Homework Set No. 7, Physics 8808.02

Deadline – Thursday, April 25, 2013

1. For the non-Abelian gauge theory with the Lagrangian

$$\mathcal{L} = -\frac{1}{4} F^a_{\mu\nu} F^{a\,\mu\nu}$$

and the field strength

$$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g\, f^{abc}\, A^b_\mu\, A^c_\nu$$

find the Feynman rules for the

a. (15 pts) 3-gluon vertex, and

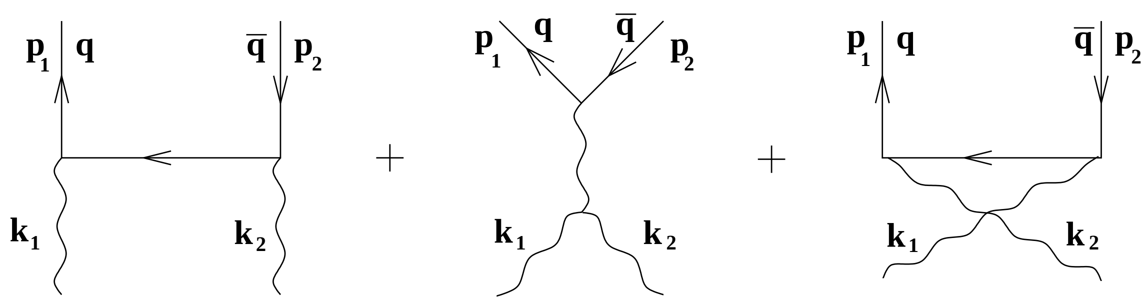
b. (15 pts) for the 4-gluon vertex,

by calculating 3- and 4-point coordinate-space Green functions correspondingly at the lowest non-trivial order using Wick contractions, and by truncating the propagators. (Your answers should agree with the rules given in class.)

2. a. (30 pts) Calculate the cross section for

$$\text{gluon} + \text{gluon} \to \text{quark} + \text{antiquark}$$

at the Born level shown in the figure below. The figure is for the *amplitude*, which needs to be squared and multiplied by appropriate factors to get the cross section.



You should find

$$\frac{d\sigma_{gg\to q\bar{q}}}{dt} = \frac{3\,\pi\,\alpha_s^2}{8\,s^2}\,(t^2+u^2)\left(\frac{4}{9\,t\,u} - \frac{1}{s^2}\right)$$

with the Mandelstam variables $s = (k_1 + k_2)^2$, $t = (k_1 - p_1)^2$, $u = (k_2 - p_1)^2$. (q and $\bar{q}$ in the figure denote the quark and the antiquark. Time flows upward.) Assume that quarks are massless.

b. (5 pts) Using the result of part **a** find the cross section $d\sigma_{q\bar{q}\to gg}/dt$ for the inverse process

$$\text{quark} + \text{antiquark} \to \text{gluon} + \text{gluon}.$$

Hints:

You may want to read Sterman pp. 233-237, where he works out the process from part **b** in much detail. If you choose to work in the $\partial_\mu A^\mu = 0$ Lorenz gauge, then following Sterman one may use the polarization sum

$$\sum_{\lambda=\pm 1} \epsilon_\mu^{*\,\lambda}(k)\, \epsilon_\nu^\lambda(k) \; = \; -g_{\mu\nu} + \frac{\bar{k}_\mu\, k_\nu + k_\mu\, \bar{k}_\nu}{k \cdot \bar{k}}, \tag{1}$$

where for $k^\mu = (k^0, \vec{k})$ we defined $\bar{k}^\mu = (k^0, -\vec{k})$. Using Eq. (1) after squaring the amplitudes one may simplify the resulting expression following Sterman's prescription.

The following γ-matrix formulas may be useful:

$$\gamma^\mu\, \gamma^\nu\, \gamma_\mu \; = \; -2\, \gamma^\nu$$

$$\gamma^\mu\, \gamma^\nu\, \gamma^\rho\, \gamma_\mu \; = \; 4\, g^{\nu\rho}$$

$$\gamma^\mu\, \gamma^\nu\, \gamma^\rho\, \gamma^\sigma\, \gamma_\mu \; = \; -2\, \gamma^\sigma\, \gamma^\rho\, \gamma^\nu$$

$$\mathrm{tr}[\gamma^\mu\, \gamma^\nu] \; = \; 4\, g^{\mu\nu}$$

$$\mathrm{tr}[\gamma^\mu\, \gamma^\nu\, \gamma^\rho\, \gamma^\sigma] \; = \; 4\, (g^{\mu\nu}\, g^{\rho\sigma} + g^{\mu\sigma}\, g^{\nu\rho} - g^{\mu\rho}\, g^{\nu\sigma}).$$

For color traces the following expressions may come in handy:

$$T^a\, T^a \; = \; C_F\, \mathbf{1}$$

with

$$C_F \; = \; \frac{N_c^2 - 1}{2\, N_c},$$

$$\mathrm{tr}[T^a T^b T^a T^b] \; = \; -\frac{C_F}{2}$$

$$f^{abc}\, f^{abd} \; = \; N_c\, \delta^{cd}.$$