Regularization & Renormalization

Field - Strength Renormalization: the Electron Self - Energy.

When deriving LSZ formula we claimed that "dressing" particle propagator leads to (for scalars):

$$\int d^4x\, e^{ip\cdot x} \langle \Psi_0 | T\, \varphi_H(x)\, \varphi_H(0) | \Psi_0 \rangle = Z \frac{i}{p^2 - m_{phys}^2 + i\varepsilon} +$$

+ multiparticle contributions.

It appears that one can write

$$\varphi_H(x) \simeq \sqrt{Z}\; \varphi_{free}(x)$$

with some free field $\varphi_{free}(x)$ with mass m_{phys}.

Def. Z is called <u>field-strength renormalization</u>.

For Dirac fields $\psi(x)$ one has:

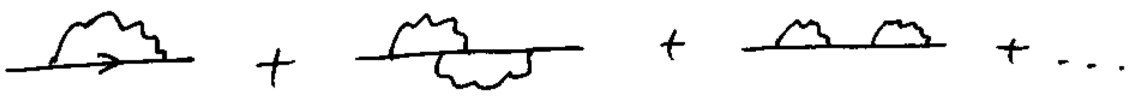
$$\int d^4x\, e^{ip\cdot x} \langle \Psi_0 | T\, \psi(x) \bar{\psi}(0) | \Psi_0 \rangle = Z_2 \frac{i(\not{p} + m_{phys})}{p^2 - m_{phys}^2 + i\varepsilon} + \text{multi-particle contrib's}$$

Z_2 ~ new notation.

Let us see how Z_2 & m_{phys} arise.

Consider electron propagator: $\xrightarrow[p]{}$.

If we dress it we need to sum graphs like:

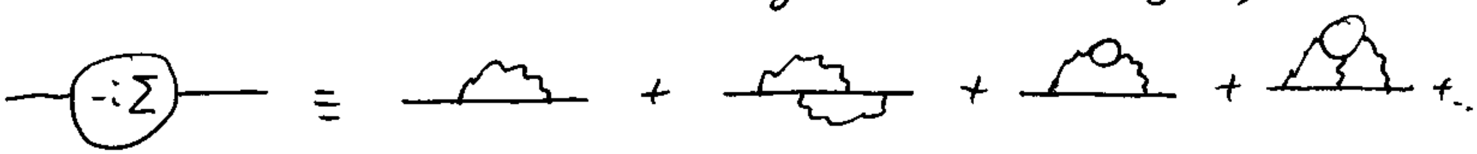


Def. <u>One-particle irreducible</u> (1PI) diagram: any diagram that can not be split into two by removing a single line.

<u>Examples</u>  ~ 1PI graphs

 ~ reducible.

<u>Define</u> $-i\Sigma(p) \equiv$ sum of all 1PI diagrams (excluding external legs)



We then write the full resummed electron propagator as:

$$\boxed{S'(p) \equiv \int d^4x \, e^{ip\cdot x} \langle \Psi_0 | T \, \psi(x) \bar{\psi}(0) | \Psi_0 \rangle}$$

$$\Rightarrow S'(p) = \frac{i}{\not{p} - m_0} + \frac{i}{\not{p} - m_0} \left(-i\Sigma(p)\right) \frac{i}{\not{p} - m_0} + \frac{i}{\not{p} - m_0} (-i\Sigma) \cdot$$

$$\cdot \frac{i}{\not{p} - m_0} (-i\Sigma) \frac{i}{\not{p} - m_0} + \ldots$$ where m_0 is the bare electron mass.

(diagrammatic representation)

$$S(p) = \frac{i}{\not p - m_0} \cdot \left[1 + \Sigma \frac{1}{\not p - m_0} + \Sigma \frac{1}{\not p - m_0} \Sigma \frac{1}{\not p - m_0} + \dots \right]$$

$$= \frac{i}{\not p - m_0} \cdot \left[1 - \Sigma \frac{1}{\not p - m_0} \right]^{-1} = \frac{i}{\not p - m_0} \left[(\not p - m_0 - \Sigma) \cdot \right.$$

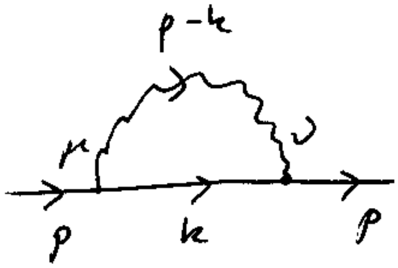
$$\left. \cdot \frac{1}{\not p - m_0} \right]^{-1} = \underset{\substack{\uparrow \\ \text{cancel}}}{\frac{i}{\not p - m_0}} \underbrace{\left(\frac{1}{\not p - m_0} \right)^{-1}}_{\not p - m_0} (\not p - m_0 - \Sigma)^{-1} =$$

$$= i \, (\not p - m_0 - \Sigma)^{-1} = \frac{i}{\not p - m_0 - \Sigma(p)}$$

$\Rightarrow$ the full dressed propagator is

$$\boxed{S(p) = \frac{i}{\not p - m_0 - \Sigma(p)}}$$

Let's calculate $\Sigma(p)$ in perturbation theory: at the lowest order we have



$$\equiv \frac{i}{\not p - m_0} \Big(-i \underset{\substack{\uparrow \\ \text{order-}e^2}}{\Sigma_2(p)} \Big) \frac{i}{\not p - m_0}$$

The diagram is

$$\int \frac{i}{\not p - m_0} (-ie\gamma^\nu) \frac{i(\not k + m_0)}{k^2 - m_0^2 + i\varepsilon} (-ie\gamma^\mu) \frac{i}{\not p - m_0} \frac{d^4k}{(2\pi)^4} \cdot \frac{-ig_{\mu\nu}}{(p-k)^2 + i\varepsilon}$$

need $i\varepsilon$ to integrate.

$$\Rightarrow -i\Sigma_2(p) = -e^2 \int \frac{d^4k}{(2\pi)^4} \gamma^\mu \frac{\not k + m_0}{k^2 - m_0^2 + i\varepsilon} \gamma_\mu \frac{1}{(p-k)^2 + i\varepsilon}.$$

$$= -e^2 \int \frac{d^4k}{(2\pi)^4} \frac{-2\not k + 4m_0}{k^2 - m_0^2 + i\varepsilon} \frac{1}{(p-k)^2 + i\varepsilon}.$$

$\Rightarrow$ Note that the answer has the structure: $f_1(p^2)\not p + f_2(p_2)$.

$\Rightarrow$ use <u>Feynman parameters</u>:

$$\frac{1}{A_1 A_2 \ldots A_n} = \int_0^1 dx_1 \ldots dx_n \, \delta\left(\sum_{i=1}^n x_i - 1\right) \frac{(n-1)!}{[x_1 A_1 + x_2 A_2 + \ldots + x_n A_n]^n}$$

induction.

which for $n = 2$ reduces to

check

$$\frac{1}{AB} = \int_0^1 dx\, dy\, \delta(1-x-y) \frac{1}{[xA + yB]^2} = \int_0^1 dx \frac{1}{[xA + (1-x)B]^2}$$

In our case $B = k^2 - m_0^2 + i\varepsilon$, $A = (p-k)^2 + i\varepsilon$

$$\Rightarrow -i\Sigma_2 = -e^2 \int \frac{d^4k}{(2\pi)^4} \cdot (-2\not k + 4m_0) \int_0^1 dx \frac{1}{[x[(p-k)^2 + i\varepsilon] + (1-x)[k^2 - m_0^2 + i\varepsilon]]^2} = -e^2 \int \frac{d^4k}{(2\pi)^4} (-2\not k + 4m_0) \cdot \int_0^1 dx \,.$$