Interacting Fields and Feynman Diagrams

The interesting and physically relevant field theories have interaction terms:

Examples:

$$\mathcal{L}_{QED} = \underbrace{\bar{\psi}\left[i\not{\partial}-m\right]\psi}_{\text{free Dirac field}} - \underbrace{\frac{1}{4}F_{\mu\nu}F^{\mu\nu}}_{\text{free gauge field}} - \underbrace{\overset{\text{coupling constant}}{e}\,\bar{\psi}\gamma^{\mu}\psi A_{\mu}}_{\text{interaction term}}$$

$$\mathcal{L} = \frac{1}{2}\partial_{\mu}\varphi\,\partial^{\mu}\varphi - \frac{m^2}{2}\varphi^2 - \underbrace{\overset{\text{coupling}}{\frac{\lambda}{4!}}\varphi^4}_{\text{interaction}}$$

"phi to the fourth" theory

$$\mathcal{L} = \frac{1}{2}\partial_{\mu}\varphi\,\partial^{\mu}\varphi - \frac{m^2}{2}\varphi^2 - \underbrace{\frac{\lambda}{3!}\varphi^3}_{\text{interactions}}$$

"phi-cubed" theory

Take φ^4 theory:

$$\mathcal{L} = \frac{1}{2}\partial_{\mu}\varphi\,\partial^{\mu}\varphi - \frac{m^2}{2}\varphi^2 - \frac{\lambda}{4!}\varphi^4$$ & can do canonical quantization:

$$\left[\varphi(\vec{x},t), \pi(\vec{y},t)\right] = i\,\delta(\vec{x}-\vec{y})$$

$\Rightarrow$ EOM are

$$\left(\Box + m^2\right)\varphi = -\frac{\lambda}{3!}\varphi^3$$ $\Rightarrow$ can not decompose the field φ into creation & annihilation operators ($\hat{a}^{\dagger}, a$)...

$\Rightarrow$ can not calculate anything...