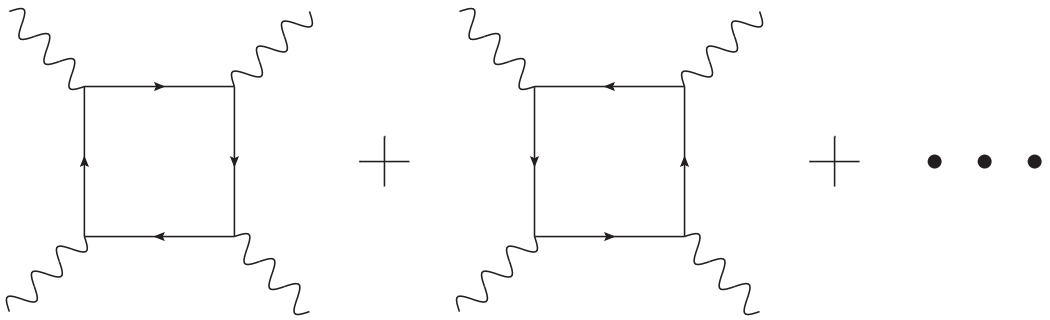
Homework Set No. 1, Physics 880.08

Deadline – Wednesday, April 13, 2011

1. (15 pts) Show that the light-by-light scattering QED diagram pictured below does not contain a UV divergence (even thought naively counting powers of momenta it seems like it is logarithmically divergent). The ellipsis in the figure represent all possible permutations of the vertices along the loop. (Hint: you may find formulas (A.41) and (A.42) in Appendix A.4 of Peskin and Schroeder useful.)



2. Using dimensional regularization find the one-loop beta-function

$$\beta(\lambda) \,=\, \mu^2 \,\frac{d\,\lambda}{d\,\mu^2}$$

of the real scalar φ^3 theory with the Lagrangian density

$$\mathcal{L} = \frac{1}{2}\,\partial_\mu\varphi_0\,\partial^\mu\varphi_0 - \frac{m_0^2}{2}\,\varphi_0^2 - \frac{\lambda_0}{3!}\,\varphi_0^3 \tag{1}$$

in **six** (!) space-time dimensions. Here φ_0 is the bare field, while λ_0 and m_0 are the bare coupling constant and the bare mass correspondingly.

a. (20 pts) Similar to how we did it in class for ϕ^4 theory (see also HW 4 from last quarter), rewrite the Lagrangian (1) in terms of renormalized physical fields φ, coupling λ, mass m and the counterterms. By calculating the one-loop propagator and vertex corrections find the divergent ($\sim 1/\epsilon$ with $\epsilon = 6 - d$) parts of the coefficients of the counterterms δ_λ and δ_Z. (Ignore the tadpole graph.) Note that we do not need the finite parts of the counterterms to find $\beta(\lambda)$!

b. (10 pts) Using the results of part **a** find $\beta(\lambda)$.