv m_2^2 - m_1^2$, $L = t$ (meters), E is in MeV.
$(eV)^2/c^4$

$$\boxed{P_{\nu_\mu \to \nu_e} = \sin^2(2\theta) \cdot \sin^2\left(\frac{1.27\,\Delta m^2 L}{E}\right)}$$

$\Rightarrow$ ν_μ can turn into ν_e & vice versa $\Rightarrow$

$\Rightarrow$ neutrino oscillations!

$|\nu_\mu(0)\rangle = \cos\theta\,|\nu_1(0)\rangle + \sin\theta\,|\nu_2(0)\rangle$

~ initially produced a true ν_μ state (EW state)

$|\nu_e(0)\rangle = -\sin\theta\,|\nu_1(0)\rangle + \cos\theta\,|\nu_2(0)\rangle$, $|\psi_i\rangle = |\nu_\mu(0)\rangle + \phi \cdot |\nu_e(0)\rangle$

~ this is a true ν_e state $\perp$ ν_μ state (not there initially)

$$|\nu_\mu(t)\rangle = e^{-iE_1 t}\cos\theta\,|\nu_1(0)\rangle + e^{-iE_2 t}\sin\theta\,|\nu_2(0)\rangle$$

$P(\nu_\mu \to \nu_\mu) =$

$$= \left|\langle \nu_\mu(0)|\nu_\mu(t)\rangle\right|^2 = \left|\cos^2\theta\, e^{-iE_1 t} + \sin^2\theta\, e^{-iE_2 t}\right|^2$$

$\Rightarrow$ the rest is like above

~ solar neutrino problem: # ν_e's from the Sun was ~3 times smaller than expected

(Ray Davies '68, John Bahcall '80)

~ SNO experiment in 2003 measured ν_e and ν_μ from the sun: total # of neutrinos was just right, in agreement with solar models $\Rightarrow$ oscillations!

& also Super-Kamiokande, KamLAND.

$\Rightarrow$ assuming 3 neutrino flavors get: (PDG):

$$\sin^2\theta_{12} \simeq 0.86 \qquad \Delta m_{12}^2 = 8\times10^{-5}\,eV^2$$

$$\sin^2\theta_{23} \geq 0.92 \qquad \Delta m_{32}^2 = 1.9 \div 3.0 \times 10^{-3}\,eV^2$$

Note the large mixing angles

$\Rightarrow$ mass hierarchy of neutrinos has not been worked out either... experiments keep on going...

Can we "fix" Standard Model to include right-handed neutrinos? Sure we can: for instance do like for quarks: postulate right-handed neutrino singlet ν_R $\Rightarrow$ $\mathcal{L}_{R.H.\nu} = G_R\left[\underset{Y=+1}{\bar{L}}\ \underset{Y=-1}{\tilde{\phi}}\ \underset{Y=0}{\nu_R} + c.c.\right] + \ldots$ $\Rightarrow$ ok.

$\Rightarrow$ VEV of $\tilde{\phi}$ is $\langle 0|\tilde{\phi}|0\rangle = \begin{pmatrix} v/\sqrt{2} \\ 0 \end{pmatrix}$

$$\Rightarrow \mathcal{L}_{RH\nu} = G_R\left[(\bar{\nu}_L\ \ e_L)\begin{pmatrix} v/\sqrt{2} \\ 0\end{pmatrix}\nu_R + c.c.\right] = G_R\left[\bar{\nu}_L\nu_R + \bar{\nu}_R\nu_L\right]\frac{v}{\sqrt{2}}$$

$$= \frac{G_R v}{\sqrt{2}}\bar{\nu}\nu \Rightarrow m_\nu = \frac{G_R v}{\sqrt{2}} \Rightarrow \text{as } m_\nu \geq 0.04\,eV,\ v \simeq 289\,G$$

$$\Rightarrow G_R = \frac{m_\nu\sqrt{2}}{v} \simeq 2\times10^{-13}$$ ~ too much fine-tuning...

(some say "why not?") ...

Quantum Chromodynamics (QCD)

~ the theory of strong interactions

~ contains: quark fields q^{if} ← color, $i = 1, 2, 3$; ← flavor, $f = u, d, s, c, b, t$

gluon fields A_μ^a ← gluon color $a = 1, 2, \ldots, 8$

$$\boxed{\mathcal{L}_{QCD} = \bar{q}^f \left[i \gamma^\mu D_\mu - m_f \right] q^f - \frac{1}{4} F_{\mu\nu}^a F^{a\mu\nu}}$$

with $D_\mu = \partial_\mu - i g A_\mu$, $\quad A_\mu = \sum_{a=1}^{8} T^a \cdot A_\mu^a$

$$F_{\mu\nu}^a = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a + g f^{abc} A_\mu^b A_\nu^c$$

Running Coupling and Asymptotic Freedom

g ~ is the coupling constant

put $m_f = 0$ in $\mathcal{L}_{QCD}$ for simplicity:

$$\left\{ \mathcal{L}_{QCD}^{m_f = 0} = \bar{q}^f i \gamma^\mu D_\mu q^f - \frac{1}{4} F_{\mu\nu}^a F^{a\mu\nu} \right\}$$

g is the only parameter for such theory.

⇒ When people do perturbation theory, infinities arise:

 $\sim \int^\mu \frac{d^4 k}{k^4} \sim \ln \mu$ with μ a UV cutoff

- problems are usually in the ultraviolet (UV) where momenta are large
- one has to introduce a UV cutoff μ $\Rightarrow$

$\Rightarrow$ $\mathcal{L}$ & observables would depend on μ:

$$\mathcal{L} = \mathcal{L}(g, \mu), \quad \underset{\substack{\uparrow \\ \text{observable}}}{\mathcal{M}} = \mathcal{M}(g, \mu).$$

$\Rightarrow$ but physics should not be dependent on any cutoff if the theory is consistent $\Rightarrow$

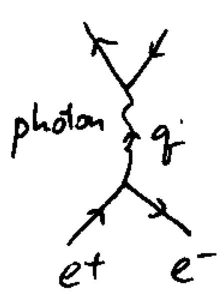
$\Rightarrow$ the only way to make it work is to have g depend on μ $\Rightarrow$ $\mathcal{L} = \mathcal{L}(g_\mu, \mu)$

$\mathcal{M} = \mathcal{M}(g_\mu, \mu)$.

$\Rightarrow$ <u>running coupling</u>: g_μ depends on momentum scale μ.

$\Rightarrow$ imagine an observable $\mathcal{M}$ which depends on a single four-momentum squared: $Q^2 = q_\mu q^\mu$

<u>Example</u>: $e^+e^- \to$ hadrons



$\Rightarrow$ the cross section depends on center of mass energy $Q^2 = q_\mu q^\mu$ $\Rightarrow$ $\sigma = \sigma(Q^2)$

in CM frame $q^\mu = (Q, \vec{0})$ $\Rightarrow$ $q^2 = Q^2$.

$Q^2 \sigma$ is dimensionless

$\Rightarrow$ in general would have $M = M(Q^2, \alpha_\mu, \mu)$

where $\alpha_\mu = \frac{g_\mu^2}{4\pi}$

$\Rightarrow$ assume that M is dimensionless $\Rightarrow M = M\left(\frac{Q^2}{\mu^2}, \alpha_\mu\right)$.

But: no physical observable should depend on μ!

$$\Rightarrow \boxed{\mu^2 \frac{d}{d\mu^2} M\left(\frac{Q^2}{\mu^2}, \alpha_\mu\right) = 0}$$

$$\Rightarrow \left[\mu^2 \frac{\partial}{\partial \mu^2} + \mu^2 \frac{d\alpha_\mu}{d\mu^2} \frac{\partial}{\partial \alpha_\mu}\right] M\left(\frac{Q^2}{\mu^2}, \alpha_\mu\right) = 0$$

Def. Beta-function of QCD: $\beta(\alpha_\mu) = \mu^2 \frac{d\alpha_\mu}{d\mu^2}$.

$\beta(\alpha_\mu)$ is dimensionless $\Rightarrow$ can not depend on μ explicitly, μ-dependence comes in through α_μ only!

$$\Rightarrow \boxed{\left[\mu^2 \frac{\partial}{\partial \mu^2} + \beta(\alpha_\mu) \frac{\partial}{\partial \alpha_\mu}\right] M\left(\frac{Q^2}{\mu^2}, \alpha_\mu\right) = 0}$$

renormalization group equation (Callan, Symanzik '70)

- tells how things change with the changing momentum scale / distance resolution

$\Rightarrow$ equivalently $\left[-Q^2 \frac{\partial}{\partial Q^2} + \beta(\alpha_\mu) \frac{\partial}{\partial \alpha_\mu}\right] M\left(\frac{Q^2}{\mu^2}, \alpha_\mu\right) = 0$.

To solve the renormalization group (RG) equation <u>define</u> $\rho(\alpha_\mu) = \int_{\alpha_0}^{\alpha_\mu} \frac{d\alpha'}{\beta(\alpha')}$

α_0 ~ arbitrary cutoff

Def. Running Coupling by:

$$\boxed{\alpha(Q^2) \equiv \rho^{-1}\left(\ln\frac{Q^2}{\mu^2} + \rho(\alpha_\mu)\right)}$$

ρ^{-1} ~ inverse function

=> note that

(i) $\alpha(\mu^2) = \alpha_\mu$

(ii) $\left[\mu^2 \frac{\partial}{\partial\mu^2} + \beta(\alpha_\mu)\frac{\partial}{\partial\alpha_\mu}\right]\alpha(Q^2) = 0$

Item (ii) is true because $\left[\mu^2\frac{\partial}{\partial\mu^2} + \beta(\alpha_\mu)\frac{\partial}{\partial\alpha_\mu}\right]\cdot$

$$\cdot\left(\ln\frac{Q^2}{\mu^2} + \rho(\alpha_\mu)\right) = -1 + \beta(\alpha_\mu)\underbrace{\frac{\partial\rho(\alpha_\mu)}{\partial\alpha_\mu}}_{1/\beta(\alpha_\mu) \text{ by definition}} = 0$$

as $M\left(\frac{Q^2}{\mu^2}, \alpha_\mu\right)$ does not depend on μ we can put $\mu = Q$ and get:

$$M\left(\frac{Q^2}{\mu^2}, \alpha_\mu\right) = M\left(\frac{Q^2}{\mu^2}, \alpha(\mu^2)\right) \overset{\mu^2\to Q^2}{=} M(1, \alpha(Q^2)) = \mathcal{M}(\alpha(Q^2))$$

=> any $\mathcal{M}$ which is a function of $\alpha(Q^2)$ only automatically satisfies RG equation.

$\Rightarrow$ We have shown that running coupling $\alpha(Q^2)$ satisfies RG equation + allows any observable dependent on it to satisfy RG equation.

$\Rightarrow$ let's find $\alpha(Q^2)$: to do this need $\rho(\alpha_\mu)$. To find $\rho(\alpha_\mu)$ need $\beta(\alpha_\mu)$ ~ the beta-function. Beta-function has to be found through an explicit (hard) calculation ~ see field theory texts like Peskin.

$\Rightarrow$ in perturbation theory one usually gets:

$$\boxed{\beta(\alpha) = -\beta_2\,\alpha^2 - \beta_3\,\alpha^3 + \ldots}$$

(perturbative / small coupling α expansion)

in QCD $\quad \beta_2 = \dfrac{11\,N_c - 2\,N_f}{12\pi}$, N_c ~ # colors

N_f ~ # flavors

(Politzer '73, Gross & Wilczek '73)

~ was probably obtained before by 't Hooft (oral communication)

$\Rightarrow$ it is <u>very important</u> that in QCD $\beta(\alpha) < 0$ ~ beta-function is negative