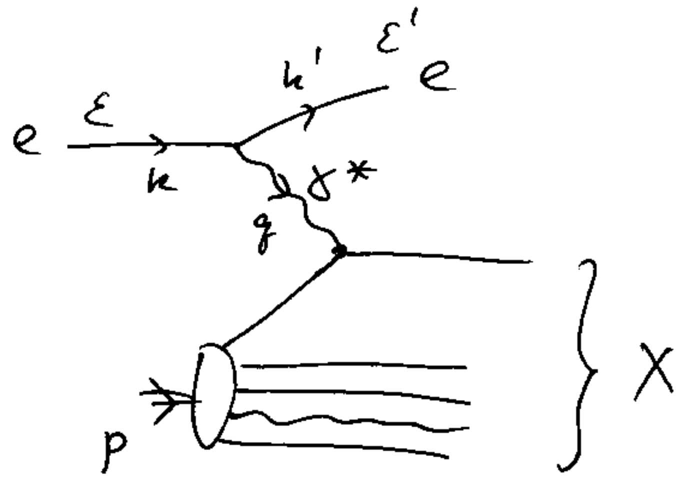
Last time: started talking about

Parton Model and DIS

Deep Inelastic Scattering (DIS).



two essential scalars:

$Q^2 = -q^2$ photon's virtuality

$x_{Bj} = \frac{Q^2}{2p\cdot q}$ Bjorken-x variable

$= \frac{Q^2}{Q^2+\hat{s}} \Rightarrow 0 \le x \le 1$

We squared the diagram & obtained the following expression for the cross-section:

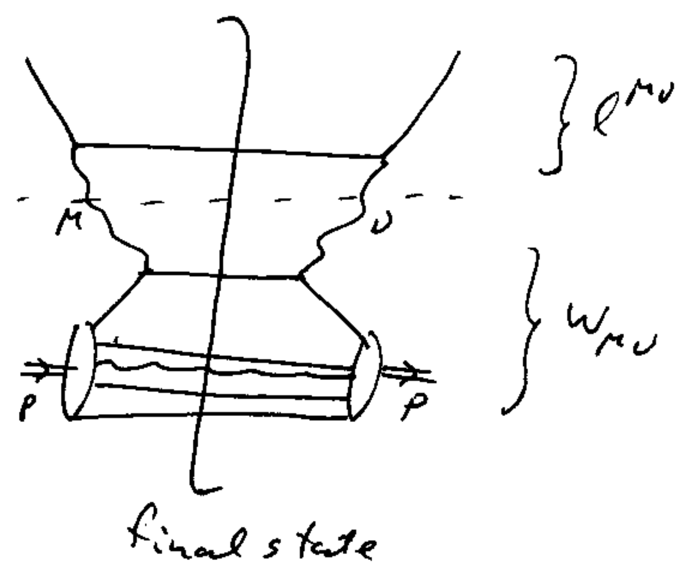
$$\frac{d\sigma}{d^3k'} = \frac{e^4}{Q^4\, 4\mathcal{E}\mathcal{E}'}\,\frac{1}{4\pi^2}\, \ell^{\mu\nu} W_{\mu\nu}$$

with

$$\ell^{\mu\nu} = 2\left(k^\mu k'^\nu + k^\nu k'^\mu - g^{\mu\nu} k\cdot k'\right)$$

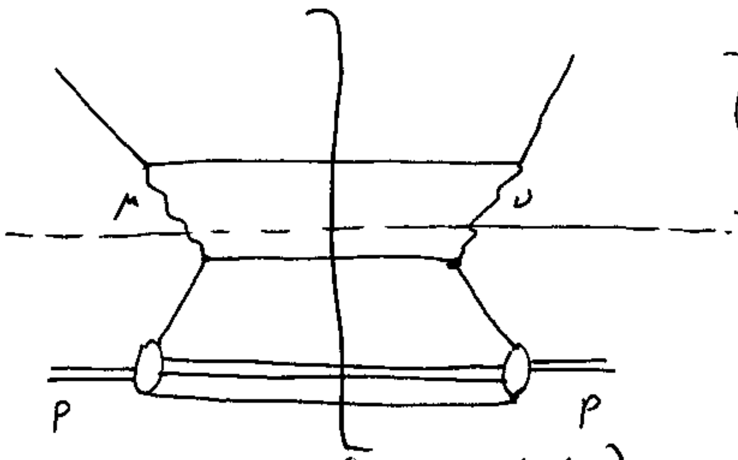
$$\boxed{W_{\mu\nu} = \frac{\mathcal{E}}{2\pi m}\int d^4x\, e^{iq\cdot x} \cdot \langle p|\, j_\mu(x)\, j_\nu(0)\,|p\rangle}$$

hadronic tensor



$$\boxed{W_{\mu\nu} = \frac{4\pi^2 E_p}{m\,(2\pi)^3}\int d^4x\; e^{iq\cdot x}\,\langle p|\, j_\mu(x)\, j_\nu(0)\,|p\rangle}$$

(averaged over σ)



$\Big\}$ $l_{\mu\nu}$ (EM part)

$\Big\}$ $W_{\mu\nu}$ (hadronic part)

cut (final state)

$$W_{\mu\nu}(p,q) = A(x,Q^2)\, p_\mu p_\nu + B(x,Q^2)\, q_\mu q_\nu + C(x,Q^2)\, g_{\mu\nu} +$$

$$+ D(x,Q^2)\,(p_\mu q_\nu + p_\nu q_\mu) + E(x,Q^2)\,(p_\mu q_\nu - p_\nu q_\mu) +$$

$$+ F(x,Q^2)\;\varepsilon_{\mu\nu\rho\sigma}\, p^\rho q^\sigma$$

$F = 0$ in $\gamma^* p$, $\gamma^* A$ (F comes from γ_5's, appears in ν DIS).

(1) $q_\mu W^{\mu\nu} = 0$ (current conservation)

$q_\mu \to \partial_\mu \Rightarrow \partial_\mu j^\mu(x) = 0$

$$A\, p_\nu\, (p\cdot q) + B\, q_\nu q^2 + C\, q_\nu + D\,(p\cdot q\; q_\nu + q^2 p_\nu)$$

$$+ E\,(p\cdot q\;\; q_\nu - q^2 p_\nu) = 0$$

(2) $q_\nu W^{\mu\nu} = 0 = A\, p_\mu\,(p\cdot q) + B\, q^2 q_\mu + C q_\mu + D(p\cdot q\; q_\mu +$

$$+ q^2 p_\mu) + E\,(p_\mu q^2 - p\cdot q\; q_\mu) = 0$$

$$(1) - (2) = 0 \quad \Rightarrow \quad E = 0.$$

as p_μ and q_μ are independent $\Rightarrow$

$$0 = A\, p\cdot q + D q^2 \qquad 0 = B q^2 + C + D\, p\cdot q$$

$$D = -A \frac{p\cdot q}{q^2} \qquad B = -\frac{1}{q^2} C + A\left(\frac{p\cdot q}{q^2}\right)^2$$

$$W_{\mu\nu} = A\left[p_\mu p_\nu - \frac{p\cdot q}{q^2}(p_\mu q_\nu + p_\nu q_\mu) + \left(\frac{p\cdot q}{q^2}\right)^2 q_\mu q_\nu \right]$$

$$+ C\left[g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right]$$

Usually one writes

$$W_{\mu\nu} = -W_1(x, Q^2)\left[g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right] + \frac{W_2(x, Q^2)}{m_p^2} \cdot$$

$$\cdot \left[p_\mu p_\nu - \frac{p\cdot q}{q^2}(p_\mu q_\nu + p_\nu q_\mu) + \left(\frac{p\cdot q}{q^2}\right)^2 q_\mu q_\nu \right]$$

W_1 & W_2 are <u>structure functions</u> (Def.)

Using $q_\mu \ell^{\mu\nu} = q_\nu \ell^{\mu\nu} = 0$ yields

$$\ell_{\mu\nu} W^{\mu\nu} = -W_1 (-4 \underbrace{k\cdot k'}_{2\varepsilon\varepsilon' \sin^2\frac{\theta}{2}}) + \frac{2W_2}{m_p^2}\underbrace{\left[2\, p\cdot k\; p\cdot k' - m^2 k\cdot k'\right]}_{2m^2\varepsilon\varepsilon' - 2m^2\varepsilon\varepsilon' \sin^2\frac{\theta}{2}}$$

$$2m^2\varepsilon\varepsilon' - 2m^2\varepsilon\varepsilon' \sin^2\frac{\theta}{2} = $$

$$= 2m^2\varepsilon\varepsilon' \cos^2\frac{\theta}{2}$$

$$q_\mu l^{\mu\nu} = (k-k')_\mu \, 2\left(k^\mu k'^\nu + k^\nu k'^\mu - g^{\mu\nu} k\cdot k'\right) =$$ (43')

$$= 2\left(k^2 k'^\nu + \cancel{k^\nu k\cdot k'} - \cancel{k^\nu k\cdot k'} - \cancel{k'\cdot k\, k'^\nu} - k^\nu k'^2 \right.$$

$$\left. + \cancel{k'^\nu k\cdot k'} \right) = 2\left(k^2 k'^\nu - k'^2 k^\nu\right) \simeq 0 \quad \text{as } k^2 \simeq k'^2 \simeq 0.$$

(neglect electron's mass), $q_\nu l^{\mu\nu} = 0$ (similar)

$$\Rightarrow l_{\mu\nu} W^{\mu\nu} = l^{\mu\nu}\left[-W_1\left(g_{\mu\nu} - \frac{\cancelto{0}{q_\mu q_\nu}}{q^2}\right) + \frac{W_2}{m_p^2} \right.$$

$$\left. \left(p_\mu p_\nu - \frac{p\cdot q}{q^2}\left(\cancelto{0}{p_\mu q_\nu} + \cancelto{0}{p_\nu q_\mu}\right) + \left(\frac{p\cdot q}{q^2}\right)^2 \cancelto{0}{q_\mu q_\nu}\right)\right]$$

$$= -l^\mu{}_\mu W_1 + \frac{W_2}{m_p^2} p_\mu p_\nu l^{\mu\nu} = \left| \begin{array}{l} \text{as } l^{\mu\nu} = 2(k^\mu k'^\nu + \\ + k^\nu k'^\mu - g^{\mu\nu} k\cdot k') \end{array} \right.$$

$$= -2\left(2k\cdot k' - \overset{g^\mu{}_\mu}{4}\, k\cdot k'\right) W_1 + \frac{W_2}{m_p^2}\, 2\left(2p\cdot k\; p\cdot k' - p^2 k\cdot k'\right)$$

$$= 4\, k\cdot k'\, W_1 + 2\frac{W_2}{m_p^2}\left(2 p\cdot k\; p\cdot k' - m_p^2\, k\cdot k'\right)$$

Remember: $k = (\varepsilon, 0, 0, \varepsilon)$, $k' = (\varepsilon', \varepsilon' \sin\theta, 0, \varepsilon' \cos\theta)$

$p = (m_p, \vec{0})$

$$\Rightarrow k\cdot k' = 2\varepsilon\varepsilon' \sin^2(\theta/2) \, ; \quad p\cdot k = m_p \varepsilon, \; p\cdot k' = \varepsilon' m_p$$

$$\ell_{\mu\nu} W^{\mu\nu} = 4 \varepsilon \varepsilon' \left[2 W_1 \sin^2 \frac{\theta}{2} + W_2 \cos^2 \frac{\theta}{2} \right]$$

$$\frac{d\sigma}{d^3k'} = \frac{e^4}{Q^4 \, 4\pi^2} \left[2 W_1 \sin^2 \frac{\theta}{2} + W_2 \cos^2 \frac{\theta}{2} \right]$$

By varying the angle θ can separate W_1 & W_2 contributions in experiments.

Usually one defines $F_1(x, Q^2) = m_p W_1(x, Q^2)$, $F_2(x, Q^2) = \nu \, W_2(x, Q^2)$

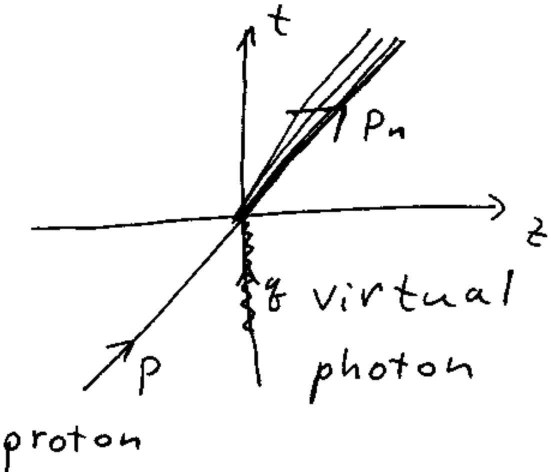
The Parton Model.

Sterman ch 14, Peskin 17.5

Go to Infinite Momentum Frame:

$$p_\mu \approx \left(p + \frac{m^2}{2p}, 0, 0, p \right)$$
$$\quad\;\; 0 \qquad\quad 1 \;\; 2 \;\; 3$$

$$q_\mu = \left(q_0, \; \underset{\sim}{q}, \; 0 \right)$$
$$\quad\;\; 0 \quad\; 1,2 \quad 3$$



Q^2 and x are 2 invariants
large, $Q \gg \Lambda_{QCD}$

$$p \cdot q = m\nu = q_0 \cdot p$$

$$\Rightarrow q_0 = \frac{m\nu}{p} \sim \text{small as } p \text{ goes large}$$

$$\Rightarrow Q^2 = -q^2 = \underset{\sim}{q}^2$$

$$F_1(x, Q^2) = m_p W_1(x, Q^2)$$

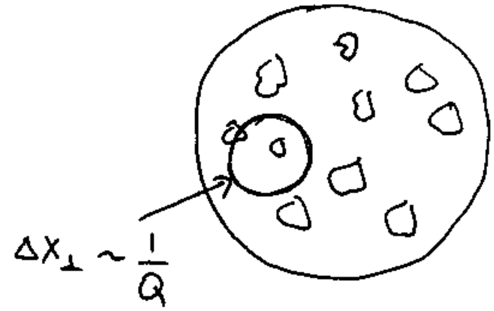
$$F_2(x, Q^2) = \nu W_2(x, Q^2) = \frac{Q^2}{2 m_p x} W_2(x, Q^2)$$

$$\boxed{\frac{d\sigma}{d^3k'} = \frac{4\alpha_{EM}^2}{Q^2}\left[2\frac{1}{m_p}F_1(x, Q^2)\sin^2\frac{\theta}{2} + \frac{2m_p x}{Q^2}F_2(x, Q^2) \cdot \cos^2(\theta/2)\right]}$$

$Q^2 = q^2 \Rightarrow$ photon acts like a <u>microscope</u> in <u>transverse</u> plane:

$\Delta x_\perp \cdot q_\perp \sim 1 \qquad (\hbar = 1)$

$\Delta x_\perp \sim \frac{1}{q_\perp} \sim \frac{1}{Q}$

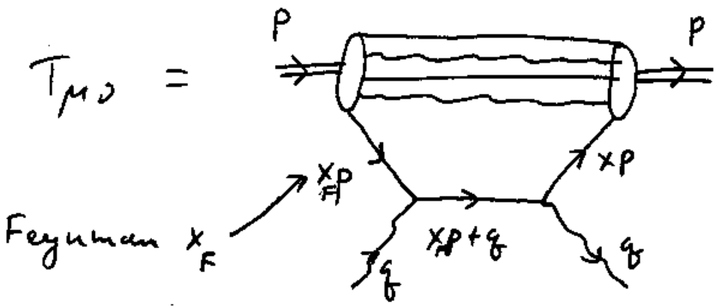


proton

large $Q \sim$ resolve just 1 quark

Define $\boxed{T_{\mu\nu} = \frac{\varepsilon_p}{2\pi m_p} \int d^4x \, e^{iq\cdot x} \, \frac{1}{2} \sum_\sigma \langle p,\sigma | T j_\mu(x) j_\nu(0) | p,\sigma \rangle}$

$W_{\mu\nu} = 2\,\mathrm{Im}\,(i\,T_{\mu\nu})$ (optical theorem)

$T_{\mu\nu} =$ 

"Forward Amplitude"

Def Feynman-x: the fraction of proton's longitudinal momentum carried by struck quark

typical interaction time in proton's rest frame is $\frac{1}{\Lambda_{QCD}} \Rightarrow$ boost to get $\frac{p}{m}\frac{1}{\Lambda} \equiv \tau_\Lambda$

int. time of DIS is $\tau_{DIS} \simeq \frac{1}{q^0}$, where

$q^0 \simeq \frac{2Q^2}{x_B p}$ $\left(= \frac{m\nu}{p} \leftarrow \text{as } m\nu = \frac{Q^2}{2x}\right)$ is struck quark's velocity: $\tau_{DIS} \simeq \frac{x_B p}{2Q^2}$.

time-ordered product: (denoted T)

$$\boxed{T\, j_\mu(x)\, j_\nu(y) \equiv \theta(x^0-y^0)\, j_\mu(x)\, j_\nu(y) + \theta(y^0-x^0)\, j_\nu(y)\, j_\mu(x)}$$

note: currents do not commute with each other in general $\Rightarrow$ not a trivial object.

$$2\,\mathrm{Im}\left(i T_{\mu\nu}\right) = 2\,\mathrm{Im}\Big[i \cdot \frac{4\pi^2 E_p}{m_p}\int d^4x\, e^{iq\cdot x}\langle p|\,\theta(x^0)\, \overset{e^{ip\cdot x} j_\mu(0) e^{-ip\cdot x}}{j_\mu(x)} \underset{\sum_n |n\rangle\langle n|}{\,} j_\nu(0) +$$

$$+\,\theta(-x^0)\, j_\nu(0)\, j_\mu(x)\,|p\rangle\Big] = 2\cdot\frac{4\pi^2 E_p}{m_p}\sum_n \mathrm{Re}\Big\{\int d^4x\, e^{iq\cdot x + ip\cdot x - ip_n\cdot x}$$

$$\cdot\,\theta(x^0)\,\langle p|j_\mu(0)|n\rangle\langle n|j_\nu(0)|p\rangle + \int d^4x\, e^{iq\cdot x + ip_n\cdot x - ip\cdot x}\,\theta(-x^0)\,\cdot$$

$$\cdot\,\langle p|j_\nu(0)|n\rangle\langle n|j_\mu(0)|p\rangle\Big\} = \sum_n 2\,\frac{4\pi^2 E_p}{m_p}\Big\{(2\pi)^3\,\delta(\vec{q}+\vec{p}-\vec{p}_n)\cdot\mathrm{Re}$$

$$\left(\frac{-1}{i(q^0+p^0-p_n^0+i\epsilon)}\langle p|j_\mu(0)|n\rangle\langle n|j_\nu(0)|p\rangle\right) + \underbrace{(2\pi)^3\,\underset{0}{\underset{\|}{\delta(\vec{q}+\vec{p}_n-\vec{p})}}\cdot}_{\substack{\text{not physical}\\ \Rightarrow \text{drop}\\ (\text{after including}\\ \delta(q^0+p_n^0-p^0))}}$$

$$\cdot\,\mathrm{Re}\left(\overset{\propto\,\delta(q^0+p_n^0-p^0)}{\frac{1}{i(q^0+p_n^0-p^0-i\epsilon)}}\langle p|j_\nu(0)|n\rangle\langle n|j_\mu(0)|p\rangle\right)\Big\}$$

$$= 2\,\frac{4\pi^2 E_p}{m_p}\sum_n (2\pi)^3\,\delta(\vec{q}+\vec{p}-\vec{p}_n)\,(-)\left(\mathrm{Im}\,\frac{1}{q^0+p^0-p_n^0+i\epsilon}\right)\cdot\langle p|j_\mu(0)|n\rangle$$

$$\langle n|j_\nu(0)|p\rangle = \Big|\ \text{as } \mathrm{Im}\,\frac{1}{x+i\epsilon} = -\pi\,\delta(x) = \frac{4\pi^2 E_p}{m_p}\sum_n (2\pi)^4\,\delta^4(q+p-p_n)$$

$$\cdot\,\langle p|j_\mu(0)|n\rangle\langle n|j_\nu(0)|p\rangle = W_{\mu\nu}\quad \text{as desired.}$$

if x is small ($\lesssim 1$) and Q is large

$\Rightarrow$ $\tau_{DIS} \ll \tau_{\Lambda}$ interaction is "instantaneous".

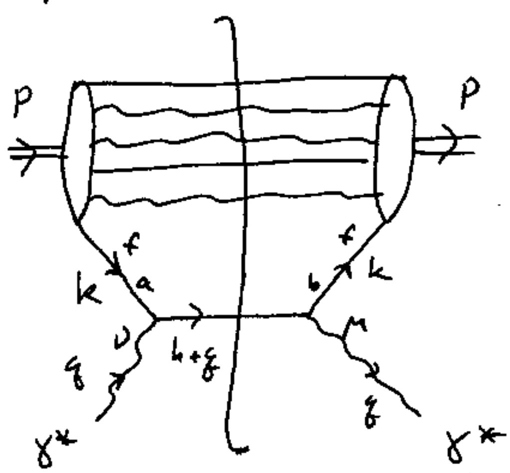
Define light cone variables:

for vector V^{μ} one has $V^{+} = \frac{1}{\sqrt{2}}(V^{0} + V^{3})$

$\underline{V} = (V^{1}, V^{2})$ $\qquad V^{-} = \frac{1}{\sqrt{2}}(V^{0} - V^{3})$

(2d transverse vector)

$$V_1 \cdot V_2 = V_{1\mu} V_2^{\mu} = V_{1+} V_{2-} + V_{1-} V_{2+} - \underline{V}_1 \cdot \underline{V}_2$$

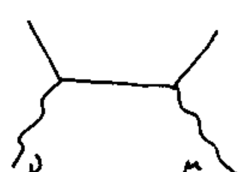
$W_{\mu\nu} =$ 

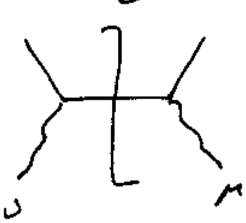
$\Big\}$ $A^{f}_{ab}(p,k)$

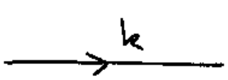
e.g.
$p_{+} = \frac{p}{\sqrt{2}}$
$p_{-} = \frac{m^2}{2\sqrt{2}\,p}$
$p_{+} \gg p_{-}$

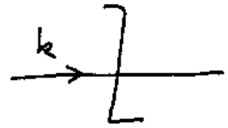
as $W_{\mu\nu} = 2\, \mathrm{Im}\,(i\, T_{\mu\nu})$

cut (Im part of a propagator)

$T_{\mu\nu} =$ 

$W_{\mu\nu} =$ 



$\frac{i}{k^2 - m^2 + i\varepsilon}$ $\Rightarrow$ 

$2\pi\,\delta^{(+)}(k^2 - m^2)$

as $2\,\mathrm{Im}\,\frac{-1}{k^2 - m^2 + i\varepsilon} = +2\pi\,\delta^{(+)}(k^2 - m^2)$