Homework Set No. 6, Physics 8808.01

Deadline – Tuesday, November 20, 2012

1. Consider real scalar φ^4-theory described by the Lagrangian density

$$\mathcal{L} = \frac{1}{2}\,\partial_\mu\varphi\,\partial^\mu\varphi - \frac{m^2}{2}\,\varphi^2 - \frac{\lambda}{4!}\,\varphi^4.$$

a. (7 pts) Draw all connected Feynman diagrams contributing to the two-point function

$$\langle\psi_0|T\varphi(x)\varphi(y)|\psi_0\rangle$$

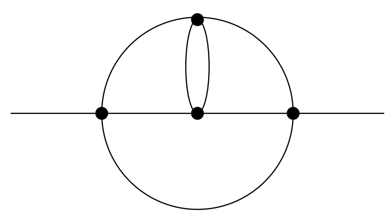
at the order λ^3. (Connected = no vacuum bubbles, no disjoint graphs.) Find the symmetry factors for all the graphs.

b. (10 pts) Draw all connected Feynman diagrams contributing to the four-point function

$$\langle\psi_0|T\varphi(x_1)\varphi(x_2)\varphi(x_3)\varphi(x_4)|\psi_0\rangle$$

up to the order λ^3. Calculate the symmetry factors. (Again, connected = no vacuum bubbles, no disjoint graphs.)

c. (3 pts) What is the symmetry factor of the following Feynman diagram?



2. Consider real scalar φ^3-theory described by the Lagrangian density

$$\mathcal{L} = \frac{1}{2}\,\partial_\mu\varphi\,\partial^\mu\varphi - \frac{m^2}{2}\,\varphi^2 - \frac{\lambda}{3!}\,\varphi^3.$$

a. (5 pts) Draw all connected Feynman diagrams contributing to the two-point function

$$\langle\psi_0|T\varphi(x)\varphi(y)|\psi_0\rangle$$

up to the order λ^4. Find the symmetry factors. (Connected = no vacuum bubbles, no disjoint graphs.)

b. (5 pts) Draw all connected Feynman diagrams contributing to the three-point function

$$\langle\psi_0|T\varphi(x_1)\varphi(x_2)\varphi(x_3)|\psi_0\rangle$$

up to the order λ^3. Calculate the symmetry factors. (Connected = no vacuum bubbles, no disjoint graphs.)

3. (15 pts) For a free real scalar field theory with mass m find the following correlator

$$G_4(x_1, x_2, x_3, x_4) \;=\; \langle 0|\mathrm{T}\{\phi(x_1)\,\phi(x_2)\,\phi(x_3)\,\phi(x_4)\}|0\rangle$$

where x_1, x_2, x_3, and x_4 are four different 4-vectors. Without using Wick's theorem, express the answer in terms of the Feynman propagators

$$\begin{aligned} D_F(x-y) \;&=\; \int \frac{d^4k}{(2\,\pi)^4}\, e^{-i\,k\cdot(x-y)}\, \frac{i}{k^2 - m^2 + i\,\epsilon} \\ &=\; \theta(x^0 - y^0)\, D(x-y) + \theta(y^0 - x^0)\, D(y-x) \end{aligned}$$

where

$$D(x-y) \;=\; \int \frac{d^3k}{(2\,\pi)^3\, 2\, E_k}\, e^{-i\,k\cdot(x-y)}.$$

(Hint: I found it easier to first consider the case when the time-ordering is, say, $x_1^0 > x_2^0 > x_3^0 > x_4^0$, obtain the answer in this case, and then generalize to other time-orderings by considering permutations of arguments.)