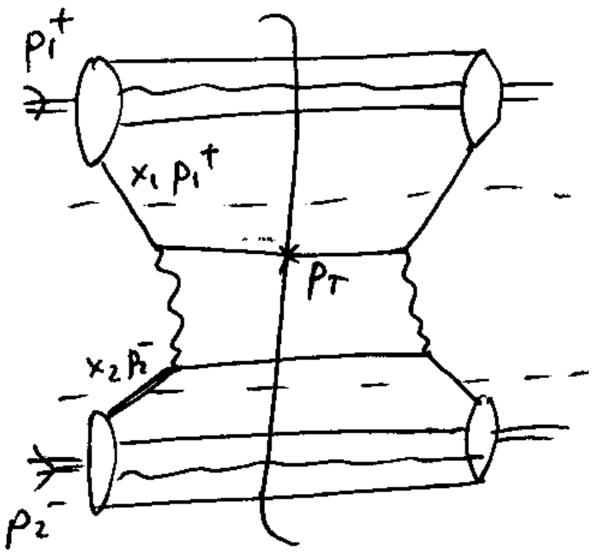
<u>Last time:</u> talked about collinear factorization for jet production:

<u>Jet Production in Hadronic Collisions (cont'd)</u>



$\}\ \rho^{f_1}(x_1, \mu^2) \sim PDF$

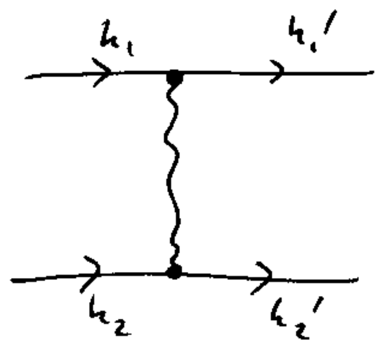
$\}\ \mathcal{E}_p \dfrac{d\hat{\sigma}_{f_1 f_2 \to f_1' f_2'}}{d^3 p} \sim$ hard cross-section

$\}\ \rho^{f_2}(x_2, \mu^2) \sim PDF$

$$\mathcal{E}_p \frac{d\sigma}{d^3 p} = \sum_{i,j} \int_0^1 dx_1 \int_0^1 dx_2\, \rho^{f_i}(x_1, \mu^2)\, \rho^{f_j}(x_2, \mu^2) \cdot \mathcal{E}_p \frac{d\hat{\sigma}_{f_i f_j \to f_i' f_j'}}{d^3 p} + o\left(\frac{\Lambda^2}{p_T^2}\right)$$

Calculated hard cross-section:

Mandelstam variables



$$\hat{s} = (k_1 + k_2)^2$$

$$\hat{t} = (k_1 - k_1')^2$$

$$\hat{u} = (k_1 - k_2')^2$$

Showed that

$$\varepsilon_p \frac{d\hat{\sigma}}{d^3p} = \frac{1}{(4\pi)^2} \frac{1}{\hat{s}} \delta(\hat{s}+\hat{t}+\hat{u}) \langle |M|^2 \rangle$$

Calculating the diagram we got:

$$\varepsilon_p \frac{d\hat{\sigma}}{d^3p} = \alpha_s^2 \frac{C_F}{N_c} \frac{1}{\hat{s}\,\hat{t}^2} \left[\hat{s}^2+\hat{u}^2\right] \delta(\hat{s}+\hat{t}+\hat{u})$$

$$\hat{s} = 2k_1 \cdot k_2 = 4\,\varepsilon_1 \varepsilon_2$$

$$\Rightarrow \varepsilon_p \frac{d\hat{\sigma}}{d^3p} = \frac{1}{\hat{s}}\,\alpha_s^2\,\delta(\hat{s}+\hat{t}+\hat{u})\,\frac{1}{\hat{t}^2}\,\frac{C_F}{N_c}\left[\hat{s}^2+\hat{u}^2\right]$$

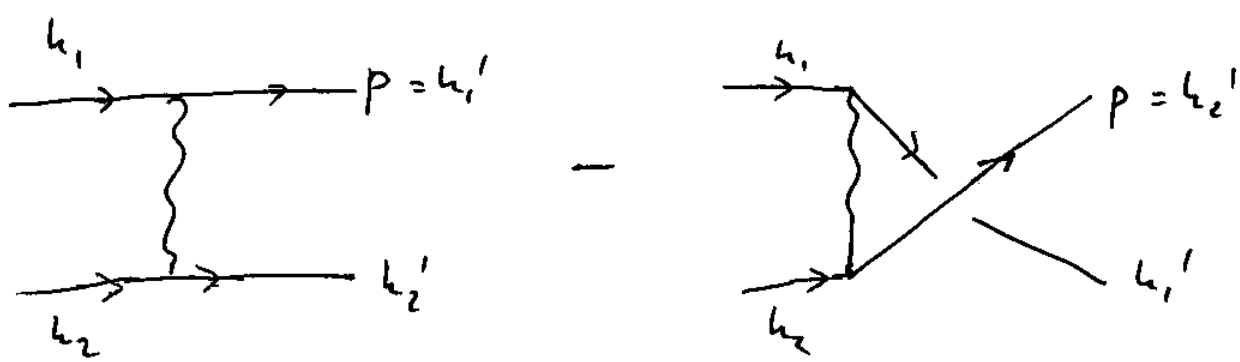
$$\Rightarrow \boxed{\varepsilon_p \frac{d\hat{\sigma}}{d^3p} = \alpha_s^2\,\frac{C_F}{N_c}\,\frac{1}{\hat{s}\,\hat{t}^2}\left[\hat{s}^2+\hat{u}^2\right]\delta(\hat{s}+\hat{t}+\hat{u})}$$

+ term with $\vec{k}_2' = \vec{p}$

$$C_F = \frac{N_c^2-1}{2N_c} = \frac{8}{6} = \frac{4}{3}$$

$$\Rightarrow \varepsilon_p \frac{d\hat{\sigma}}{d^3p} = \frac{4}{9}\,\alpha_s^2\,\frac{1}{\hat{s}\,\hat{t}^2}\left[\hat{s}^2+\hat{u}^2\right]\delta(\hat{s}+\hat{t}+\hat{u}) + \ldots$$

However, one has to be careful, we forgot the term with $\vec{k}_2' = \vec{p}$. Diagrammatically we have



$\hat{t}_{old} = (k_1 - p)^2 = (k_1 - k_1')^2$ here $\hat{t}_{NEW} = (k_1 - k_1')^2 = (k_2 - p)^2 = \hat{u}_{old} = \hat{u}$

$\hat{u}_{old} = (k_1 - k_2')^2 = (k_2 - p)^2$ $\hat{u}_{New} = (k_1 - p)^2 = \hat{t}_{old} = \hat{t}$

$\Rightarrow$ the second graph squared is obtained by replacing

$$\hat{t} \leftrightarrow \hat{u}.$$

The final answer reads:

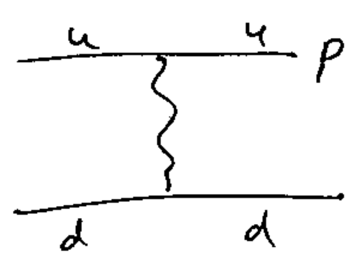
$$\boxed{E_p \frac{d\hat{\sigma}_{qq\to qq}}{d^3p} = \alpha_s^2 \frac{C_F}{N_c} \frac{1}{\hat{s}} \delta(\hat{s}+\hat{t}+\hat{u}) \left[\frac{\hat{s}^2+\hat{u}^2}{\hat{t}^2} + \frac{\hat{s}^2+\hat{t}^2}{\hat{u}^2} - \frac{2}{N_c} \frac{\hat{s}^2}{\hat{t}\,\hat{u}} \right]}$$

interference term we did not calculate. (see next page)

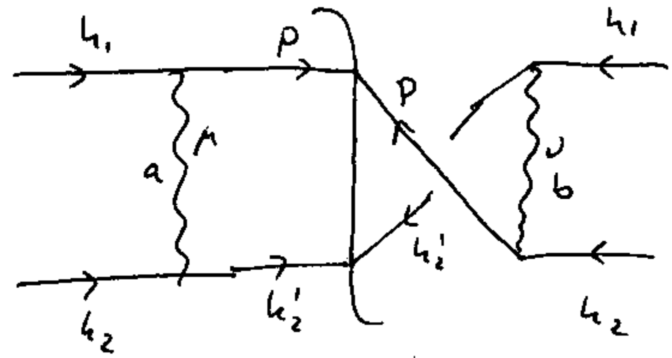
To find jet production x-section need to calculate hard cross section $\hat{\sigma}$ & convolute it with 2 PDF's.

If quarks are distinguishable => no crossing term, no interference =>

$$\boxed{E_p \frac{d\hat{\sigma}_{ud\to ud}}{d^3p} = \alpha_s^2 \frac{C_F}{N_c} \frac{1}{\hat{s}} \delta(\hat{s}+\hat{t}+\hat{u}) \frac{\hat{s}^2+\hat{u}^2}{\hat{t}^2}}$$



Interference term:



color factor:

$$\frac{1}{N_c^2}\, tr\left[T^a T^b T^a T^b\right] =$$

$$= \frac{1}{N_c^2}\left(-\frac{1}{2N_c}\right)\underbrace{tr\left[T^b T^b\right]}_{C_F N_c} = -\frac{1}{2}\,\frac{C_F}{N_c^2}.$$

crossing graph comes with a "−"!

$$\Rightarrow \underset{+c.c.}{2}\langle |M|^2\rangle = (-)\,2\,\underbrace{\frac{1}{(k_1-p)^2}}_{\hat t}\,\underbrace{\frac{1}{(k_2'-k_1)^2}}_{\hat u}\cdot\frac{1}{4}g^4\cdot\left(-\frac{1}{2}\right)\frac{C_F}{N_c^2}\cdot tr\left[\not p\gamma^\mu\not k_1\gamma^\nu\cdot\not k_2'\gamma^\mu\not k_2\gamma^\nu\right] =$$

| as $\gamma^\mu\gamma^\nu\gamma^\rho\gamma^\sigma\gamma_\mu = -2\gamma^\sigma\gamma^\rho\gamma^\nu$

$$= 2\,\frac{-1}{\hat t\hat u}\left(\frac{-g^4}{8}\right)\frac{C_F}{N_c^2}\cdot(-2)\; tr\left[\not p\,\underbrace{\gamma^\mu\not k_1\not k_2\gamma^\mu}_{4k_1\cdot k_2}\not k_2'\right] =$$

$$= 2\,\frac{-1}{\hat t\hat u}\,g^4\,\frac{C_F}{N_c^2}\cdot\underbrace{k_1\cdot k_2}_{\hat s/2}\;4\,\underbrace{p\cdot k_2'}_{\hat s/2} = -2\,\frac{1}{\hat t\hat u}\,g^4\,\frac{C_F}{N_c^2}\,\hat s^2$$

giving the missing factor:

$$E_p\frac{d\hat\sigma}{d^3p} = \frac{1}{4(2\pi)^2}\,\frac{1}{\hat s}\,\delta(\hat s+\hat t+\hat u)\cdot(-2)\,\frac{\hat s^2}{\hat t\hat u}\,\frac{C_F}{N_c^2}\,g^4 = -2\,\frac{\hat s^2}{\hat t\hat u}\cdot\alpha_s^2\cdot$$

$$\cdot\frac{C_F}{N_c^2}\cdot\frac{1}{\hat s}\,\delta(\hat s+\hat t+\hat u)$$ as desired.