φ^4 theory:

$$\Gamma^4_{reg}(s,t,u) = -i\lambda\mu^\varepsilon\left[1 + \frac{\lambda}{32\pi^2}\int_0^1 dx\left[\ln\left(\frac{m^2 - x(1-x)s}{\mu^2}\right) - \left(\frac{2}{\varepsilon} - \gamma + \ln 4\pi\right) + (s\leftrightarrow t) + (s\leftrightarrow u)\right]\right].$$

$$\Gamma^4_{ren} = \Gamma^4_{reg} - i\delta_\lambda$$

$$\Rightarrow \delta_\lambda^{\overline{MS}} = \lambda\mu^\varepsilon\,\frac{\lambda}{32\pi^2}\,3\left(\frac{2}{\varepsilon} - \gamma + \ln 4\pi\right)$$

$$\Rightarrow \boxed{\Gamma^4_{ren} = -i\lambda\underset{\text{drop}}{\mu^\varepsilon}\left[1 + \frac{\lambda}{32\pi^2}\int_0^1 dx\,\ln\left(\frac{m^2 - x(1-x)s}{\mu^2}\right) + s\leftrightarrow u + s\leftrightarrow t\right]}.$$

Callan-Symanzik: (put $m=0$ for simplicity)

$$\left[\mu^2\frac{\partial}{\partial\mu^2} + \beta(\lambda_r)\frac{\partial}{\partial\lambda_r} - 4\gamma(\lambda_r)\right]\Gamma^4_{ren} = 0$$

$\gamma(\lambda) = \mu^2 \dfrac{d\ln\sqrt{Z}}{d\mu^2} = 0$ as $Z = 1$ at the one-loop order. C-S eqn. is satisfied if:

$$+i\frac{3\lambda^2}{32\pi^2} + \beta(\lambda)(-i) = 0 \Rightarrow \boxed{\beta_{\varphi^4}(\lambda) = \frac{3\lambda^2}{32\pi^2}}$$

same as before

Def: $\mu \sim$ renormalization scale.

On-shell-like renormalization: demand: $\Gamma^4(s=t=u=\mu^2) = -i\lambda$

$$\Rightarrow \Gamma^4_{ren} = -i\lambda\left[1 + \frac{\lambda}{32\pi^2}\int_0^1 dx\left(\ln\left[\frac{-x(1-x)s}{x(1-x)\mu^2}\right] + (s\leftrightarrow t) + (s\leftrightarrow u) + \text{finite}\right)\right]. \quad (m=0).$$

Again, using Callan-Symanzik equation get the same β-function.