Last time: Correlators in Free Field Theory (cont'd)

Scalar Field

$$D(x-y) \equiv \langle 0|\varphi(x)\,\varphi(y)|0\rangle = \int \frac{d^4k}{(2\pi)^4}\, e^{-ik\cdot(x-y)}\,(2\pi)\,\delta^{(+)}(k^2-m^2)$$

Def. Time-ordered product:

$$T\,\varphi(x)\,\varphi(y) \equiv \theta(x^0-y^0)\,\varphi(x)\,\varphi(y) + \theta(y^0-x^0)\,\varphi(y)\,\varphi(x)$$

Def. Feynman propagator

$$D_F(x-y) \equiv \langle 0|\,T\,\varphi(x)\,\varphi(y)|0\rangle = \int \frac{d^4k}{(2\pi)^4}\, e^{-ik\cdot(x-y)}\,\frac{i}{k^2-m^2+i\varepsilon}$$

Def. Retarded Green function:

$$D_R(x-y) \equiv \theta(x^0-y^0)\,\langle 0|[\varphi(x),\varphi(y)]|0\rangle = \int \frac{d^4k}{(2\pi)^4}\, e^{-ik\cdot(x-y)}\,\frac{i}{k^2-m^2+i\varepsilon k^0}.$$

We transformed $D_R(x-y)$ into coordinate space (for $m=0$):

$$D_R(x-y)\Big|_{m=0} = \frac{-i}{2\pi}\,\theta(x^0-y^0)\,\delta\big((x-y)^2\big).$$

Note that D_R & D_F are Green functions of K-G eqn:

$$(\Box_x+m^2)\,D_{F,R}(x-y) = -i\,\delta^{(4)}(x-y).$$

$$x \xleftarrow{\;k\;} y$$

propagator notation

Dirac Field (cont'd)

Def. Feynman propagator:

$$\boxed{S_F(x-y) \equiv \langle 0 | T\, \psi(x) \bar{\psi}(y) | 0 \rangle}$$

$$\boxed{T\, \psi_\alpha(x) \bar{\psi}_\beta(y) \equiv \theta(x^0-y^0)\, \psi_\alpha(x) \bar{\psi}_\beta(y) - \theta(y^0-x^0) \cdot \bar{\psi}_\beta(y) \psi_\alpha(x).}$$

(time-ordered product)

Dirac Field

Feynman propagator is

$$\boxed{S_F(x-y) = \langle 0 | T\, \psi(x)\, \bar{\psi}(y) | 0 \rangle}$$

where time-ordering is

$$\boxed{T\, \psi_\alpha(x)\, \bar{\psi}_\beta(y) = \theta(x^0-y^0)\, \psi_\alpha(x)\, \bar{\psi}_\beta(y) - \theta(y^0-x^0)\, \bar{\psi}_\beta(y)\, \psi_\alpha(x).}$$

Plug in

$$\psi(x) = \int \frac{d^3k}{(2\pi)^3 2\varepsilon_k} \sum_{r=1}^{2} \left\{ \hat{b}_{\vec{k},r}\, u_r(\vec{k})\, e^{-ik\cdot x} + \hat{d}^+_{\vec{k},r}\, v_r(\vec{k})\, e^{ik\cdot x} \right\}$$

only $\hat{b}\hat{b}^+$ contribute

into $\langle 0 | \psi(x)_\alpha\, \bar{\psi}(y)_\beta | 0 \rangle = \int \frac{d^3k}{(2\pi)^3 2\varepsilon_k} \sum_{r=1}^{2} u_{r,\alpha}(\vec{k})\cdot$

spinor indices

$$\cdot\, \bar{u}_{r,\beta}(\vec{k})\, e^{-ik\cdot(x-y)} = \left(\text{as } \sum_r u_{r,\alpha}(\vec{k})\, \bar{u}_{r,\beta}(\vec{k}) = (\gamma\cdot k+m)_{\alpha\beta} \right) =$$

$$= \int \frac{d^3k}{(2\pi)^3 2\varepsilon_k} (\gamma\cdot k+m)_{\alpha\beta}\cdot e^{-ik\cdot(x-y)}$$

$\hat{d}\hat{d}^+$ contribute

Similarly $\langle 0 | \bar{\psi}(y)_\beta\, \psi_\alpha(x) | 0 \rangle = \int \frac{d^3k}{(2\pi)^3 2\varepsilon_k} \sum_{r=1}^{2} \bar{v}_{r,\beta}(\vec{k})\cdot$

$$\cdot\, v_{r,\alpha}(\vec{k})\, e^{ik(x-y)} = \int \frac{d^3k}{(2\pi)^3 2\varepsilon_k} (\gamma\cdot k-m)_{\alpha\beta}\, e^{ik\cdot(x-y)}$$

$$S_F(x-y) = \theta(x^0-y^0)\langle 0|\psi(x)\bar{\psi}(y)|0\rangle - \theta(y^0-x^0)\langle 0|\bar{\psi}(y)\psi(x)|0\rangle$$

$$S_F(x-y) = \int \frac{d^3k}{(2\pi)^3 2E_k}\left[\theta(x^0-y^0)(\gamma\cdot k+m)e^{-ik\cdot(x-y)} - \theta(y^0-x^0)e^{ik\cdot(x-y)}(\gamma\cdot k-m)\right]$$

$$= (\text{can show}) = \int \frac{d^4k}{(2\pi)^4} e^{-ik\cdot(x-y)} \frac{i(\gamma\cdot k+m)}{k^2-m^2+i\varepsilon}$$

$$\Rightarrow \quad \boxed{S_F(x-y) = \int \frac{d^4k}{(2\pi)^4} e^{-ik\cdot(x-y)} \frac{i(\gamma\cdot k+m)}{k^2-m^2+i\varepsilon}}$$

Sometimes will use shorthand notation

$$\boxed{\not{k} \equiv k\cdot\gamma}$$

Note that $\left[i\gamma^\mu\partial_\mu - m\right] S_F(x-y) = \int \frac{d^4k}{(2\pi)^4} e^{-ik\cdot(x-y)} \cdot i \cdot$

$$\cdot \frac{(\gamma\cdot k-m)(\gamma\cdot k+m)}{k^2-m^2+i\varepsilon} = i\int \frac{d^4k}{(2\pi)^4} e^{-ik\cdot(x-y)} = i\,\delta(x-y)$$

$$\boxed{\left[i\not{\partial}_x - m\right] S_F(x-y) = i\,\delta(x-y)}$$

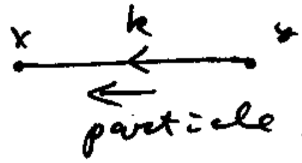
Also a Green function, this time of Dirac operator.

$\hat{b}^+ \sim$ create particles

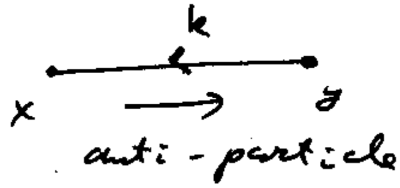
$\hat{d}^+ \sim$ —"— anti-particles

$\Rightarrow$ for $x^0 > y^0$ had only $\hat{b}\ \hat{b}^+ \Rightarrow$ particle

propagation from y to x :



for $x^0 < y^0 \Rightarrow \hat{d}\ \hat{d}^+ \Rightarrow$ anti-particle propagation from x to y :



(Note: momentum flow $\neq$ particle # flow.)

Gauge Fields

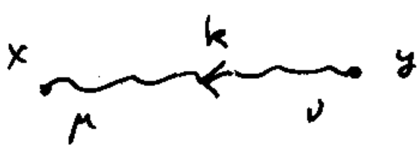
Will derive the expression later. Here just note that the propagator is (in $\partial_\mu A^\mu = 0$ gauge)

$$\boxed{\begin{aligned} D_{\mu\nu}(x-y) &\equiv \langle 0 | T A_\mu(x) A_\nu(y) | 0 \rangle = \\ &= \int \frac{d^4k}{(2\pi)^4}\, e^{-ik\cdot(x-y)} \frac{-i}{k^2+i\varepsilon}\left[g_{\mu\nu} - \left(1-\frac{1}{\lambda}\right)\frac{k_\mu k_\nu}{k^2}\right] \end{aligned}}$$

$\lambda = 1$ Feynman gauge

$\lambda = \infty$ Landau gauge

Feynman gauge: $\tilde{D}_{\mu\nu}(k) = \dfrac{-i}{k^2+i\varepsilon}\, g_{\mu\nu}$ simple



~ usually denoted by wiggly line