High Energy QCD Journal Club: starting 4/1/2013

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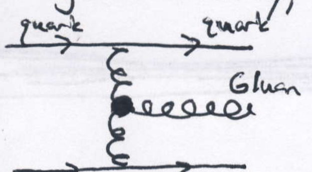
k_T-factorization, particle production in pA-collisions and the KLN model

Introduction:

Consider gluon production at lowest order, in quark-quark collisions. We have already seen that the scattering cross section for this process can be written

$$\sigma_{qq\to qqG} = \frac{2\alpha_s^3 C_F}{\pi^2}\int \frac{d^2k_\perp\, d^2q_\perp}{k_\perp^2 q_\perp^2 (\vec{k}_\perp-\vec{q}_\perp)^2}\int_0^Y dy \qquad (1)$$

Diagrammatically, this process (the amplitude) looks like this:



$\vec{q}_\perp$: transverse momentum of one of virtual gluons entering the Lipatov vertex in the diagram

$\vec{k}_\perp$: transverse momentum of outgoing gluon in diagram

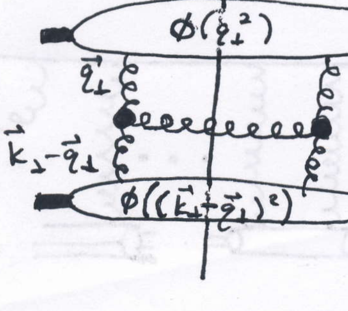
The differential cross section is then, by this result,

$$\frac{d\sigma}{d^2k_T\, dy} = \frac{2\alpha_s^3 C_F}{\pi^2}\frac{1}{k_T^2}\int \frac{d^2q_\perp}{q_\perp^2(\vec{k}_\perp-\vec{q}_\perp)^2} \qquad (k_T^2 = \vec{k}_\perp^2) \qquad (2)$$

We have seen in previous lectures that we may write the unintegrated gluon distribution for a nucleus as $\phi_{LO} \approx \frac{A\alpha_s C_F}{\pi}\frac{1}{k_T^2}$; setting $A=1$, we can write (2) as

$$\frac{d\sigma}{d^2k_T\, dy} = \frac{2\alpha_s}{C_F}\frac{1}{k_T^2}\int d^2q_\perp\, \phi_{LO}(q_T^2)\,\phi_{LO}((\vec{k}_\perp-\vec{q}_\perp)^2), \qquad (3)$$

which we visualize as



→ This means the process factorizes into two independent gluon distributions which only interact through a Lipatov vertex. This factorization persists for onium-onium scattering.

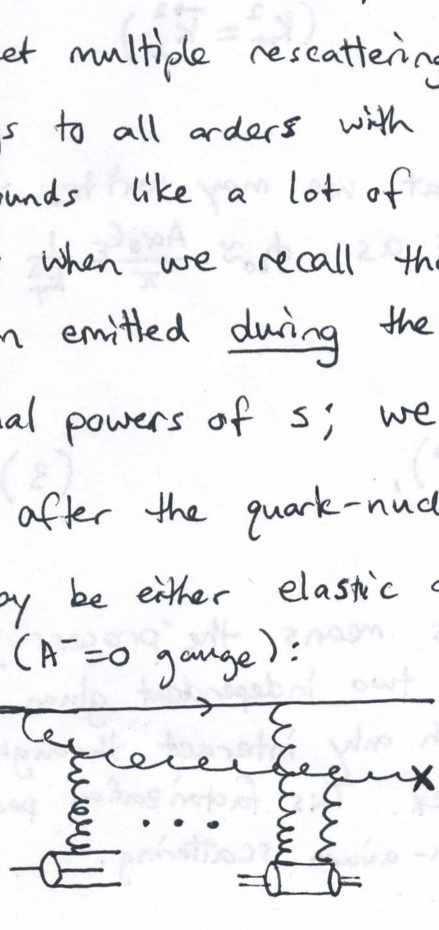
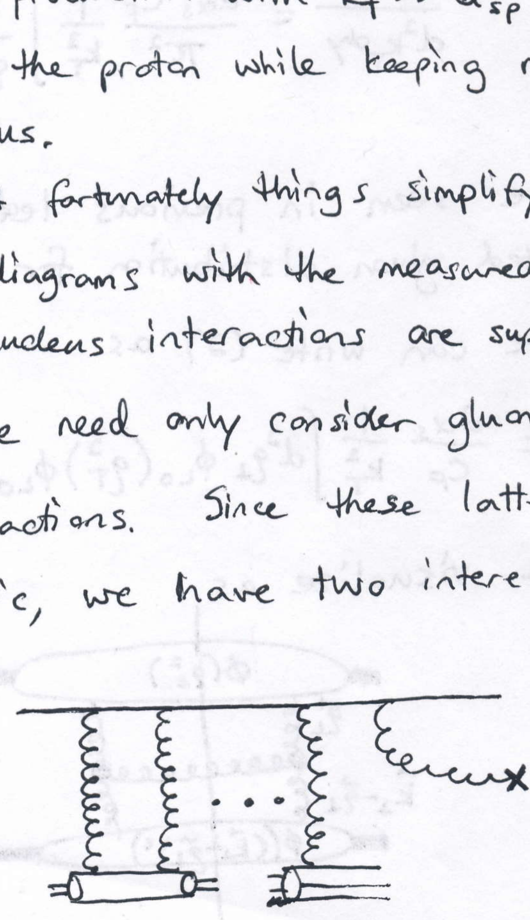
If we integrate (3) over $d^2k_\perp$ and $d^2q_{\perp}$, we find (after introducing some IR-cutoffs Λ to regulate the integrals) that $d\sigma/dy \sim \frac{1}{\Lambda^2} \ln \Lambda$

→ We therefore have an IR-singularity! However, we have already encountered this sort of problem before, and the standard prescription for its treatment is by now familiar: we must incorporate the long-distance effects of saturation physics which will serve to screen out the IR-singularity, so that the complications of non-pQCD become irrelevant (or, at least, less relevant than they would otherwise have been). This is our next task.

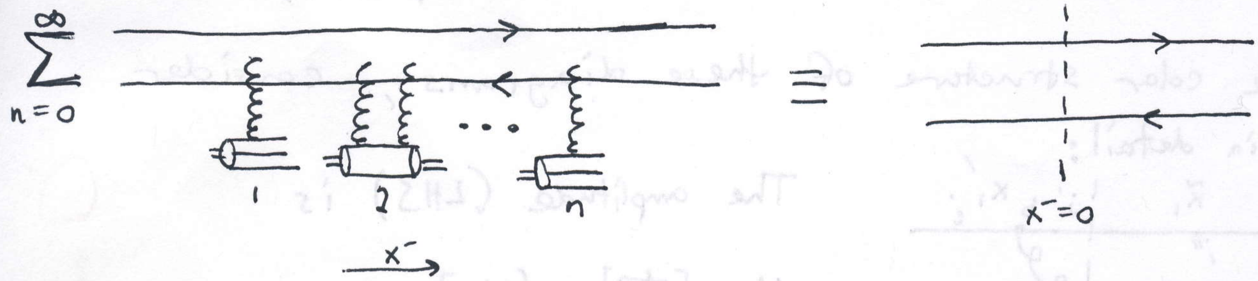
Gluon production in quark-nucleus scattering

Recall that the saturation scale $\sim A^{1/3}$, so $Q_{sp} \ll Q_{sA}$, for a large nucleus N. In this case, for gluon production with $k_T \gg Q_{sp}$, we can neglect multiple rescatterings with the proton while keeping multiple rescatterings to all orders with the nucleus.

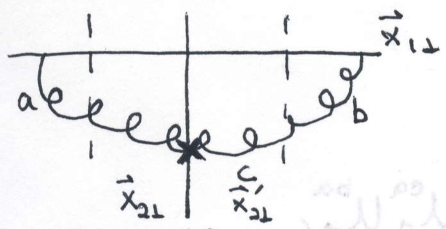
This sounds like a lot of work, but fortunately things simplify considerably when we recall that any diagrams with the measured final-state gluon emitted during the quark-nucleus interactions are suppressed by additional powers of s; we therefore need only consider gluons emitted before or after the quark-nucleus interactions. Since these latter interactions may be either elastic or inelastic, we have two interesting LCPT diagrams ($A^-=0$ gauge):

 and 

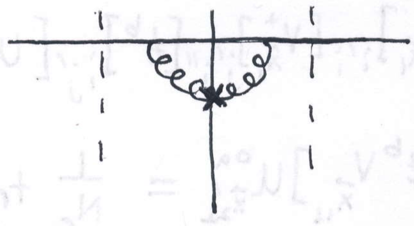
Since we are only considering gluon production before or after the quark-nucleus interactions, we may condense our diagrammatic notation slightly:



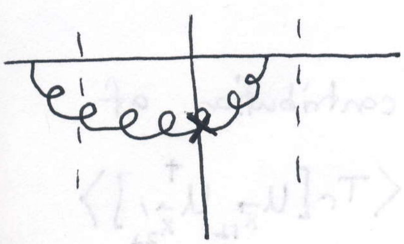
where the sum is over any number of rescatterings off of individual nucleons, elastically or inelastically. In this new notation, the differential cross section is dominated by two "quadratic" terms and two "cross-terms":

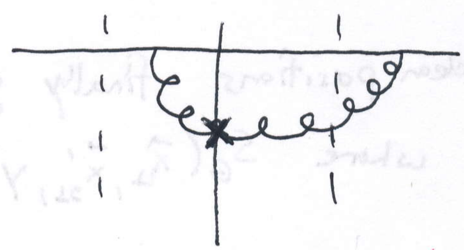


(A)


(B)

"quadratic" terms


(C)


(D)

"cross" terms

* Note that B ↔ D in the book's notation

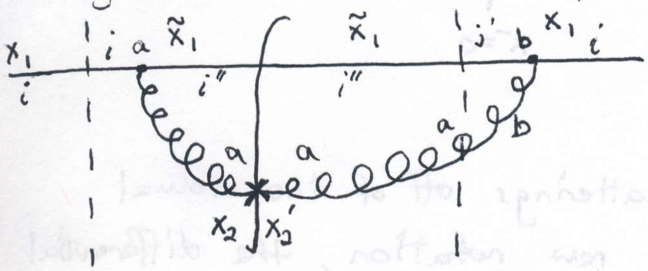
One can argue that, having calculated these diagrams, the result may be used to find $\frac{d\sigma}{d^2k_T dy}$ in the following way:

$$\frac{d\sigma}{d^2k_T dy} = \frac{1}{2(2\pi)^3}\int d^2x_2\, d^2x_{2'}\, d^2x_1\, e^{-i\vec{k}_\perp\cdot\vec{x}_{22'}}\langle A(\vec{x}_{23},\vec{x}_{13})\, A^*(\vec{x}_{2'3},\vec{x}_{13})\rangle, \qquad (4)$$

where the $\langle\ldots\rangle$ indicates an average* of the total amplitude squared over all positions of the nucleon. We must calculate these diagrams in order to compute (4).

* average $\equiv \int d^2x_3\, T(\vec{x}_{3\perp})$

The soft gluon emissions in these diagrams bring in factors of $\frac{i g t^a}{\pi} \frac{\vec\epsilon_\perp^{\,\lambda *} \cdot \vec x_{21}}{x_{21}^2}$ (with $a \to a'$ and $a \to b$ in the C.C. amplitude). To understand the color structure of these diagrams, consider diagram D in detail:



The amplitude (LHS) is

$$M \sim [t^a]_{i''' i''} [V_{\tilde x_1}]_{i'' i} \text{, while the RHS is}$$

$$M^* \sim \left([V_{\tilde x_1}]_{i'' j} [t^b]_{j i'} [U_{x_2'}]_{ab} \right)^*$$

$$\sim [V^\dagger_{\tilde x_1}]_{j i''} [t^b]_{i' j} [U_{x_2'}]_{ba}$$

$$\Rightarrow |M|^2 \sim [t^a]_{i' i''} [V_{\tilde x_1}]_{i'' i} [V^\dagger_{\tilde x_1}]_{j i''} [t^b]_{i j} [U_{x_2'}]_{ba}$$

$$\Rightarrow \frac{1}{N_c} \mathrm{tr}\left[t^a V^\dagger_{\vec x_{11}} t^b V_{\vec x_{11}} \right] U^{ba}_{\vec x_{21}'} = \frac{1}{N_c} \mathrm{tr}[t^b t^c] \, U^{ca}_{\vec x_{12}} U^{ba}_{\vec x_{21}'}$$

average over colors

$\rightarrow$ averaging over nucleon positions finally gives a contribution of $C_F S_G(\vec x_{12}, \vec x_{21}', Y)$, where $S_G(\vec x_{12}, \vec x_{21}', Y) \equiv \frac{1}{N_c^2 - 1} \left\langle \mathrm{Tr}\left[U_{\vec x_{12}} U^\dagger_{\vec x_{21}'} \right] \right\rangle$

In the end, when we have properly accounted for minus signs amongst the various diagrams and summed over gluon polarizations, we find that

$$\frac{d\sigma^{qA}}{d^2k_T\,dy} = \frac{1}{(2\pi)^2}\int d^2x_2\,d^2x_{2'}\,d^2x_1\,e^{-i\vec{k}_1\cdot\vec{x}_{22'}}\,\frac{\alpha_s C_F}{\pi^2}\left(\frac{\vec{x}_{21}\cdot\vec{x}_{2'1}}{x_{21}^2\,x_{2'1}^2}\right)$$

$$\cdot\left[S_G(\vec{x}_{2\perp},\vec{x}_{2'\perp},y) - S_G(\vec{x}_{1\perp},\vec{x}_{2'\perp},y) - S_G(\vec{x}_{2\perp},\vec{x}_{1\perp},y) + 1\right],$$

where S_G is the S-matrix element for a gluon dipole. If we define $N_G(\ ,\ ,\ ) = 1 - S_G(\ ,\ ,\ )$, then this expression becomes

$$\frac{d\sigma^{qA}}{d^2k_T\,dy} = \frac{1}{(2\pi)^2}\int d^2x_2\,d^2x_{2'}\,d^2x_1\,e^{-i\vec{k}_1\cdot\vec{x}_{22'}}\,\frac{\alpha_s C_F}{\pi^2}\left(\frac{\vec{x}_{21}\cdot\vec{x}_{2'1}}{x_{21}^2\,x_{2'1}^2}\right)$$

$$\cdot\left[N_G(\vec{x}_{1\perp},\vec{x}_{2'\perp},y) - N_G(\vec{x}_{2\perp},\vec{x}_{2'\perp},y) + N_G(\vec{x}_{2\perp},\vec{x}_{1\perp},y)\right]$$

Integrate in the first term over $d^2x_{2\perp}$, in the second term over $d^2x_{1\perp}$ and in the third term over $d^2x_{2'\perp}$ to get

$$\frac{d\sigma^{qA}}{d^2k_T\,dy} = \frac{\alpha_s C_F}{2\pi^3}\int d^2x_2\,d^2x_{2'}\,N_G(\vec{x}_{2\perp},\vec{x}_{2'\perp},y)\,e^{-i\vec{k}_\perp\cdot\vec{x}_{22'}}\left(2i\,\frac{\vec{k}_\perp}{k_\perp^2}\cdot\frac{\vec{x}_{22'}}{x_{22'}^2} - \ln\frac{1}{\Lambda\,x_{22'}}\right),$$

using $\int d^2q_\perp\,e^{i\vec{q}_\perp\cdot\vec{x}_\perp}\,\frac{\vec{q}_\perp}{q_\perp^2} = 2\pi i\,\frac{\vec{x}_\perp}{x_\perp^2}$, $\int d^2y_\perp\,\frac{\vec{y}_\perp\cdot(\vec{y}_\perp+\vec{x}_\perp)}{y_\perp^2(\vec{y}_\perp+\vec{x}_\perp)^2} = 2\pi\ln\frac{1}{x_\perp\Lambda}$

and relabeling coordinate indices appropriately (Λ is some IR cutoff). This expression does not diverge at $\vec{x}_{2\perp} = \vec{x}_{2'\perp}$, since $N_G = 0$ here (a zero-size dipole does not interact), so we can rewrite this as

$$\frac{d\sigma^{qA}}{d^2k_T\,dy} = \frac{\alpha_s C_F}{2\pi^3}\int d^2x_2\,d^2x_{2'}\,N_G(\vec{x}_{2\perp},\vec{x}_{2'\perp},y)\,\vec{\nabla}^2_{\vec{x}_{22'}}\left(e^{-i\vec{k}_\perp\cdot\vec{x}_{22'}}\ln\frac{1}{\Lambda\,x_{22'}}\right)$$

In the same way that we have rewritten a portion of this expression in terms of the gluon dipole-nucleus forward scattering amplitude, we can write the rest of it in terms of the forward scattering amplitude for the incident quark-gluon dipole (n_G^q) to get, after some algebra and integration by parts,

$$\frac{d\sigma^{qA}}{d^2k_T dy} = \frac{C_F}{\alpha_s \pi (2\pi)^3} \frac{1}{k_T^2} \int d^2B_\perp d^2b_\perp d^2x_\perp \left[\nabla^2_{\vec{x}_\perp} n_G^q(\vec{x}_\perp, \vec{B}_\perp - \vec{b}_\perp, 0)\right] \cdot e^{-i\vec{k}_\perp \cdot \vec{x}_\perp} \left[\nabla^2_{\vec{x}_\perp} N_G(\vec{x}_\perp, \vec{b}, y)\right],$$

where $\vec{x}_\perp \equiv \vec{x}_{22'}$ and $\vec{b}_\perp$ and $\vec{B}_\perp$ are the transverse impact parameters of the produced gluon and the incident quark, respectively.

Finally, if we define the unintegrated gluon distributions

$$\phi_A(k_T^2) = \frac{C_F}{\alpha_s (2\pi)^3} \int d^2b_\perp d^2x_\perp e^{-i\vec{k}_\perp \cdot \vec{x}_\perp} \nabla^2_{\vec{x}_\perp} N_G(\vec{x}_\perp, \vec{b}_\perp, 0) \tag{5}$$

and

$$\phi_p(k_T^2) = \frac{C_F}{\alpha_s (2\pi)^3} \int d^2b_\perp d^2x_\perp e^{-i\vec{k}_\perp \cdot \vec{x}_\perp} \nabla^2_{\vec{x}_\perp} n_G(\vec{x}_\perp, \vec{b}_\perp, 0)$$

for the nucleus and proton (quark), respectively, then we may write

$$\frac{d\sigma^{pA}}{d^2k_T dy} = \frac{2\alpha_s}{C_F} \frac{1}{k_T^2} \int d^2q_\perp \phi_p(q_T^2) \phi_A((\vec{k}_\perp - \vec{q}_\perp)^2) \tag{6}$$

$\rightarrow$ k_T-factorized form persists!! This is unexpected, since we are used to seeing this happen when we can write the relevant diagrams as distribution functions interacting only through a Lipatov vertex (squared); it is not clear whether a gauge exists in which the diagrams (A)-(D) can be drawn in such a separated form. For this reason, a complete understanding of the origin of this factorization is still lacking.

We can also show

$$\left.\frac{d\sigma^{pA}}{d^2k_T dy}\right|_{k_T \gg Q_{SG}} \approx \frac{8\alpha_s^3 C_F A}{\pi} \frac{1}{k_T^4} \ln \frac{k_T}{\Lambda}$$

$$\left.\frac{d\sigma^{pA}}{d^2k_T dy}\right|_{k_T \ll Q_{SG}} \approx \frac{\alpha_s C_F S_\perp}{\pi^2} \frac{1}{k_T^2}$$

($S_\perp$: transverse area of nucleus)

Notice that, for large k_T ($\gg Q_{SG}$), the coherence length is small compared to the size of the nucleus, so the produced gluons are only weakly screened by the local color charge density.

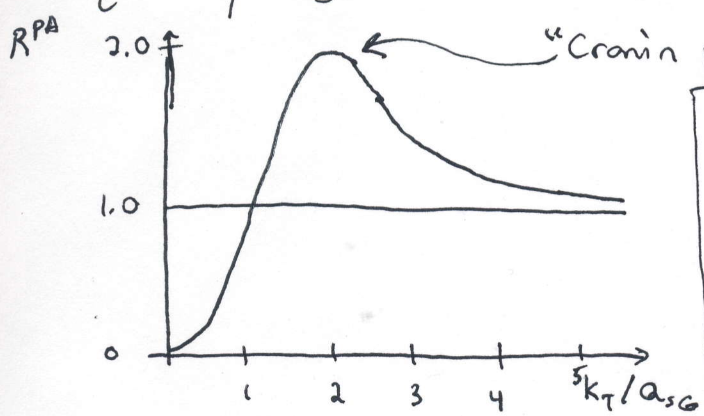
However, for small k_T ($\ll Q_{SG}$), the gluons are strongly screened and saturation physics works well, softening the steep IR divergence we saw earlier to $\frac{d\sigma^{pA}}{dy} \sim \ln\left(\frac{Q_{SG}}{\Lambda}\right)$.

To help visualize these results, we define the ratio

$$R^{pA}(k_T, y) = \frac{d\sigma^{pA}/d^2k_T dy}{A\, d\sigma^{pp}/d^2k_T dy},$$

known as the <u>nuclear modification factor</u>.

Deviations of this quantity from unity represent collective nuclear effects due to saturation effects in the collision. A plot of this quantity against k_T/Q_{SG} looks like



This is reasonable: low-k_T gluons are strongly screened (or "shadowed") by the nucleus, leading to an enhancement (the "Cronin peak" or "Cronin effect") at $k_T \sim Q_{SG}$. This effect has been confirmed for hadron production in pA collisions.