$$\underline{\text{Cross section}} = \frac{\text{event prob. per unit volume \& time}}{(\text{incident flux})\times(\text{target density})}$$

$$\overset{=|\psi_f\rangle}{|\text{final state}\rangle} = \mathcal{S}\;\overset{=|\psi_i\rangle}{|\text{initial state}\rangle}$$

Def. ⟶ ↖ $\mathcal{S}$ - matrix (time-evolution operator)

$$|\psi_f\rangle = \mathcal{S}|\psi_i\rangle = \left[1 + (\mathcal{S}-1)\right]|\psi_i\rangle = |\psi_i\rangle + $$

$$+ (S-1)|\psi_i\rangle$$

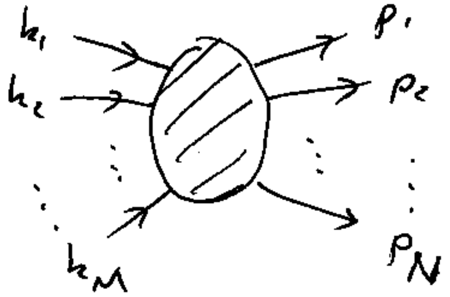
⇒ the interaction is in $S-1$ ⇒

Def. T-matrix is defined by $\boxed{\mathcal{S} = \mathbb{1} + i\,\mathbb{T}}$

Unitarity: $\mathcal{S}^+\mathcal{S} = 1 \Rightarrow 1 - iT^+ + iT + T^+T = 1$

$$\Rightarrow i(T-T^+) = -T^+T \Rightarrow \boxed{2\,\mathrm{Im}\,T = T^+T} \Rightarrow \text{optical theorem}$$

Def. <u>Scattering amplitude:</u>



⇒ initial state is $|k_1, k_2, \dots, k_M\rangle$

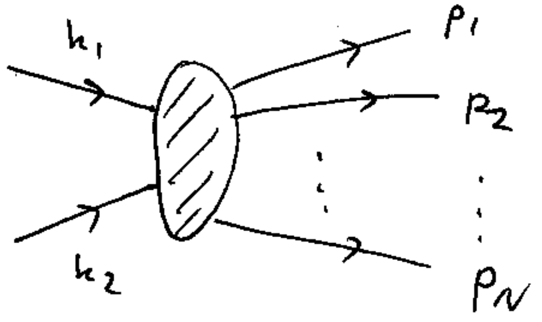
final (outgoing) free-particle state $|p_1, \dots, p_N\rangle$

⇒ transition amplitud $\mathcal{M}$ is defined by

$$\langle p_1,\dots,p_N|\mathcal{S}-1|k_1,\dots,k_M\rangle = i\langle p_1,\dots,p_N|T|k_1,\dots,k_M\rangle \equiv$$

$$\equiv (2\pi)^4\,\delta^{(4)}(p_1+\dots+p_N-k_1-\dots-k_M)\; i\,\mathcal{M}(k_1,\dots,k_M;\,p_1,\dots,p_N)$$

For simplicity consider $2 \to N$ process:



$\Rightarrow$ one can show that the cross section for the process is:

$$\boxed{\begin{aligned} d\sigma = \frac{1}{2\mathcal{E}_{k_1}\, 2\mathcal{E}_{k_2}\, |\vec{v}_1 - \vec{v}_2|} \prod_{i=1}^{N} \frac{d^3 p_i}{(2\pi)^3\, 2\mathcal{E}_i} \, |M(k_1, k_2; p_1, \ldots, p_N)|^2 \\ \cdot (2\pi)^4\, \delta^{(4)}\Big(k_1 + k_2 - \sum_{j=1}^{N} p_j\Big) \end{aligned}}$$

(see Peskin & Schroeder, Ryder, ...)

$\vec{v}_1, \vec{v}_2$ ~ 3-velocities of the incoming particles.

Is this object Lorentz-invariant?

$$\frac{d^3 p_i}{(2\pi)^3\, 2\mathcal{E}_i} = \int \frac{d^4 p_i}{(2\pi)^4}\, 2\pi\, \delta(p_i^2 - m_i^2)\, \theta(p_i^0) \sim \text{Lorentz-inv.}$$

$\delta^{(4)}$ is $\mathcal{L}$-inv. (why?)

$|M|^2$ is $\mathcal{L}$ inv. (see definition)

What about $\mathcal{E}_{k_1} \mathcal{E}_{k_2} |\vec{v}_1 - \vec{v}_2|$? Note that $v_i = \frac{\vec{k}_i}{\mathcal{E}_{k_i}}$

$$\Rightarrow \mathcal{E}_{k_1} \mathcal{E}_{k_2} |\vec{v}_1 - \vec{v}_2| = \mathcal{E}_{k_1} \mathcal{E}_{k_2} \left| \frac{\vec{k}_1}{\mathcal{E}_{k_1}} - \frac{\vec{k}_2}{\mathcal{E}_{k_2}} \right| = |\mathcal{E}_{k_2} \vec{k}_1 - \mathcal{E}_{k_1} \vec{k}_2|$$