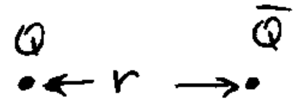
<u>Last time</u>: finished talking about Feynman rules in QCD using covariant $(\partial \cdot A = 0)$ and light-cone $(A^+ = 0)$ gauges:

covariant gauge $(\partial_\mu A^\mu = 0)$ $\Rightarrow$ need ghost field

light-cone gauge $\left(A^+ = \frac{A^0 + A^3}{\sqrt{2}} = 0\right)$ $\Rightarrow$ no ghost, different gluon propagator.

Heavy Quark Potential & Confinement (cont'd)

$\Rightarrow$ only one scale $\sim r$ $\Rightarrow$



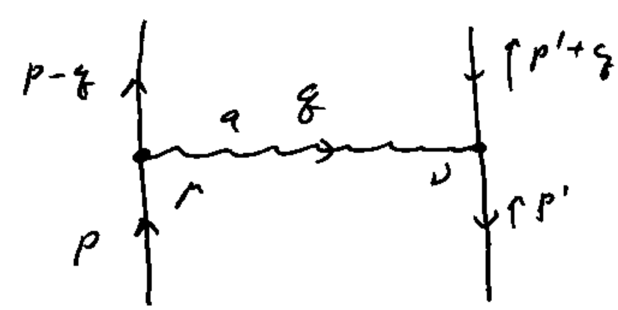
$\Rightarrow$ $\alpha_s = \alpha_s(1/r^2)$ $\Rightarrow$

if $r \ll \frac{1}{\Lambda_{QCD}}$ $\Rightarrow$ perturbative, $\alpha_s \ll 1$

if $r \gtrsim \frac{1}{\Lambda_{QCD}}$ $\Rightarrow$ non-perturbative, $\alpha_s >> 1$.

Short Distances $\left(\alpha_s \ll 1,\ r \ll \frac{1}{\Lambda_{QCD}}\right)$

$\Rightarrow$ use pert. theory



$\Rightarrow$ we got

$$V(r) = -g^2 C_F \int \frac{d^3q}{(2\pi)^3} \frac{e^{i\vec{q}\cdot\vec{r}}}{\vec{q}^{\,2}}$$

with $C_F = T^a T^a = \frac{N_c^2 - 1}{2N_c}$

Quark mass M is very large $\Rightarrow$

$$(p-q)^2 = M^2 \Rightarrow M^2 - 2\,p\cdot q + q^2 = M^2$$

$$\Rightarrow p\cdot q \simeq M\cdot q^0 \quad \Rightarrow M^2 - 2M\cdot q^0 = M^2$$

$$(q^2 \ll p\cdot q) \Rightarrow \quad q^0 = 0 \quad \Rightarrow q^2 = -|\vec{q}|^2 .$$

$$\bar{u}_{s'}(p-q)\,\gamma^\mu u_s(p) \overset{\text{static case}}{\approx} g^{\mu 0}\cdot \bar{u}_{s'}(p-q)\,\gamma^0 u_s(p)$$

$$= g^{\mu 0}\, u^+_{s'}(p-q)\, u_s(p) = g^{\mu 0}\cdot 2M\,\delta^{ss'}$$

similarly $\bar{v}_r(p')\gamma^\nu v_{r'}(p'+q) = g^{\nu 0}\, 2M\,\delta^{rr'}$.

$$iM = -g^2 \int \frac{d^3q}{(2\pi)^3}\cdot \underbrace{2M\,\delta^{ss'}\delta^{rr'}}_{\text{norm}}\ \frac{1}{\vec{q}^{\,2}}\ \underbrace{tr(T^aT^a)\cdot\frac{1}{N_c}}_{\frac{N_c^2-1}{2N_c}\equiv C_F}$$

To get the potential need to turn d^3q into Fourier transform. Fixing the normalization write

choose $\vec{r} = r\hat{z}$ in polar coord's

$$V(r) = -g^2 C_F \int \frac{d^3q}{(2\pi)^3}\ \frac{e^{i\vec{q}\cdot\vec{r}}}{\vec{q}^{\,2}} = -g^2 C_F \int_0^\infty \frac{\cancel{q^2}\,dq}{(2\pi)^3}\cdot$$

$$\int_{-1}^{1} d\cos\theta \underbrace{\int_0^{2\pi} d\varphi}_{2\pi}\ \frac{1}{\cancel{q^2}}\, e^{iqr\cos\theta} = -g^2\frac{C_F}{(2\pi)^2}\int_0^\infty dq\,\cdot$$

$$\cdot\frac{1}{iqr}\left(e^{iqr} - e^{-iqr}\right) = -\frac{g^2 C_F}{4\pi^2}\frac{1}{ir}\int_0^\infty \frac{dq}{q}\left(e^{iqr} - e^{-iqr}\right)$$

$$= -\frac{g^2}{4\pi^2} C_F \frac{1}{ir} \; \frac{1}{2} \int_{-\infty}^{\infty} \frac{dq}{q + i\varepsilon} \left(\underbrace{e^{iqr}}_{\substack{\text{close} \\ \text{in upper} \\ \text{half-plane}}} - \underbrace{e^{-iqr}}_{\substack{\text{close in l.h. plane} \\ \Rightarrow \text{ zero}}} \right)$$

$$= -\frac{g^2}{4\pi^{\cancel{2}}} \cdot \frac{C_F}{\cancel{i}r} \cdot \frac{1}{\cancel{2}} \cdot \cancel{2\pi i} = \Big| \; \alpha_s \equiv \frac{g^2}{4\pi} \; = -\frac{\alpha_s C_F}{r}$$

$$\Rightarrow \boxed{ V_{QCD}(r) \Big|_{r\Lambda \ll 1} \simeq -\frac{\alpha_s C_F}{r} }$$

$\Rightarrow$ attractive Coulomb potential!

just like in QED

$$\Rightarrow \; C_F = \frac{N_c^2 - 1}{2 N_c} = \frac{8}{2 \cdot 3} = \frac{4}{3}$$

$$\Rightarrow \boxed{ V_{QCD}(r) \Big|_{r\Lambda \ll 1} \simeq -\frac{4}{3} \frac{\alpha_s}{r} }$$

$\Rightarrow$ if one drops color factor of $4/3$ and replaces

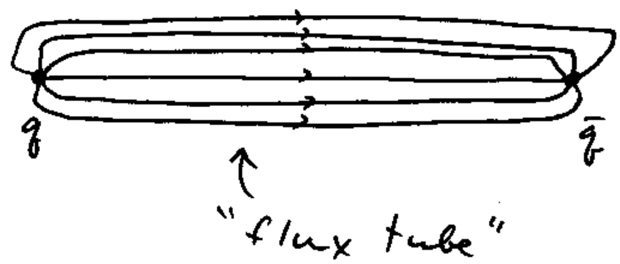
$\alpha_s \to \alpha_{EM}$ $\Rightarrow$ get QED Coulomb potential

$$V_{QED}(r) = -\frac{\alpha_{EM}}{r}.$$

Longer Distances: $r\Lambda_{QCD} \gtrsim 1 \Rightarrow$

$\alpha_s = \alpha_s\left(\frac{1}{r^2}\right) \sim \alpha_s(\Lambda_{QCD}^2) \sim 1 \Rightarrow$ perturbative approach breaks down as α_s is not small anymore!

Qualitative picture of what happens: draw force lines as:



$\sim$ constant force in-between, inside the flux tube

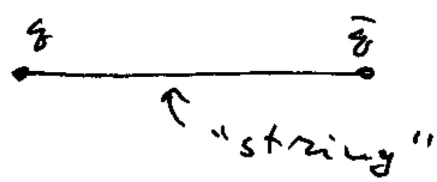
$\Rightarrow V(r) \propto F \cdot r \Rightarrow V(r)\Big|_{r\Lambda_{QCD} \gg 1} \simeq \sigma r$

(F ← force)

dimensions of σ ~ mass squared, $\sigma \simeq \Lambda_{QCD}^2$.

$\Rightarrow$ think of a flux tube as a relativistic string: σ is string tension:

$\sigma \simeq 1 \, \frac{GeV}{fm} \simeq \frac{1}{5} \, GeV^2$



Relativistic particle: the action is proportional to proper time τ, such that

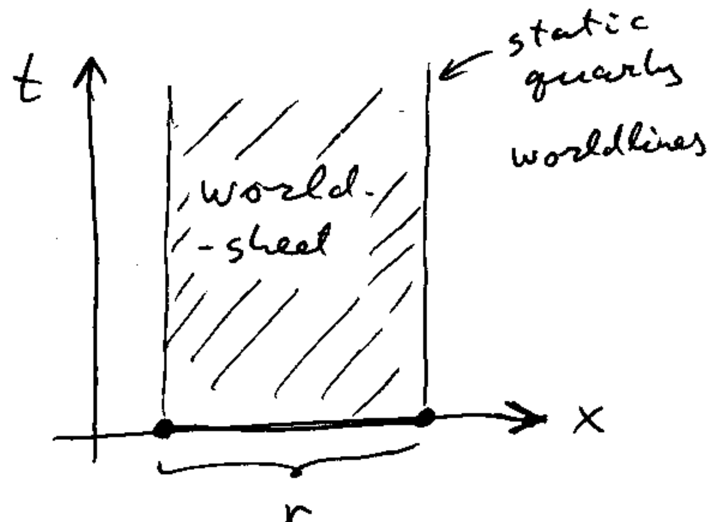
$$S_{particle} = -mc^2 \int d\tau .$$

Relativistic string: the action is proportional to "proper area" of a world-sheet:

$$S_{string} = -\sigma \cdot (\text{Area}).$$

(put $c=1$ for simplicity).

Consider a static string between 2 quarks:



to find classical configuration need to extremize the action S_{string} $\Rightarrow$ minimize the area of string worldsheet.

$\Rightarrow$ obviously min. is achieved for straight string with the action $S_{string}^{classical} = -\sigma \cdot \int dt \cdot \int_0^r dx$

$$= -\sigma \int dt \cdot r = \int dt \cdot L = \int dt \, (\underbrace{K}_{\substack{= \\ 0}} - V(r)) = \int dt \, [-V(r)]$$

as no motion

$\Rightarrow$ $V(r) = \sigma r$ as desired!

(note the difference from non-relativistic string in classical mechanics which has $V(r) \sim \frac{1}{2} k r^2 \Rightarrow$ force $= kr$)

$\Rightarrow$ the attractive force is <u>constant</u>: $F = \sigma$.

$\Rightarrow$ We know that
$$\begin{cases} V(r)\big|_{r\Lambda \ll 1} \simeq -\frac{4}{3}\frac{\alpha_s}{r} \\ V(r)\big|_{r\Lambda \gg 1} \simeq \sigma r \end{cases}$$

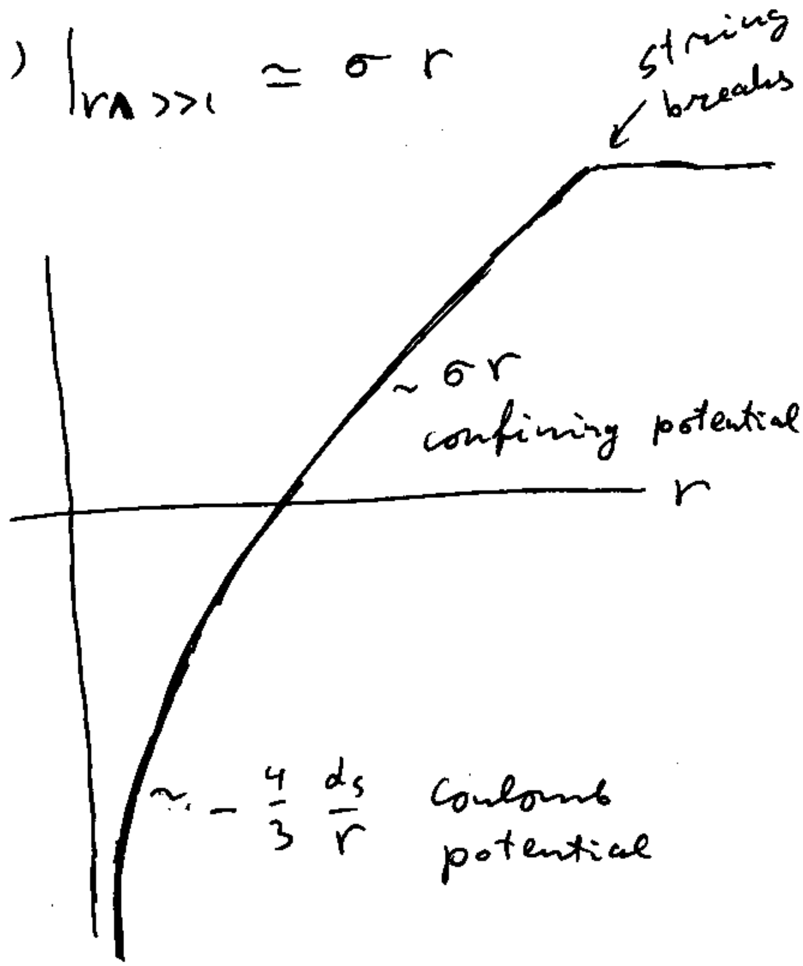
The full potential is:

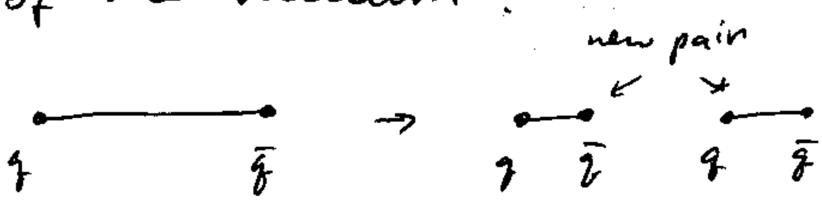
(see attached lattice data handout)

Linear potential is <u>confining</u>: quarks can not escape.

If string breaks $\Rightarrow$

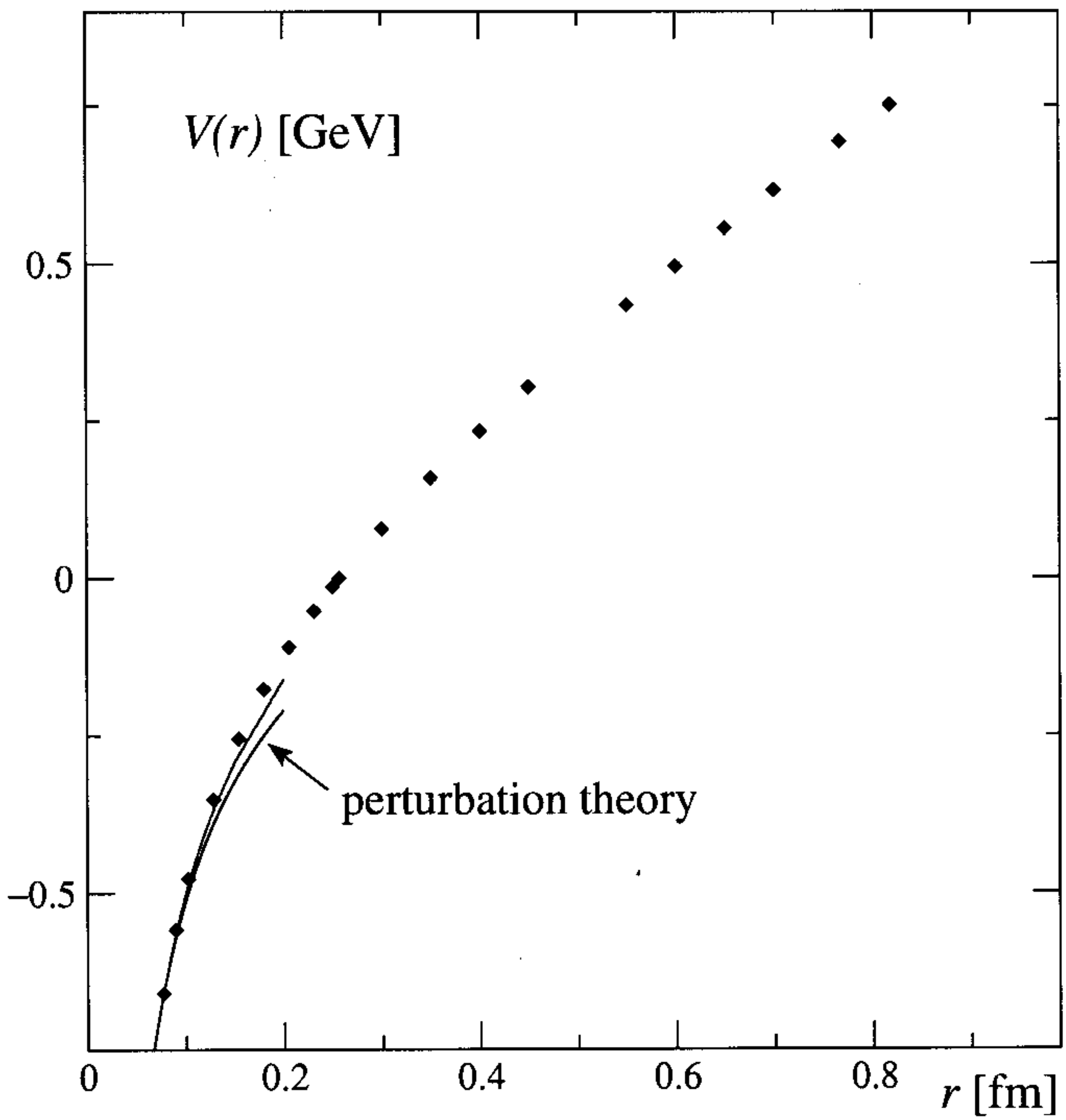
$\Rightarrow$ get $q\bar{q}$ pair out of the vacuum:





Lattice QCD: data points

perturbative QCD: solid lines + band.



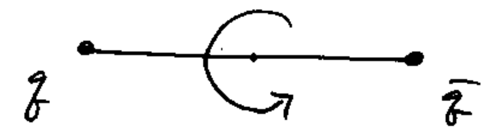
Good interpolation:

$$\boxed{V(r) \simeq -\frac{4}{3}\frac{\alpha_s}{r} + \sigma r}$$

"Cornell potential".

String model works amazingly well: think of $q\bar{q}$ state as a meson. If the meson has spin => think of an ultra-relativistic rotating string:

$d \sim$ string length



if q & $\bar{q}$ rotate with velocity $= 1$ (UR quarks) => $v = \frac{r}{d/2} = \frac{2r}{d}$.

$r \sim$ distance from string element to rot. center

$v \sim$ velocity of string element.

$$\Rightarrow M = \int \frac{dm}{\sqrt{1-v^2}} = 2\int_0^{d/2} \frac{\sigma\, dr}{\sqrt{1-v^2}} = 2\sigma \cdot \int_0^{d/2} \frac{dr}{\sqrt{1-\left(\frac{2r}{d}\right)^2}} = 2\sigma \cdot \frac{d}{2} \cdot \underbrace{\int_0^1 \frac{d\xi}{\sqrt{1-\xi^2}}}_{\substack{\pi/2 \\ \shortparallel \\ \left(\arcsin \xi\right)\Big|_0^1}} = \frac{\pi}{2}\sigma d.$$

The angular momentum (meson's spin)

is
$$J = \int \frac{r v\, dm}{\sqrt{1-v^2}} = 2\sigma \int_0^{d/2} \frac{r v\, dr}{\sqrt{1-v^2}} =$$

$$= 2\sigma \int_0^{d/2} \frac{dr \cdot (2r/d) \cdot r}{\sqrt{1-\left(\frac{2r}{d}\right)^2}} = 2\sigma \left(\frac{d}{2}\right)^2 \underbrace{\int_0^1 d\xi \frac{\xi^2}{\sqrt{1-\xi^2}}}_{\pi/4} =$$

$$= \frac{\sigma d^2}{2} \cdot \frac{\pi}{4} = \frac{\pi \sigma d^2}{8}.$$

$\Rightarrow$ meson mass $\boxed{M = \frac{\pi}{2} \sigma d}$ Gasiorowicz & Rosner '81

meson spin $\boxed{J = \frac{\pi \sigma d^2}{8}}$

$$\Rightarrow J = \frac{\pi}{8} \sigma \cdot \left(\frac{2M}{\pi \sigma}\right)^2 = \frac{1}{2\pi\sigma} M^2$$

$\Rightarrow$ $J = \frac{1}{2\pi\sigma} M^2$ is an example of a Regge trajectory

In general, on the basis of phenomenological evidence, people noticed that

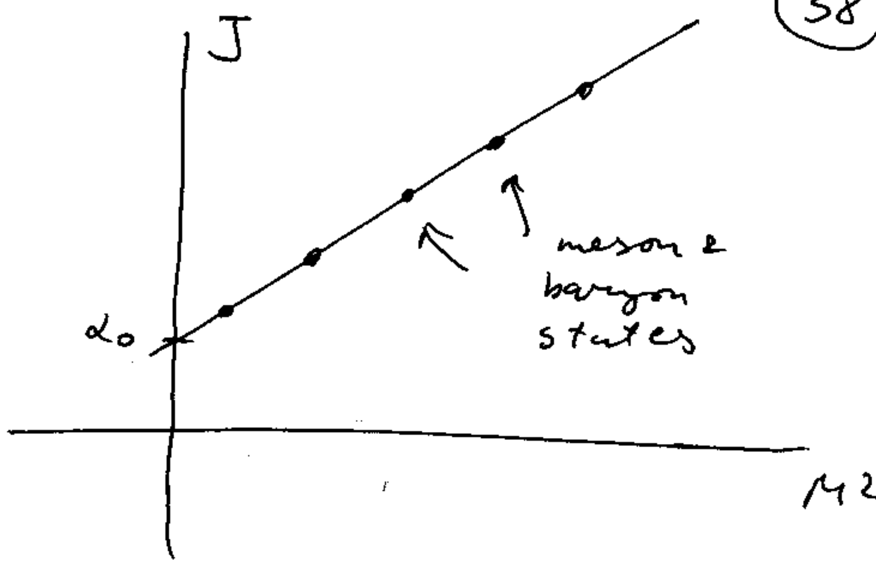
$$\boxed{J = \alpha_0 + \alpha' M_J^2}$$

Chew & Frautschi '61

$\alpha_0 \sim$ intercept

$\alpha' \sim$ slope

Regge trajectory:

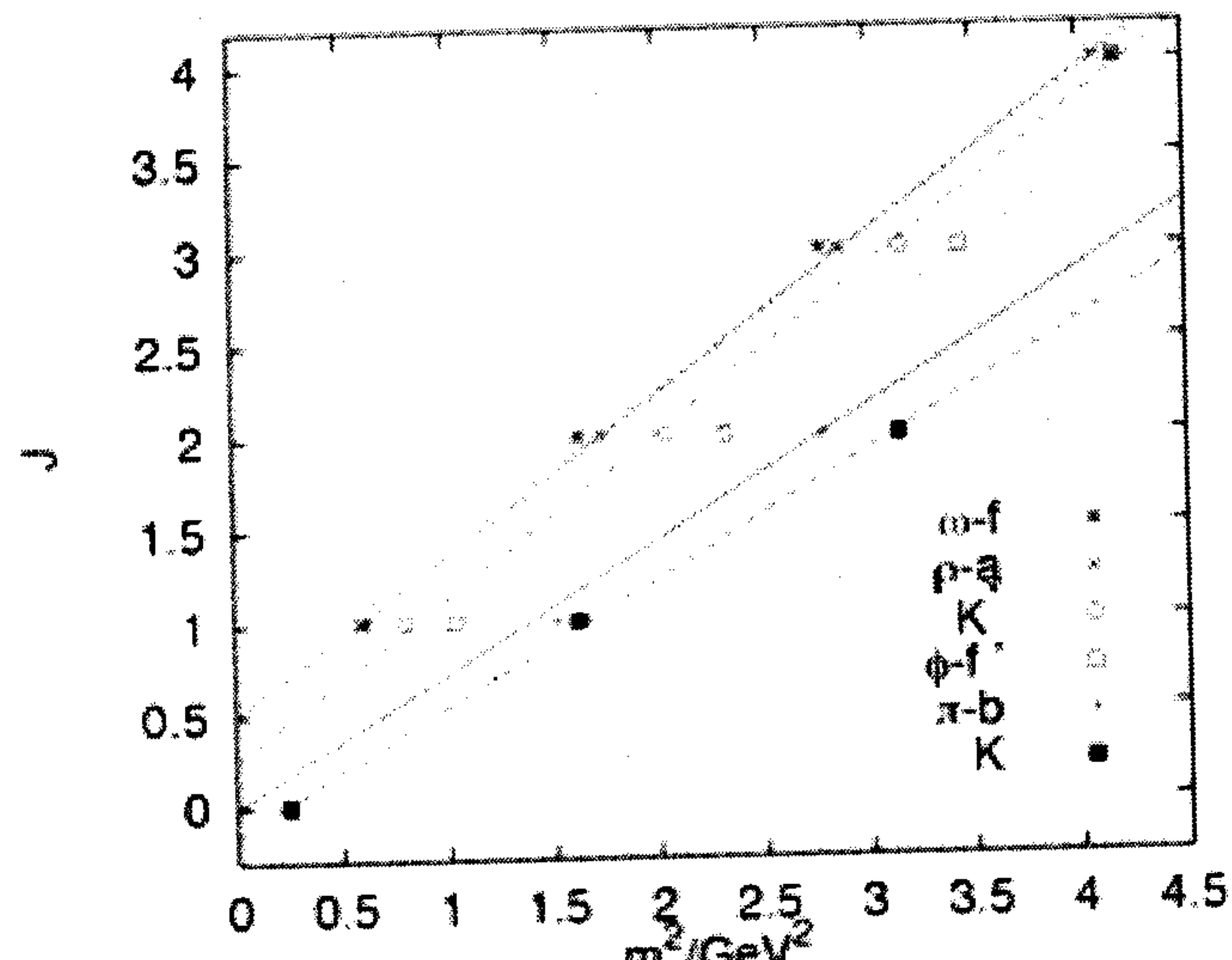


we got $\alpha' = \frac{1}{2\pi\sigma}$

or $\boxed{\sigma = \frac{1}{2\pi\alpha'}}$. $\alpha' = \frac{1}{2\pi\sigma} \approx \frac{5}{2\pi}\ GeV^{-2}$

⇒ successes of string approximation to strong interaction data led to proposal of string theory as the theory of strong interactions in the '60's.

⇒ that idea was killed by $e^+e^- \to$ hadrons & (mostly) DIS data & string theory moved on to gravity in '84.


J
m^2/GeV^2
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
ω-f
ρ-a
K
φ-f '
π-b
K