<u>Last time</u>: Proved that the denominator in Gell-Mann-Low f-la

$$\langle \Psi_0 | T\, \varphi_H(x_1) \dots \varphi_H(x_n) | \Psi_0 \rangle = \frac{\langle 0 | T\, \varphi_I(x_1) \dots \varphi_I(x_n)\, e^{-i\int dt\, H_I(t)} | 0 \rangle}{\langle 0 | T\, e^{-i\int dt\, H_I(t)} | 0 \rangle}$$

cancels disconnected diagrams like  leaving only connected terms:

$$\langle \Psi_0 | T\, \varphi_H(x_1) \dots \varphi_H(x_n) | \Psi_0 \rangle = \langle 0 | T\, \varphi_I(x_1) \dots \varphi_I(x_n)\, e^{+i\int d^4z\, \mathcal{L}_I} | 0 \rangle_{conn}$$

Introduced rules for calculating symmetry factors:

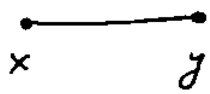
$$\frac{1}{S} = \frac{1}{S_1}\,\frac{1}{S_2} \qquad \left(\text{for } \frac{\lambda}{p!}\varphi^p\text{-theory}\right)$$

$S_1 = \left(\begin{matrix}\text{2 for each line which begins} \\ \text{\& ends at the same vertex}\end{matrix}\right) \otimes \left(\begin{matrix}m! \text{ for each } m \\ \text{identical lines}\end{matrix}\right)$

S_2 = # of exchanges of vertices that do not destroy time-ordering in the graph.

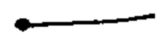
Formulated Feynman rules for n-point functions in coordinate space. Started formulating Feynman rules in momentum space.

Feynman rules for φ^4-theory in coordinate space:
(for correlation functions)

① Each propagator gives 

$= D_F(x-y)$

② Each vertex gives  $= -i\lambda \int d^4 z$

③ Each external point  $= 1$.

④ Divide by symmetry factors.

⑤ Keep connected diagrams only.

Often it is important to find observables in momentum space. It is also easier to calculate Feynman diagrams in momentum space.

(Def.) n-point "Green function":

Take $G(x_1, x_2, \dots, x_n) = \langle \Psi_0 | T\{\varphi(x_1)\varphi(x_2)\dots\varphi(x_n)\} | \Psi_0 \rangle$.

In momentum space write:

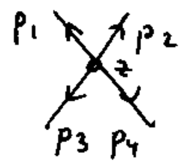
$$\tilde{G}(p_1, p_2, \dots, p_n) = \int d^4x_1\, d^4x_2 \dots d^4x_n\, e^{ip_1\cdot x_1 + ip_2\cdot x_2 + \dots + ip_n\cdot x_n}$$

$$G(x_1, x_2, \dots, x_n)$$ is the "Green function" in momentum space.

$\Rightarrow$ Each propagator $D_F(x-y) = \int \frac{d^4k}{(2\pi)^4} \frac{i}{k^2-m^2+i\varepsilon} e^{-ik\cdot(x-y)}$

gives $\frac{i}{k^2-m^2+i\varepsilon}$ in momentum space.

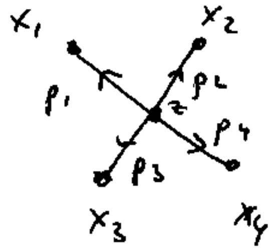
$\Rightarrow$ Each vertex gives: $-i\lambda \int d^4z\, e^{ip_1\cdot z + ip_2 z + ip_3 z + ip_4 z} =$



$= -i\lambda (2\pi)^4 \delta^{(4)}(p_1+p_2+p_3+p_4)$

~ conservation of energy & momentum.

<u>Example</u>



$= \int d^4x_1\, d^4x_2\, d^4x_3\, d^4x_4\, e^{ip_1x_1+ip_2x_2+ip_3x_3+ip_4x_4} \cdot (-i\lambda)\int d^4z\, D_F(x_1-z)\, D_F(x_2-z)\, D_F(x_3-z) \cdot$

$\cdot D_F(x_4-z) = \left| \tilde{x}_i = x_i - z \right. = \int d^4\tilde{x}_1\, d^4\tilde{x}_2\, d^4\tilde{x}_3\, d^4\tilde{x}_4 \cdot$

$\cdot e^{ip_1\tilde{x}_1+ip_2\tilde{x}_2+ip_3\tilde{x}_3+ip_4\tilde{x}_4} D_F(\tilde{x}_1)\, D_F(\tilde{x}_2)\, D_F(\tilde{x}_3)\, D_F(\tilde{x}_4)$

$(-i\lambda)\int d^4z\, e^{ip_1z_1+ip_2z_2+ip_3z_3+ip_4z_4} = \frac{i}{p_1^2-m^2+i\varepsilon} \cdot$

$\cdot \frac{i}{p_2^2-m^2+i\varepsilon} \frac{i}{p_3^2-m^2+i\varepsilon} \frac{i}{p_4^2-m^2+i\varepsilon} (-i\lambda) \underbrace{(2\pi)^4 \delta^{(4)}(p_1+p_2+p_3+p_4)}$.

overall factor of energy-momentum conservation (usually dropped) ~ see later

Feynman rules for φ^4 theory in momentum space

(for correlation fn's)

① Each propagator gives  $= \dfrac{i}{k^2 - m^2 + i\varepsilon}$

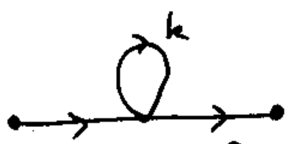
② Each vertex gives  $= -i\lambda$

③ Make sure 4-momentum is conserved at each vertex. Integrate over each indepent (internal) momentum with the measure $\dfrac{d^4k}{(2\pi)^4}$.

④ Divide by symmetry factors.

⑤ Keep connected diagrams only.

Example



$= -i\lambda \cdot \frac{1}{2} \cdot \left(\frac{i}{p^2 - m^2 + i\varepsilon}\right)^2 \cdot \int \frac{d^4k}{(2\pi)^4} \cdot \frac{i}{k^2 - m^2 + i\varepsilon}$.

↑ symmetry factor (the $\frac{1}{2}$)

r immediately see that the integral is divergent

⇒ diagram is ∞ ⇒ have to address this later.

OK, we can calculate correlators, but what does it give us in terms of physics?