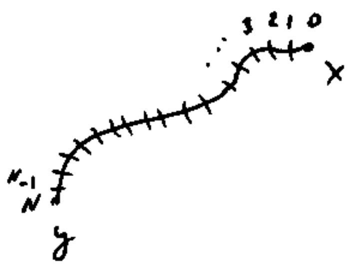
Last time | Wilson lines, loops & Heavy Quark Potential

Def. $$W_c(x,y) \equiv P_c \exp\left\{ ig \int_y^x dx'_\mu A^\mu(x') \right\}$$

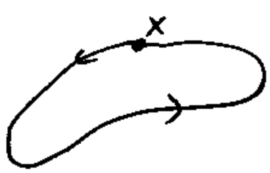


$$= \prod_{i=1}^{N} \left[\mathbb{1} + ig \Delta x_i^\mu A_\mu(x_i) \right]$$

$$x_0^\mu = x^\mu, \quad x_N^\mu = y^\mu.$$

Under gauge transformations $W_c(x,y) \to S(x) W_c(x,y) S^{-1}(y)$

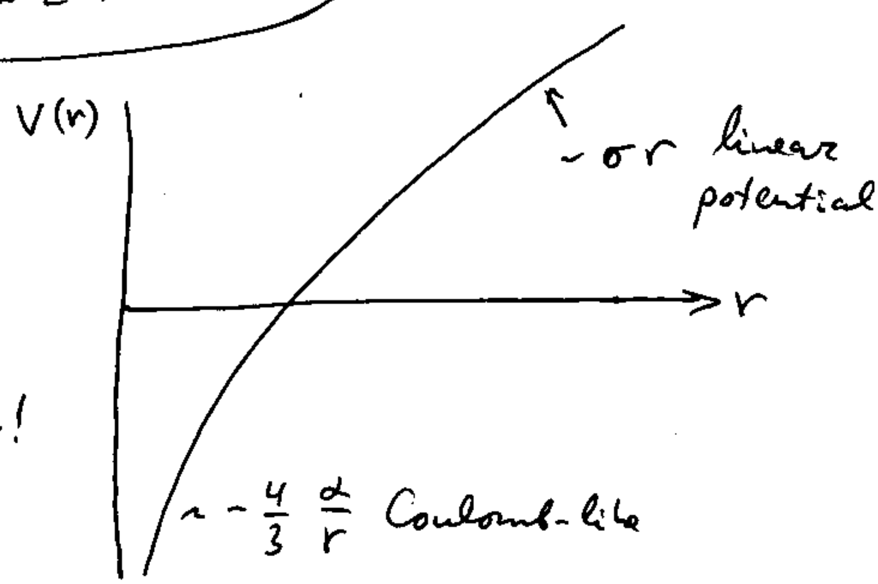
$\Rightarrow$ $\mathrm{tr}\left[W_c(x,x) \right]$ is gauge-invariant



↖ Wilson loop

Heavy Quark-Antiquark Potential: $\overset{\bullet}{Q} \leftarrow r \rightarrow \overset{\bullet}{\bar{Q}}$

$$V(r) = \lim_{T\to\infty} \left[\frac{i}{T} \ln \langle W \rangle \right]$$



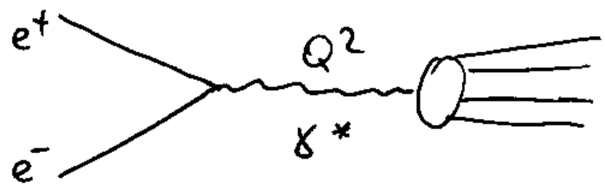
$V(r)\big|_{r \gg \frac{1}{\Lambda_{QCD}}} \simeq \sigma r, \quad \sigma \simeq \Lambda_{QCD}^2$

~ linear confining potential!

The Cross Section for $e^+e^- \to$ hadrons.

$\Rightarrow$ consider e^+e^- annihilation:

$$e^+e^- \to \begin{pmatrix}\text{virtual}\\ \text{photon}\end{pmatrix} \to \text{hadrons}$$



Define the ratio $R(Q^2) = \dfrac{\sigma_{e^+e^- \to \text{hadrons}}}{\sigma_{e^+e^- \to \mu^+\mu^-}}$

$R(Q^2)$ is dimensionless $\Rightarrow R = R\left(\frac{Q^2}{\mu^2}, \alpha_\mu\right)$

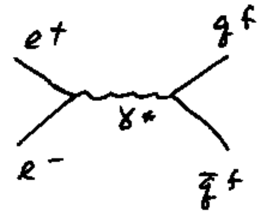
if $m_f = 0. \Rightarrow R = R\left(\frac{Q^2}{\mu^2}, \alpha_\mu\right) = (\text{put } \mu = Q) =$

$= R(1, \alpha(Q^2)) = R(\alpha(Q^2))$ ~ function of r.c. only

$\Rightarrow$ write a perturbative expansion for it:

$$R(\alpha(Q^2)) = R(0) + R_1\,\alpha(Q^2) + R_2\,\alpha^2(Q^2) + \ldots$$

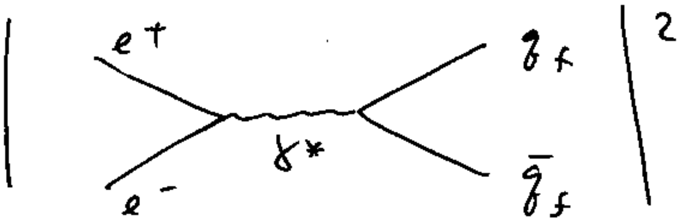
$R(0)$ is easy to get: put $\alpha(Q^2) = 0$.

- $\sigma_{e^+e^- \to \text{hadrons}} \propto \left| \right.$  $\left. \right|^2 = \left| \right.$  $\left. \right|^2$

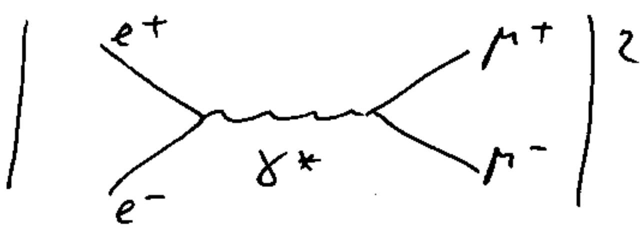
+ higher order QCD corrections

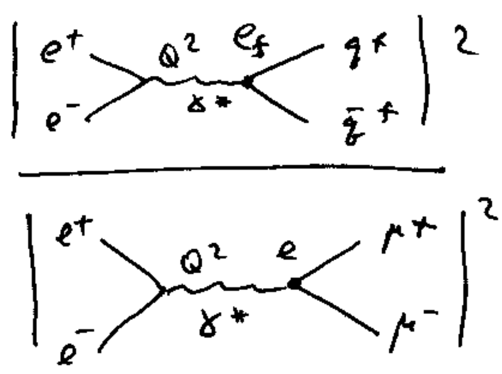
$\Rightarrow$ if $\alpha_s = 0$ $\Rightarrow$ drop higher order corrections

$\Rightarrow$ $\sigma_{e^+e^- \to \text{hadrons}} \simeq \sigma_{e^+e^- \to \text{quarks}} \propto$

$\propto$ 

On the other hand, with high precision

$\sigma_{e^+e^- \to \mu^+\mu^-} \propto$ 

$\Rightarrow R(0) =$ 

$= 3 \sum_f e_f^2$

neglect q & μ masses.

↑ # of quark colors

Where to terminate the sum over flavors depends on Q^2: if $Q^2 < 4m_c^2$ $\Rightarrow$ $Q < 2m_c \simeq 3\,\text{GeV}$

$\Rightarrow$ need only u, d, s (3 flavors)

$$\Rightarrow R(Q < 2m_c,\ Q > 2m_s) = 3\Big(\underbrace{\left(\tfrac{2}{3}\right)^2}_{u} + \underbrace{\left(\tfrac{1}{3}\right)^2}_{d} + \underbrace{\left(\tfrac{1}{3}\right)^2}_{s}\Big) =$$

$$= 3\left(\frac{4}{9} + \frac{1}{9} + \frac{1}{9}\right) = 2$$

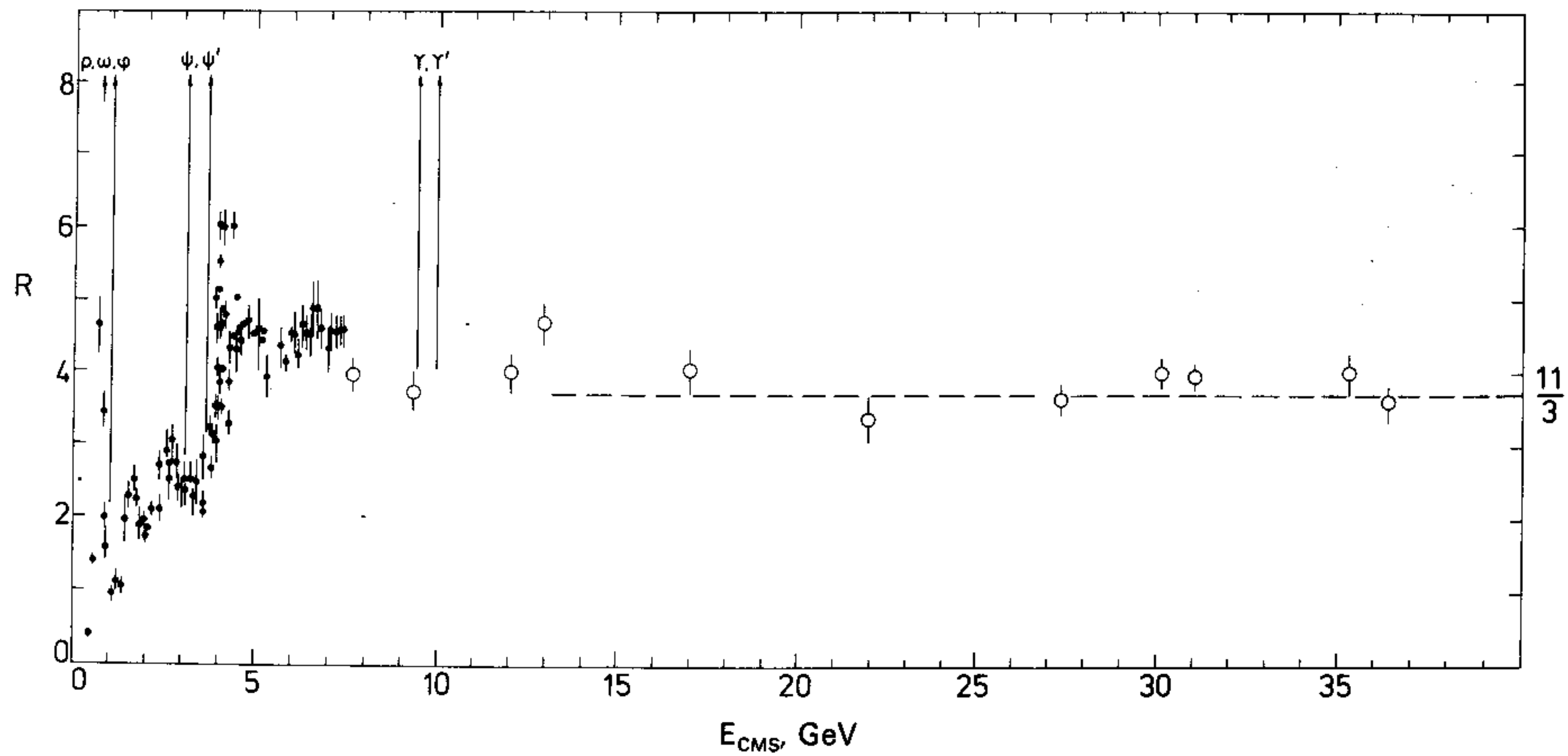


Figure 8.3 The ratio R of the cross-section for $e^+e^- \to$ hadrons, divided by that for $e^+e^- \to \mu^+\mu^-$. The fact that R is constant above 10-GeV CMS energy is a proof of the pointlike nature of hadron constituents. The predicted value of R, assuming that the primary process is formation of a quark-antiquark pair, is $\frac{11}{3}$ if pairs of u, d, s, c, b quarks are excited and they have three color degrees of freedom. The data come from many storage-ring experiments. At high energy (> 10 GeV CMS) it is from the PETRA ring at DESY, Hamburg.

take $Q > 2m_b \simeq 8.5\,\mathrm{GeV}$ $\Rightarrow$ e.g. $Q \simeq 80\,\mathrm{GeV}$

$$\Rightarrow R = 3\left(\left(\frac{2}{3}\right)^2 + \left(\frac{1}{3}\right)^2 + \left(\frac{1}{3}\right)^2 + \left(\frac{2}{3}\right)^2 + \left(\frac{1}{3}\right)^2\right) = \frac{11}{3}$$

$$\quad\ u \qquad\quad d \qquad\quad s \qquad\quad c \qquad\quad b$$

$\Rightarrow$ amazingly close to data (see attachement)

$\Rightarrow$ if one includes higher order corrections

get $$R(\alpha(Q^2)) = 3 \sum e_f^2 \left\{ 1 + \frac{\alpha(Q^2)}{\pi} + (1.986 - 0.115 N_f) \cdot \left(\frac{\alpha}{\pi}\right)^2 + \ldots \right\}$$

$\Rightarrow$ in reality quarks become hadrons, which is a non-perturbative process ...

$\Rightarrow$ $e^+e^- \to$ hadrons gives direct evidence for quarks as fermions with 3 colors and fractional electric charges