$$\langle 0 | T\{ \varphi_I(x_1)\, \varphi_I(x_2) \cdots \varphi_I(x_n) \} | 0 \rangle$$

in general.

Wick's Theorem.

The principle is simple: expand φ_I into $\hat{a}$ & $\hat{a}^+$ & calculate. Let us devise a general procedure:

$$\varphi_I(x) = \underbrace{\int \frac{d^3k}{(2\pi)^3 2\omega_k} \left[\hat{a}_{\vec{k}}\, e^{-ik\cdot x}\right.}_{\varphi_I^+} + \underbrace{\left.\hat{a}^+_{\vec{k}}\, e^{ik\cdot x} \right]}_{\varphi_I^-} \equiv \varphi_I^+(x) + \varphi_I^-(x).$$

Note that $\varphi_I^+ |0\rangle = 0$, $\langle 0 | \varphi_I^- = 0$.

(Def.) Normal-ordering: put all $\hat{a}$'s to the right of all $\hat{a}^+$'s. Denoted by $: \ldots :$.

Example: $:\varphi_I(x)\,\varphi_I(y): \; = \; :\left(\varphi_I^+(x) + \varphi_I^-(x)\right)\left(\varphi_I^+(y) + \varphi_I^-(y)\right):$

$$= :\left[\varphi_I^+(x)\varphi_I^+(y) + \varphi_I^-(x)\,\varphi_I^+(y) + \varphi_I^+(x)\varphi_I^-(y) + \varphi_I^-(x)\varphi_I^-(y) \right]:$$

$$= \varphi_I^+(x)\,\varphi_I^+(y) + \varphi_I^-(x)\,\varphi_I^+(y) + \underbrace{\varphi_I^-(y)\,\varphi_I^+(x)}_{\text{changed order!}} + \varphi_I^-(x)\varphi_I^-(y)$$

(dropped the commutator)

Note that $\langle 0| : \varphi_I(x_1) \dots \varphi_I(x_n) : |0\rangle = 0$. (VEV=0)

Example: $:\hat{a}_{\vec{k}_1}\hat{a}^+_{\vec{k}_2}: \; = \hat{a}^+_{\vec{k}_2}\,\hat{a}_{\vec{k}_1}$

$$:\hat{a}_{\vec{k}_1}\,\hat{a}^+_{\vec{k}_2}\,\hat{a}_{\vec{k}_3}: \; = \hat{a}^+_{\vec{k}_2}\,\hat{a}_{\vec{k}_1}\,\hat{a}_{\vec{k}_3}$$

Def. Contraction (or Wick contraction) of two fields:

$$\boxed{\overbracket{\varphi(x)\,\varphi(y)} \equiv T\,\varphi(x)\,\varphi(y) - \,:\varphi(x)\,\varphi(y):}$$

=> can see that $\langle 0|\overbracket{\varphi(x)\,\varphi(y)}|0\rangle = \langle 0|T\,\varphi(x)\,\varphi(y)|0\rangle$

as $\langle 0|:\varphi(x)\,\varphi(0):|0\rangle = 0$.

=> contraction is the propagator!

$$T\,\varphi_I(x)\,\varphi_I(y) = \theta(x^0-y^0)\,\varphi_I(x)\,\varphi_I(y) + \theta(y^0-x^0)\,\varphi_I(y)\,\varphi_I(x).$$

$$\Rightarrow T\,\varphi_I(x)\,\varphi_I(y) - \,:\varphi(x)\,\varphi(y): \; = \theta(x^0-y^0)\left[\varphi_I^+(x)+\varphi_I^-(x)\right]\cdot$$

$$\cdot\left[\varphi_I^+(y)+\varphi_I^-(y)\right] + \theta(y^0-x^0)\left[\varphi_I^+(y)+\varphi_I^-(y)\right]\left[\varphi_I^+(x)+\varphi_I^-(x)\right]$$

$$-\left(\varphi_I^+(x)\,\varphi_I^+(y) + \varphi_I^-(x)\,\varphi_I^+(y) + \varphi_I^-(y)\,\varphi_I^+(x) + \varphi_I^-(x)\,\varphi_I^-(y)\right) =$$

$$= \theta(x^0-y^0)\cdot\left[\varphi_I^+(x),\, \varphi_I^-(y)\right] + \theta(y^0-x^0)\left[\varphi_I^+(y),\varphi_I^-(x)\right]$$