High Energy QCD Journal Club: starting 4/1/2013

Christopher Plumberg - Notes

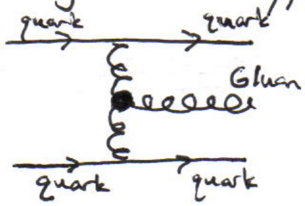
k_T-factorization, particle production in pA-collisions and the KLN model

Introduction:

Consider gluon production at lowest order, in quark-quark collisions. We have already seen that the scattering cross section for this process can be written

$$\sigma_{qq\to qqG} = \frac{2\alpha_s^3 C_F}{\pi^2}\int \frac{d^2k_\perp\, d^2q_\perp}{k_\perp^2 q_\perp^2 (\vec{k}_\perp-\vec{q}_\perp)^2}\int_0^Y dy \qquad (1)$$

Diagrammatically, this process (the amplitude) looks like this:



$\vec{q}_\perp$: transverse momentum of one of virtual gluons entering the Lipatov vertex in the diagram

$\vec{k}_\perp$: transverse momentum of outgoing gluon in diagram

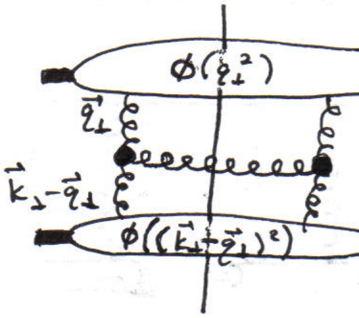
The differential cross section is then, by this result,

$$\frac{d\sigma}{d^2k_T\, dy} = \frac{2\alpha_s^3 C_F}{\pi^2}\frac{1}{k_T^2}\int \frac{d^2q_\perp}{q_\perp^2(\vec{k}_\perp-\vec{q}_\perp)^2} \qquad (k_T^2 = \vec{k}_\perp^2) \qquad (2)$$

We have seen in previous lectures that we may write the unintegrated gluon distribution for a nucleus as $\phi_{LO} \simeq \frac{A\alpha_s C_F}{\pi}\frac{1}{k_T^2}$; setting $A=1$, we can write (2) as

$$\frac{d\sigma}{d^2k_T\, dy} = \frac{2\alpha_s}{C_F}\frac{1}{k_T^2}\int d^2q_\perp\, \phi_{LO}(q_T^2)\,\phi_{LO}((\vec{k}_\perp-\vec{q}_\perp)^2), \qquad (3)$$

which we visualize as



$\rightarrow$ This means the process factorizes into two independent gluon distributions which only interact through a Lipatov vertex. This factorization persists for onium-onium scattering.

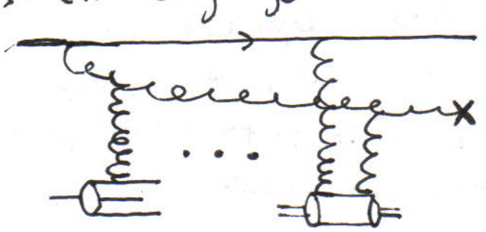
If we integrate (3) over $d^2k_\perp$ and $d^2q_\perp$, we find (after introducing some IR-cutoffs Λ to regulate the integrals) that $d\sigma/dy \sim \frac{1}{\Lambda^2}\ln\Lambda$

$\rightarrow$ We therefore have an IR-singularity! However, we have already encountered this sort of problem before, and the standard prescription for its treatment is by now familiar: we must incorporate the long-distance effects of saturation physics which will serve to screen out the IR-singularity, so that the complications of non-pQCD become irrelevant (or, at least, less relevant than they would otherwise have been). This is our next task.

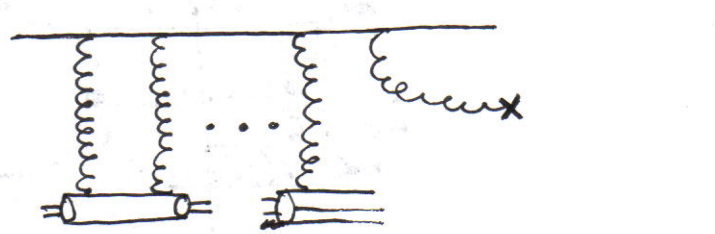
Gluon production in quark-nucleus scattering

Recall that the saturation scale $\sim A^{1/3}$, so $Q_{sp} \ll Q_{sA}$, for a large nucleus N. In this case, for gluon production with $k_T \gg Q_{sp}$, we can neglect multiple rescatterings with the proton while keeping multiple rescatterings to all orders with the nucleus.

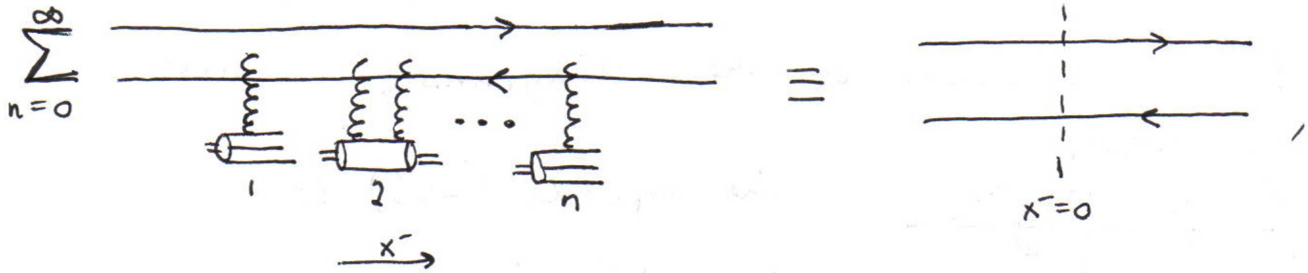
This sounds like a lot of work, but fortunately things simplify considerably when we recall that any diagrams with the measured final-state gluon emitted during the quark-nucleus interactions are suppressed by additional powers of s; we therefore need only consider gluons emitted before or after the quark-nucleus interactions. Since these latter interactions may be either elastic or inelastic, we have two interesting LCPT diagrams ($A^-=0$ gauge):



and



Since we are only considering gluon production before or after the quark–nucleus interactions, we may condense our diagrammatic notation slightly:



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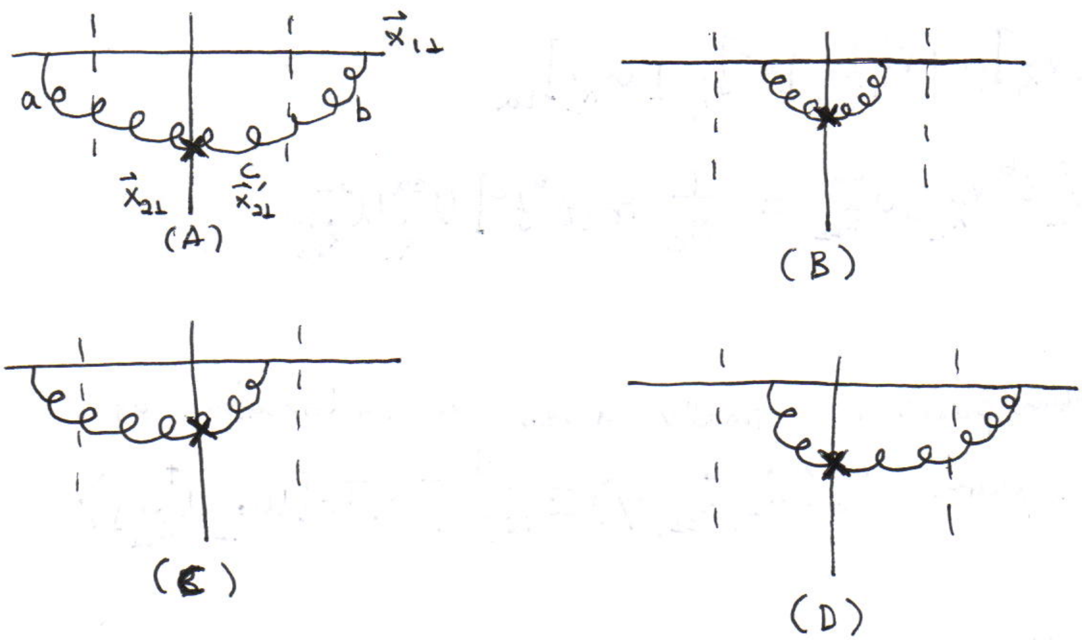
where the sum is over any number of rescatterings off of individual nucleons, elastically or inelastically. In this new notation, the differential cross section is dominated by two "quadratic" terms and two "cross-terms":



"quadratic" terms

"cross" terms

* Note that B ↔ D in the book's notation

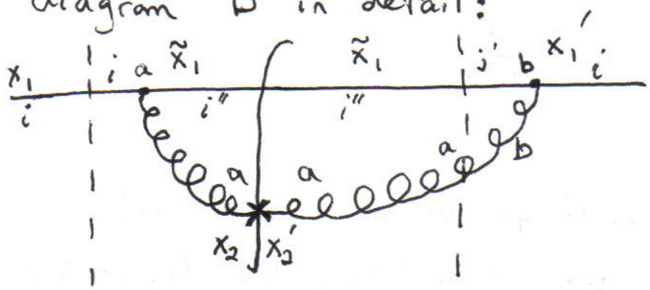
One can argue that, having calculated these diagrams, the result may be used to find $\frac{d\sigma}{d^2k_T\,dy}$ in the following way:

$$\frac{d\sigma}{d^2k_T\,dy} = \frac{1}{2(2\pi)^3}\int d^2x_2\, d^2x_{2'}\, d^2x_1\, e^{-i\vec{k}_\perp\cdot\vec{x}_{22'}} \langle A(\vec{x}_{23},\vec{x}_{13})\, A^*(\vec{x}_{2'3},\vec{x}_{13})\rangle, \qquad (4)$$

where the $\langle \ldots \rangle$ indicates an average* of the total amplitude squared over all positions of the nucleon. We must calculate these diagrams in order to compute (4).

* average $\equiv \int d^2x_3\, T(\vec{x}_{3\perp})$

The soft gluon emissions in these diagrams bring in factors of $\frac{i g t^a}{\pi} \frac{\vec{\epsilon}_\perp^{\,\lambda *} \cdot \vec{x}_{21}}{x_{21}^2}$ (with $a \to a'$ and $a \to b$ in the c.c. amplitude). To understand the color structure of these diagrams, consider diagram D in detail:



The amplitude (LHS) is

$M \sim [t^a]_{i''' i} [V_{\tilde{x}_1}]_{i' i'''}$, while the RHS is

$$M^* \sim \left([V_{\tilde{x}_1}]_{i''' j'} [t^b]_{j' i} [U_{x_2'}]_{ab}\right)^*$$

$$\sim [V^\dagger_{\tilde{x}_1}]_{j' i'''} [t^b]_{i j'} [U_{x_2'}]_{ba}$$

$$\Rightarrow |M|^2 \sim [t^a]_{i''' i} [V_{\tilde{x}_1}]_{i' i'''} [V^\dagger_{\tilde{x}_1}]_{j' i'''} [t^b]_{i j'} [U_{x_2'}]_{ba}$$

$$\Rightarrow \frac{1}{N_c} \mathrm{tr}\left[t^a V^\dagger_{\vec{x}_{1\perp}} t^b V_{\vec{x}_{1\perp}}\right] U^{ba}_{\vec{x}_{2\perp}'} = \frac{1}{N_c} \mathrm{tr}[t^b t^c]\, U^{ca}_{\vec{x}_{1\perp}} U^{ba}_{\vec{x}_{2\perp}'}$$

average over colors

$\rightarrow$ averaging over nucleon positions finally gives a contribution of $C_F S_G(\vec{x}_{1\perp}, \vec{x}_{2\perp}', Y)$, where $S_G(\vec{x}_{1\perp}, \vec{x}_{2\perp}', Y) \equiv \frac{1}{N_c^2 - 1} \left\langle \mathrm{Tr}\left[U_{\vec{x}_{1\perp}} U^\dagger_{\vec{x}_{2\perp}'}\right] \right\rangle$

In the end, when we have properly accounted for minus signs amongst the various diagrams and summed over gluon polarizations, we find that

$$\frac{d\sigma^{qA}}{d^2k_T\,dy} = \frac{1}{(2\pi)^2}\int d^2x_2\, d^2x_{2'}\, d^2x_1\, e^{-i\vec{k}_1\cdot\vec{x}_{22'}}\, \frac{\alpha_s C_F}{\pi^2}\left(\frac{\vec{x}_{21}\cdot\vec{x}_{2'1}}{x_{21}^2\, x_{2'1}^2}\right)$$

$$\cdot\left[S_G(\vec{x}_{2\perp},\vec{x}_{2'\perp},Y) - S_G(\vec{x}_{1\perp},\vec{x}_{2'\perp},Y) - S_G(\vec{x}_{2\perp},\vec{x}_{1\perp},Y) + 1\right],$$

where S_G is the S-matrix element for a gluon dipole. If we define $N_G(\ ,\ ,\ ) = 1 - S_G(\ ,\ ,\ )$, then this expression becomes

$$\frac{d\sigma^{qA}}{d^2k_T\,dy} = \frac{1}{(2\pi)^2}\int d^2x_2\, d^2x_{2'}\, d^2x_1\, e^{-i\vec{k}_1\cdot\vec{x}_{22'}}\, \frac{\alpha_s C_F}{\pi^2}\left(\frac{\vec{x}_{21}\cdot\vec{x}_{2'1}}{x_{21}^2\, x_{2'1}^2}\right)$$

$$\cdot\left[N_G(\vec{x}_{1\perp},\vec{x}_{2'\perp},Y) - N_G(\vec{x}_{2\perp},\vec{x}_{2'\perp},Y) + N_G(\vec{x}_{2\perp},\vec{x}_{1\perp},Y)\right]$$

Integrate in the first term over $d^2x_{2\perp}$, in the second term over $d^2x_{1\perp}$ and in the third term over $d^2x_{2'\perp}$ to get

$$\frac{d\sigma^{qA}}{d^2k_T\,dy} = \frac{\alpha_s C_F}{2\pi^3}\int d^2x_2\, d^2x_{2'}\, N_G(\vec{x}_{2\perp},\vec{x}_{2'\perp},Y)\, e^{-i\vec{k}_\perp\cdot\vec{x}_{22'}}\left(2i\,\frac{\vec{k}_\perp}{k_\perp^2}\cdot\frac{\vec{x}_{22'}}{x_{22'}^2} - \ln\frac{1}{\Lambda\, x_{22'}}\right),$$

using $\int d^2q_\perp\, e^{i\vec{q}_\perp\cdot\vec{x}_\perp}\,\frac{\vec{q}_\perp}{q_\perp^2} = 2\pi i\,\frac{\vec{x}_\perp}{x_\perp^2}$, $\int d^2y_\perp\,\frac{\vec{y}_\perp\cdot(\vec{y}_\perp+\vec{x}_\perp)}{y_\perp^2(\vec{y}_\perp+\vec{x}_\perp)^2} = 2\pi\ln\frac{1}{x_\perp\Lambda}$

and relabeling coordinate indices appropriately (Λ is some IR cutoff). This expression does not diverge at $\vec{x}_{2\perp} = \vec{x}_{2'\perp}$, since $N_G = 0$ here (a zero-size dipole does not interact), so we can rewrite this as

$$\frac{d\sigma^{qA}}{d^2k_T\,dy} = \frac{\alpha_s C_F}{2\pi^3}\int d^2x_2\, d^2x_{2'}\, N_G(\vec{x}_{2\perp},\vec{x}_{2'\perp},Y)\, \nabla^2_{\vec{x}_{22'}}\left(e^{-i\vec{k}_\perp\cdot\vec{x}_{22'}}\ln\frac{1}{\Lambda\, x_{22'}}\right)$$

In the same way that we have rewritten a portion of this expression in terms of the gluon dipole-nucleus forward scattering amplitude, we can write the rest of it in terms of the forward scattering amplitude for the incident quark-gluon dipole (n_G^q) to get, after some algebra and integration by parts,

$$\frac{d\sigma^{qA}}{d^2k_T\,dy} = \frac{C_F}{\alpha_s\pi(2\pi)^3}\frac{1}{k_T^2}\int d^2B_\perp\, d^2b_\perp\, d^2x_\perp \left[\nabla^2_{\vec{x}_\perp} n^q_G(\vec{x}_\perp, \vec{B}_\perp - \vec{b}_\perp, 0)\right] \cdot e^{-i\vec{k}_\perp\cdot\vec{x}_\perp}\left[\nabla^2_{\vec{x}_\perp} N_G(\vec{x}_\perp, \vec{b}, Y)\right],$$

where $\vec{x}_\perp \equiv \vec{x}_{22'}$ and $\vec{b}_\perp$ and $\vec{B}_\perp$ are the transverse impact parameters of the produced gluon and the incident quark, respectively. Finally, if we define the unintegrated gluon distributions

$$\phi_A(k_T^2) = \frac{C_F}{\alpha_s(2\pi)^3}\int d^2b_\perp\, d^2x_\perp\, e^{-i\vec{k}_\perp\cdot\vec{x}_\perp}\, \nabla^2_{\vec{x}_\perp} N_G(\vec{x}_\perp, \vec{b}_\perp, 0)$$

and

$$\phi_p(k_T^2) = \frac{C_F}{\alpha_s(2\pi)^3}\int d^2b_\perp\, d^2x_\perp\, e^{-i\vec{k}_\perp\cdot\vec{x}_\perp}\, \nabla^2_{\vec{x}_\perp} n_G(\vec{x}_\perp, \vec{b}_\perp, 0) \qquad (5)$$

for the nucleus and proton (quark), respectively, then we may write

$$\frac{d\sigma^{pA}}{d^2k_T\,dy} = \frac{2\alpha_s}{C_F}\frac{1}{k_T^2}\int d^2q_\perp\, \phi_p(q_T^2)\, \Phi_A((\vec{k}_\perp - \vec{q}_\perp)^2) \qquad (6)$$

$\longrightarrow$ k_T-factorized form persists!! This is unexpected, since we are used to seeing this happen when we can write the relevant diagrams as distribution functions interacting only through a Lipatov vertex (squared); it is not clear whether a gauge exists in which the diagrams (A)-(D) can be drawn in such a separated form. For this reason, a complete understanding of the origin of this factorization is still lacking.

We can also show

$$\left.\frac{d\sigma^{pA}}{d^2k_T\,dy}\right|_{k_T \gg Q_{sG}} \approx \frac{8\alpha_s^3 C_F A}{\pi}\frac{1}{k_T^4}\ln\frac{k_T}{\Lambda}$$

$$\left.\frac{d\sigma^{pA}}{d^2k_T\,dy}\right|_{k_T \ll Q_{sG}} \approx \frac{\alpha_s C_F S_\perp}{\pi^2}\frac{1}{k_T^2} \qquad (S_\perp\text{: transverse area of nucleus})$$

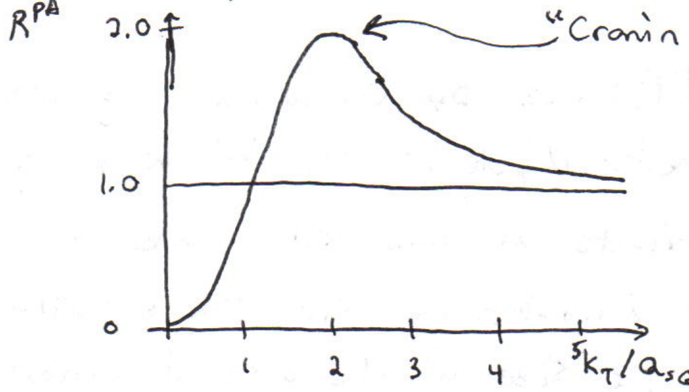
Notice that, for large k_T ($\gg Q_{sG}$), the coherence length is small compared to the size of the nucleus, so the produced gluons are only weakly screened by the local color charge density. However, for small k_T ($\ll Q_{sG}$), the gluons are strongly screened and saturation physics works well, softening the steep IR divergence we saw earlier to $\frac{d\sigma^{pA}}{dy} \sim \ln\left(\frac{Q_{sG}}{\Lambda}\right)$.

To help visualize these results, we define the ratio

$$R^{pA}(k_T, y) = \frac{d\sigma^{pA}/d^2k_T\,dy}{A\,d\sigma^{pp}/d^2k_T\,dy},$$

known as the <u>nuclear modification factor</u>.

Deviations of this quantity from unity represent collective nuclear effects due to saturation effects in the collision. A plot of this quantity against k_T/Q_{sG} looks like



This is reasonable: low-k_T gluons are strongly screened (or "shadowed") by the nucleus, leading to an enhancement (the "Cronin peak" or "Cronin effect") at $k_T \sim Q_{sG}$. This effect has been confirmed for hadron production in pA collisions.

We have demonstrated that k_T-factorization persists in pA-scattering when we include saturation physics while working in the quasi-classical MV/GGM approximation, but we want to include nonlinear LLA small-x effects in our results as well. We do this by accounting for the differences in rapidity between the target nucleus, the projectile (quark, dipole, proton, etc.) and the measured outgoing gluon via the JIMWLK/BK evolution equation. We take the nucleus to have rapidity 0, the projectile rapidity Y, and the emitted gluon rapidity y, with $Y \gg y \gg 0$. We need to consider nonlinear evolution effects from 0 to y (target to gluon) and then y to Y (gluon to projectile).

1 - target to gluon:

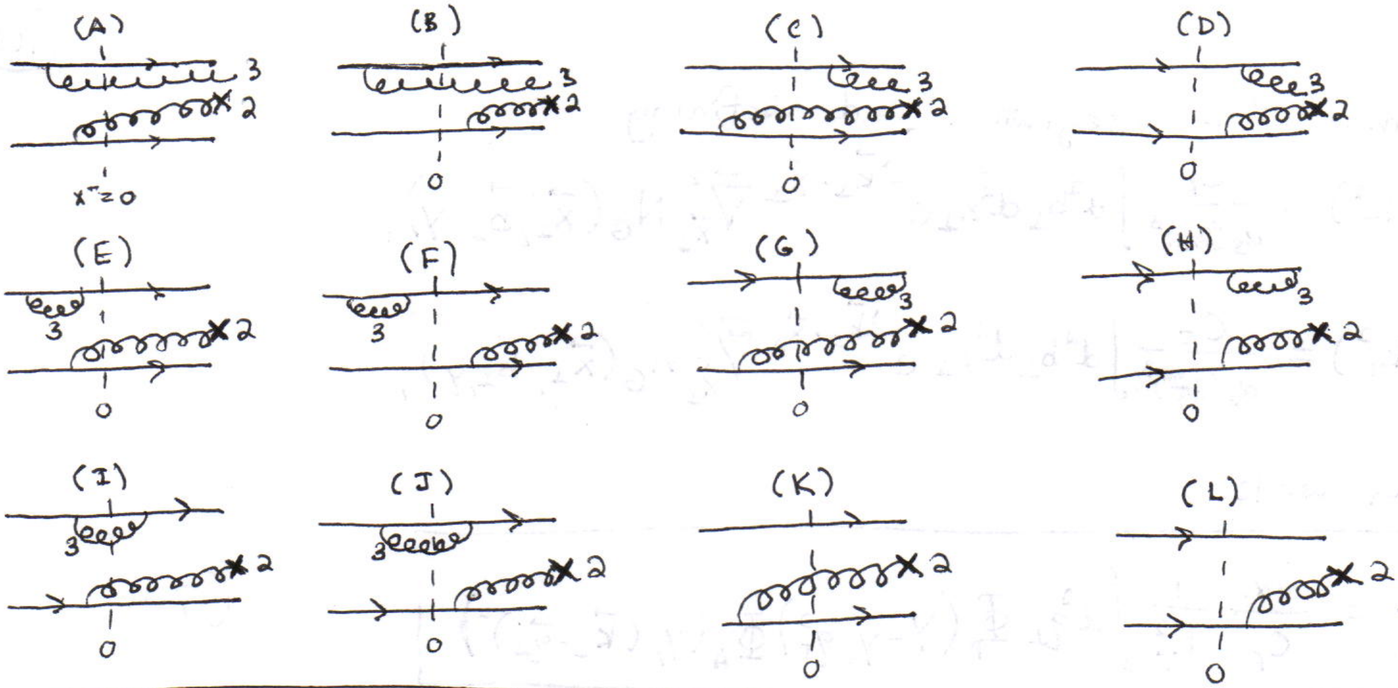
we have actually already taken care of this before by allowing the gluon to have some rapidity $y \neq 0$. To include nonlinear effects, we only have to realize that both GGM multiple rescatterings <u>and</u> these effects contribute to the forward scattering N_G. As $N_c \to \infty$, one can show that

$$N_G(\vec{x}_\perp, \vec{b}_\perp, y) = 2N(\vec{x}_\perp, \vec{b}_\perp, y) - N^2(\vec{x}_\perp, \vec{b}_\perp, y),$$

where N is the quark dipole amplitude.

2 - gluon to projectile:

we can account for the rapidity difference by considering sequential steps of small-x evolution in the Mueller dipole model. We work in the LLA, and the diagrams which contribute at this order are shown on the next page. Recall, from the derivation of the BK equation using the Mueller dipole model, that the size of the overall contribution of a given diagram depends on the ordering of the momenta in



that diagram. Here, one can show that, in order to generate an LLA contribution, one must have the harder gluons emitted first if $x^- < 0$, while the harder gluons must be emitted later for $x^- > 0$. In these diagrams, $z_3 \gg z_2$, i.e., gluon 3 is harder than gluon 2.

Each step in small-x evolution generates a new gluon dipole, so multiple steps will generate a whole distribution of such dipoles. We can formally combine all of these steps into an integral over the differential cross section, weighted by this distribution function, i.e.,:

$$\frac{d\sigma^{S}(\vec{x}_{10})}{d^2k_T\, dy\, d^2B_\perp} \longrightarrow \int d^2b_\perp\, d^2x_{1'0'}\, n_1(\vec{x}_{10}, \vec{x}_{1'0'}, \vec{B}_\perp - \vec{b}_\perp, Y-y) \times \frac{d\sigma^{S}(\vec{x}_{1'0'})}{d^2k_T\, dy\, d^2b_\perp}$$

Here, we consider $S \to q\bar{q}A$, but the form is similar for $S \to qA$ and $S \to pA$.

Performing this integration and defining

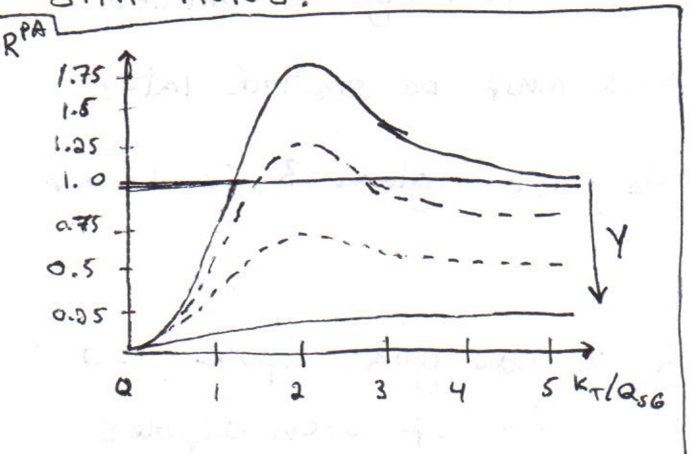
$$\Phi_A(y, k_T^2) = \frac{C_F}{\alpha_s (2\pi)^3} \int d^2 b_\perp \, d^2 x_\perp \, e^{-i\vec{k}_\perp \cdot \vec{x}_\perp} \, \vec{\nabla}^2_{\vec{x}_\perp} N_G(\vec{x}_\perp, \vec{b}_\perp, y),$$

$$\Phi_p(y, k_T^2) = \frac{C_F}{\alpha_s (2\pi)^3} \int d^2 b_\perp \, d^2 x_\perp \, e^{-i\vec{k}_\perp \cdot \vec{x}_\perp} \, \vec{\nabla}^2_{\vec{x}_\perp} n_G(\vec{x}_\perp, \vec{b}_\perp, y),$$

we can write

$$\boxed{\frac{d\sigma^{pA}}{d^2k_T\,dy} = \frac{2\alpha_s}{C_F} \frac{1}{k_T^2} \int d^2 q_\perp \, \Phi_p(Y-y, q_T^2) \, \Phi_A(y, (\vec{k}_\perp - \vec{q}_\perp)^2)} \qquad (7)$$

→ having included the LLA corrections, the k_T-factorization still holds. We now see that, for increasing rapidity, the small-x evolution has the effect of suppressing the nuclear modification factor, indicating drastic modifications from the quasi-classical approach used above.

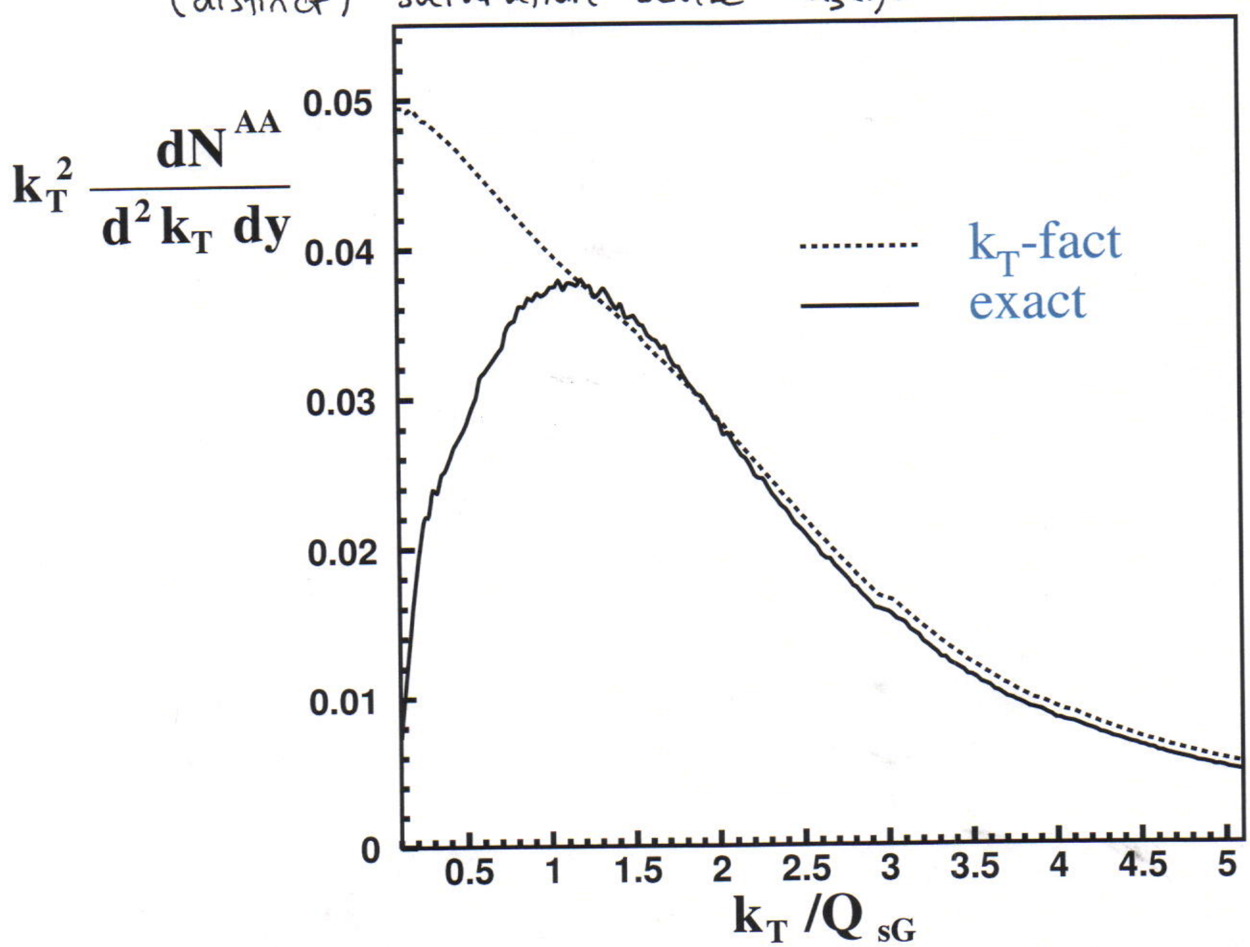


Nucleus-nucleus collisions: does k_T-factorization still hold?

- in typical collision (e.g., AA), both nucleus wavefunctions are fully saturated
- low-k_T gluons are heavily screened as a result
- thus, (7) is unlikely to be the correct description
- the natural generalization of (7) to the nucleus-nucleus case does not agree with numerical calculations at low-k_T.

Still, we can make a reasonable approximation by cutting off the q_T-integral at an upper bound proportional to k_T, the so-called Kharzeev-Levin-Nardi approach (or KLN model). This approach includes the following steps:

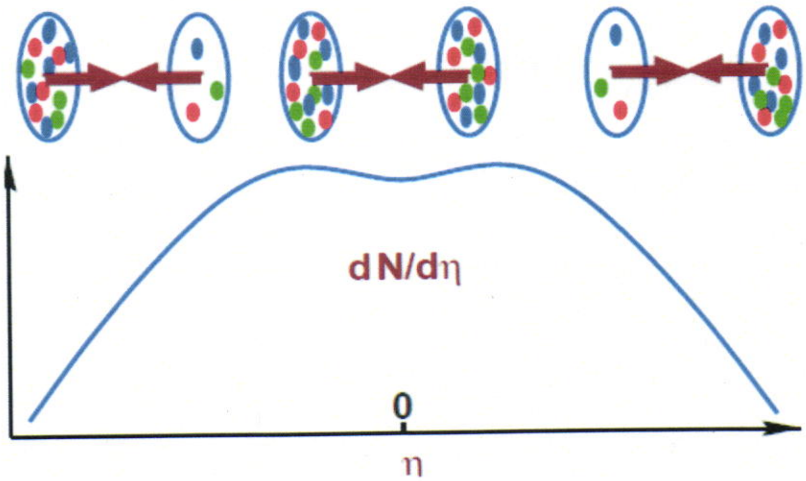
- Assume k_T-factorized form for inclusive production, but terminate integration at upper bound $\propto k_T$
- Define $xG(x, Q^2) = \int^{Q^2} dk_T^2 \, \varphi(x, k_T^2) =$ simple ansatz
- Write $\frac{dN}{dy}$ as integral over product of gluon densities in both nuclei
- N.B.: for arbitrary C.O.M. rapidity η, each nucleus as its own (distinct) saturation scale $Q_s^a(\eta)$

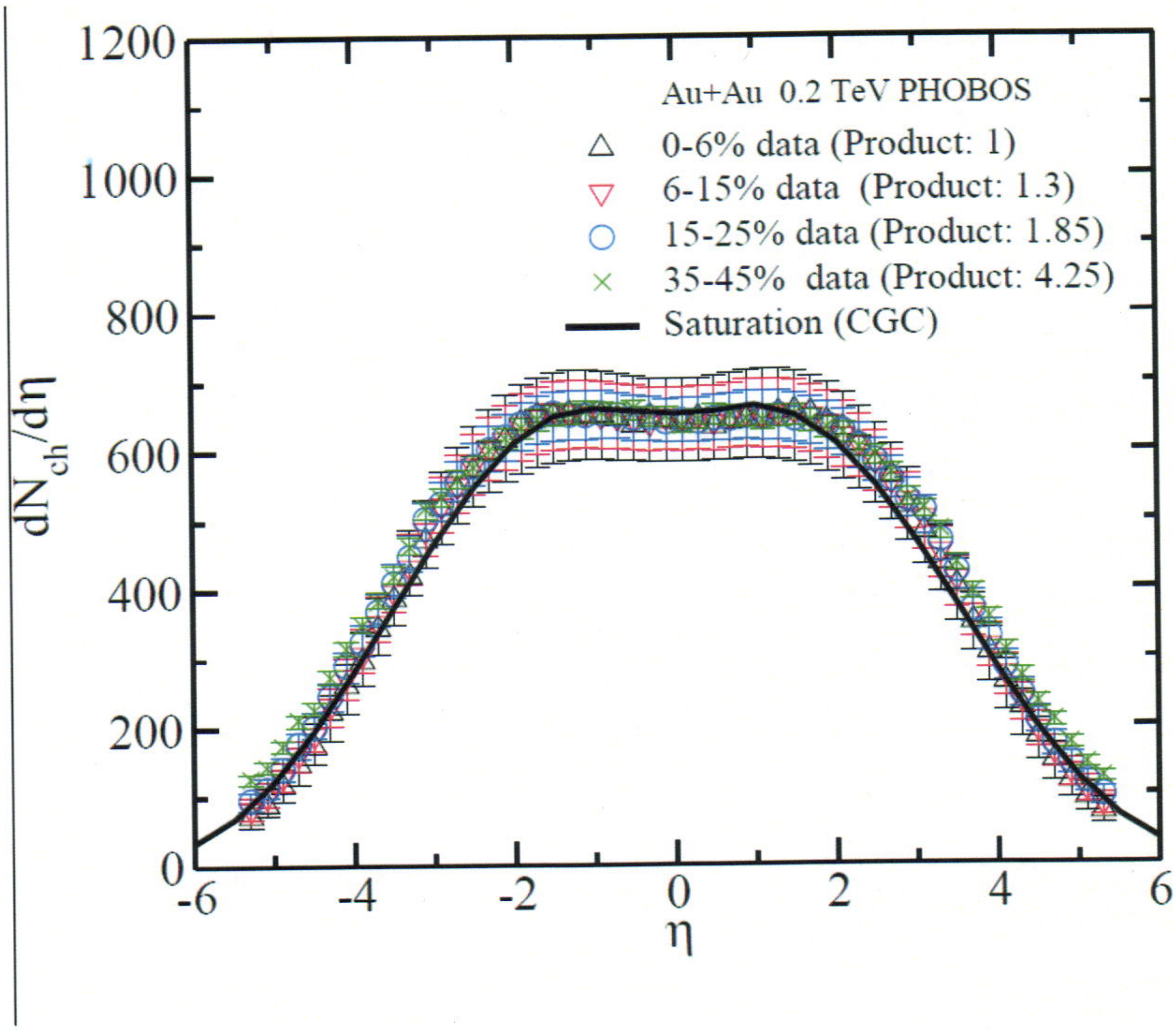


- Finally, compute multiplicities and compare with data (see figures)

Actual comparisons seem to work quite well. One can also compute the p_T-distribution in heavy-ion collisions by taking the CGC (saturation physics) results as initial conditions for hydro-dynamical evolution. One finds excellent agreement with data. See, e.g., arxiv: hep-ph/0408039 for a helpful review on this and further references.

A + A:


dN/dη
0
η


Au+Au 0.2 TeV PHOBOS
0-6% data (Product: 1)
6-15% data (Product: 1.3)
15-25% data (Product: 1.85)
35-45% data (Product: 4.25)
Saturation (CGC)
dN_ch/dη
η
1200
1000
800
600
400
200
0
-6
-4
-2
0
2
4
6