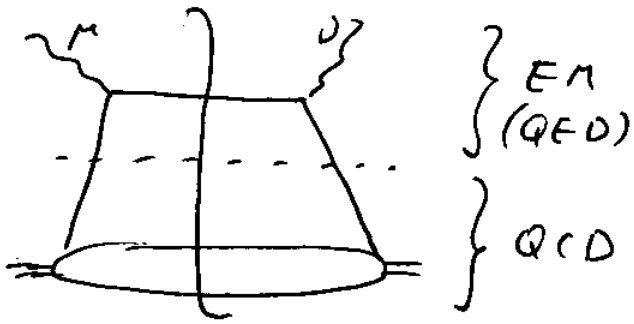
Particle Production in High Energy Hadronic Collisions.

Collinear Factorization

When we considered DIS above, we have factorized EM & QCD parts of the diagram:

We got

$$F_2(x,Q^2) = \sum_f e_f^2 x q^f(x)$$

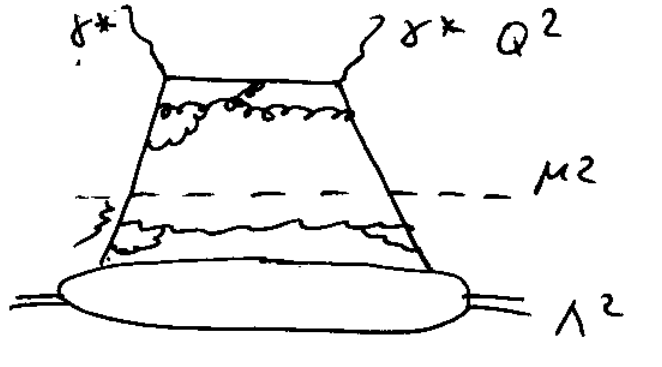


$\Rightarrow$ this is an example of collinear factorization

In general one writes:

$$F_2(x,Q^2) = \sum_{f,\bar{f},\text{gluons}} \int_0^1 d\xi \; C_2^f\left(\frac{x}{\xi}, Q^2, \mu^2\right) p^f(\xi,\mu^2) + \text{higher twists} + O\left(\frac{m^2}{Q^2}\right)$$

C_2^f ~ coefficient ftn (may contain QCD corrections at higher orders)



$p^f = \{q^f, q^{\bar{f}}, G\}$; ξ - momentum fraction of the parton in p^f.

μ^2 is called factorization scale.

Note: C_2^f is perturbatively calculable, p^f is not (though one has DGLAP for p^f)

$\Lambda^2 \lesssim \mu^2 \lesssim Q^2$ => but F_2 does not depend on μ^2, it is arbitrary => $\mu^2 \frac{d}{d\mu^2} F_2(x, Q^2) = 0$

Write $F_2 = C_2^f \otimes p^f$

$$\Rightarrow \mu^2 \frac{d}{d\mu^2} F_2 = 0 = \left(\mu^2 \frac{d}{d\mu^2} C_2^f\right) \otimes p^f + C_2^f \otimes \mu^2 \frac{d}{d\mu^2} p^f$$

=> what happens (separation of variables, C_2 only depends on Q^2, only p^f depends on Λ^2):

$$\mu^2 \frac{d}{d\mu^2} p_f = \gamma(\alpha_s) \otimes p_f$$

~ DGLAP evolution

γ ~ splitting function.

$$\mu^2 \frac{d}{d\mu^2} C_2^f = -\gamma(\alpha_s) \otimes C_2^f$$

$$\Rightarrow \mu^2 \frac{d}{d\mu^2} F_2 = -\gamma \otimes C_2^f \otimes p^f + C_2^f \otimes \gamma \otimes p^f = 0.$$

as desired.

=> can "place" corrections into PDF or coefficient function

Collinear factorization in DIS is a theorem which can be proven $\Rightarrow$ must be right! (at large-Q^2 only!)

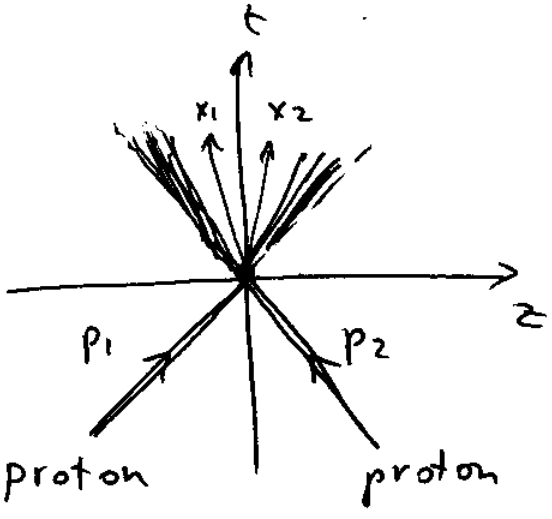
~ At LO have $C_2^f = \delta\left(\frac{x}{\xi} - 1\right) e_f^2$, f = quarks only

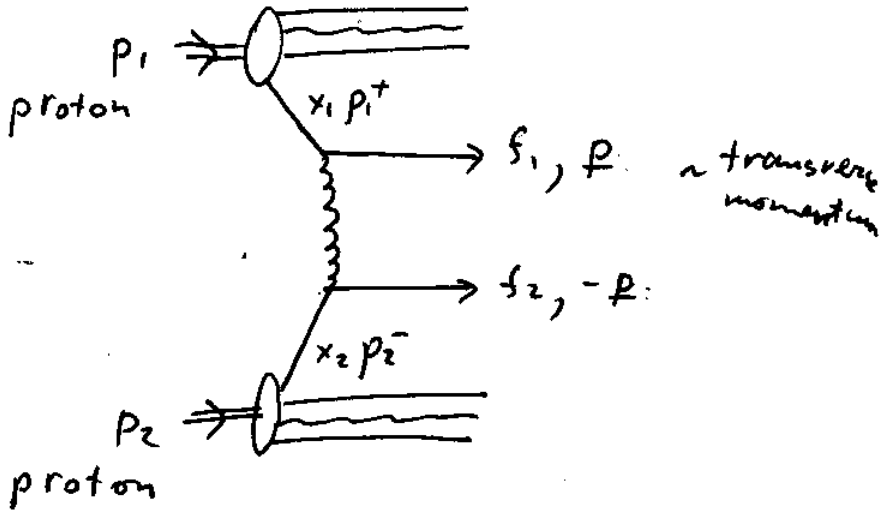
$$\Rightarrow F_2(x, Q^2) = \sum_f \underbrace{\int_0^1 d\xi \; \delta\left(\frac{x}{\xi} - 1\right)}_{x} e_f^2 \; q^f(\xi)$$

$$= \sum_f e_f^2 \, x q^f(x) \quad \text{as expected!}$$

Jet Production in Hadronic Collisions.

Collinear factorization also applies to hadron-hadron collisions. Consider quark production:





~ collision happens very fast on proton's time scales $\Rightarrow$ factorization.