Quantum Field Theory II (Physics 880.08)

All the same as before, except:

Class meets: MW 10:30 a.m. – 12:18 p.m.

Smith 1180

website: http://www.physics.ohio-state.edu/~yuri/880-08-2.php

Brief Review of 1st Quarter

We considered various classical field theories for particles with spin $\emptyset$, $\frac{1}{2}$, 1, ...

Spin-$\emptyset$: $$\mathcal{L} = \frac{1}{2}\, \partial_\mu \varphi\, \partial^\mu \varphi - \frac{m^2}{2} \varphi^2 - U(\varphi)$$

$U(\varphi) = 0$ ~ free scalar field theory

$$\left.\begin{array}{l} U(\varphi) = \frac{\lambda}{3!}\varphi^3 \sim \varphi^3\text{-theory} \\ U(\varphi) = \frac{\lambda}{4!}\varphi^4 \sim \varphi^4\text{-theory} \end{array}\right\} \text{interacting theories.}$$

~ talked about symmetries & conservation laws for classical field theories

<u>Spin-1/2:</u> $\boxed{\mathcal{L} = \bar{\psi}\left[i\gamma^\mu \partial_\mu - m \right]\psi}$

free Dirac Lagrangian

$\psi = \begin{pmatrix} \psi_1 \\ \psi_2 \\ \psi_3 \\ \psi_4 \end{pmatrix}$ ~ Dirac spinor

γ^μ ~ Dirac matrices, $\gamma^0 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$, $\gamma^i = \begin{pmatrix} 0 & \sigma^i \\ -\sigma^i & 0 \end{pmatrix}$.

(Dirac representation)

<u>Spin-1:</u> $\boxed{\mathcal{L}_{gauge} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu}}$ free vector field

A_μ ~ vector field, $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$

Interactions: QED

$$\boxed{\mathcal{L}_{QED} = \bar{\psi}\left[i\gamma^\mu D_\mu - m \right]\psi - \frac{1}{4} F_{\mu\nu} F^{\mu\nu}}$$

$D_\mu = \partial_\mu + ieA_\mu$ ~ covariant derivative

We <u>quantized</u> free fields: scalar, spinor, vector:

e.g. scalar field: promote φ, $\pi = \frac{\delta \mathcal{L}}{\delta \dot{\varphi}}$ to operators with

$$\boxed{\begin{aligned} &[\varphi(\vec{x},t), \pi(\vec{y},t)] = i\,\delta^3(\vec{x}-\vec{y}) \\ &[\varphi(\vec{x},t), \varphi(\vec{y},t)] = [\pi(\vec{x},t), \pi(\vec{y},t)] = 0 \end{aligned}}$$

Postulating that the Hamiltonian H generates time evolution got $[\Box + m^2]\varphi = 0 \Rightarrow$ K-G equation for operator $\varphi \Rightarrow$ solved if

$$\varphi(\vec{x},t) = \int \frac{d^3k}{(2\pi)^3 2\varepsilon_k} \left[\hat{a}_{\vec{k}}\, e^{-ik\cdot x} + \hat{a}^+_{\vec{k}}\, e^{ik\cdot x} \right]$$

$$\Rightarrow \quad [\hat{a}_{\vec{k}}, \hat{a}^+_{\vec{k}'}] = (2\pi)^3\, 2\varepsilon_k\, \delta(\vec{k}-\vec{k}')$$

$$[\hat{a}_{\vec{k}}, \hat{a}_{\vec{k}'}] = [\hat{a}^+_{\vec{k}}, \hat{a}^+_{\vec{k}'}] = 0.$$

$$H = \int \frac{d^3k}{(2\pi)^3 2\varepsilon_k}\, \varepsilon_k\, \hat{a}^+_{\vec{k}}\, \hat{a}_{\vec{k}}$$

$|0\rangle$ ~ vacuum

$$\left.\begin{array}{l} \hat{a}^+_{\vec{k}}|0\rangle \sim \text{one-particle state} \\ \hat{a}^+_{\vec{k}_1}\hat{a}^+_{\vec{k}_2}|0\rangle \sim \text{2-particle state} \\ \vdots \qquad\qquad \vdots \end{array}\right\} \text{Fock states}$$

Similar canonical quantization can be carried out for vector fields (A_μ) & spinor fields (ψ), except need anti-commutators for ψ.

Correlators in Free Field Theories:

$$D(x-y) \equiv \langle 0|\varphi(x)\varphi(y)|0\rangle = \int \frac{d^4k}{(2\pi)^4} e^{-ik\cdot(x-y)} 2\pi\, \delta^{(+)}(k^2-m^2)$$

Time-ordered product:

$$T\varphi(x)\varphi(y) \equiv \theta(x^0-y^0)\varphi(x)\varphi(y) + \theta(y^0-x^0)\varphi(y)\varphi(x)$$

Feynman propagator:

$$\boxed{D_F(x-y) \equiv \langle 0|T\varphi(x)\varphi(y)|0\rangle = \int \frac{d^4k}{(2\pi)^4} e^{-ik\cdot(x-y)} \frac{i}{k^2-m^2+i\varepsilon}}.$$

$$(\Box + m^2) D_F(x-y) = -i\,\delta^4(x-y)$$

Dirac field:

$$\boxed{S_F(x-y) = \langle 0|T\psi(x)\bar{\psi}(y)|0\rangle = \int \frac{d^4k}{(2\pi)^4} e^{-ik\cdot(x-y)} \frac{i(\gamma\cdot k+m)}{k^2-m^2+i\varepsilon}}$$

Vector field: $(m=0)$

$$\boxed{D_{\mu\nu}(x-y) = \langle 0|TA_\mu(x)A_\nu(y)|0\rangle = \int \frac{d^4k}{(2\pi)^4} e^{-ik\cdot(x-y)} \frac{-i}{k^2+i\varepsilon} \cdot \left[g_{\mu\nu} + (\lambda-1)\frac{k_\mu k_\nu}{k^2}\right]}.$$

$\lambda = 1$ Feynman gauge

$\lambda = 0$ Landau gauge

Interacting Fields and Feynman Diagrams (cont'd)

Interaction Picture & Correlation Functions (cont'd)

consider φ^4 - theory:

$$\mathcal{L} = \frac{1}{2}\partial_\mu\varphi\,\partial^\mu\varphi - \frac{m^2}{2}\varphi^2 - \frac{\lambda}{4!}\varphi^4.$$

Work in the interaction picture: $H = H_0 + H_{int}$ (H,S)

$$\Rightarrow \boxed{\begin{aligned} \hat{O}_I(\vec{x},t) &= e^{iH_0(t-t_0)}\hat{O}_S(\vec{x})\,e^{-iH_0(t-t_0)} \\ |\psi(t)\rangle_I &= U(t,t')\,|\psi(t')\rangle_I \end{aligned}} \Rightarrow (\Box + m^2)\varphi_I = 0$$

like free field th'y

where $U(t,t_0) = e^{iH_0(t-t_0)}\,e^{-iH(t-t_0)}$

↖ Ham. in H. or S. pictures

$$\boxed{i\partial_t U(t,t') = H_I(t)\,U(t,t')}$$

with $H_I(t) = e^{iH_0(t-t_0)}\,H_{int}\,e^{-iH_0(t-t_0)}$.

$$\boxed{U(t,t') = T\exp\left\{-i\int_{t'}^{t} dt''\,H_I(t'')\right\}}$$

~ unitary time-evolution operator.

$U(t_1,t_2)\,U^\dagger(t_1,t_2) = \mathbb{1}$

$U^\dagger(t_1,t_2) = U(t_2,t_1)$

$$H_I(t) = \int d^3x\,\frac{\lambda}{4!}\varphi_I^4 \sim \text{for } \varphi^4 \text{ theory.}$$

What about correlation functions in interacting theory?

We showed that

$$\langle \Psi_0 | T\, \varphi_H(x)\, \varphi_H(y) | \Psi_0 \rangle = \frac{\langle 0 | T\left\{ \varphi_I(x)\, \varphi_I(y)\, e^{-i\int_{-\infty}^{\infty} dt\, H_I(t)} \right\} | 0 \rangle}{\langle 0 | T\, e^{-i\int_{-\infty}^{\infty} dt\, H_I(t)} | 0 \rangle}$$

and that this is true in general:

$$\langle \Psi_0 | T\left\{ \varphi_H(x_1) \ldots \varphi_H(x_n) \right\} | \Psi_0 \rangle = \frac{\langle 0 | T\left\{ \varphi_I(x_1) \ldots \varphi_I(x_n)\, e^{-i\int_{-\infty}^{\infty} dt\, H_I(t)} \right\} | 0 \rangle}{\langle 0 | T\, e^{-i\int_{-\infty}^{\infty} dt\, H_I(t)} | 0 \rangle}$$

Gell-Mann-Low f-la

$|\Psi_0\rangle$ ~ vacuum of interacting th'y

$|0\rangle$ ~ vacuum of free th'y (perturbative vacuum).

(Note that $|\Psi_0\rangle \neq |0\rangle$ in general.)

Wick's Theorem

Def. Normal ordering ~ move all $\hat{a}^+$ left of all $\hat{a}$.

$$:\hat{a}_{\vec{k}}\,\hat{a}^+_{\vec{p}}: = \hat{a}^+_{\vec{p}}\,\hat{a}_{\vec{k}}$$

Def. Contraction $\quad \wick{\c\varphi(x)\c\varphi(y)} = T\varphi(x)\varphi(y) - :\varphi(x)\varphi(y):$

Note that $\wick{\c\varphi(x)\c\varphi(y)} = D_F(x-y) = \langle 0|T\varphi(x)\varphi(y)|0\rangle$

~ contraction is propagator

Wick's th'm main consequence:

$$\langle 0|T\varphi(x_1)\varphi(x_2)\ldots\varphi(x_n)|0\rangle = \wick{\c\varphi_1\c\varphi_2}\;\wick{\c\varphi_3\c\varphi_4}\;\ldots\;\wick{\c\varphi_{n-1}\c\varphi_n}$$

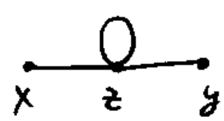
+ other perm's (even n only)

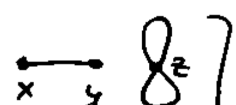
Feynman Rules for φ^4-theory

Using Gell-Mann–Low f-la and Wick's theorem we can evaluate correlators order-by-order in λ:

$$\langle\Psi_0|T\varphi_H(x)\varphi_H(y)|\Psi_0\rangle = \frac{\langle 0|T\varphi_I(x)\varphi_I(y)e^{-i\frac{\lambda}{4!}\int d^4z\,\varphi_I^4(z)}|0\rangle}{\langle 0|Te^{-i\frac{\lambda}{4!}\int d^4z'\,\varphi_I^4(z')}|0\rangle}$$

Numerator $= \langle 0|T\varphi(x)\varphi(y)|0\rangle - i\frac{\lambda}{4!}\int d^4z\,\langle 0|T\varphi(x)\varphi(y)\varphi^4(z)|0\rangle + \ldots =$ 

$- i\lambda\Big[\frac{1}{2}$ 

$+\frac{1}{8}$ 

$\Big]$

$+ O(\lambda^2)$

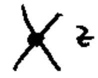
connected   vacuum bubble

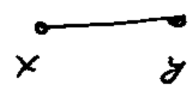
$\Rightarrow$ ditto for denominator

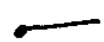
One can show that

$$\boxed{\langle \Psi_0 | T\, \varphi_H(x_1)\, \varphi_H(x_2) \ldots \varphi_H(x_n) | \Psi_0 \rangle = \langle 0 | T\, \varphi(x_1)\, \varphi(x_2) \ldots \varphi(x_n) \cdot e^{-i \frac{\lambda}{4!} \int d^4z\, \varphi^4(z)} | 0 \rangle_{\text{connected}}}$$

<u>Feynman rules for φ^4 theory</u> (Green fcn's, coord. space)

1. Draw all connected diagrams.
2. Each vertex gives $-i\lambda \int d^4z$ = 

3. Each propagator gives $D_F(x-y)$ = 

4. Include symmetry factors.
5. Each external vertex gives  $= 1$.