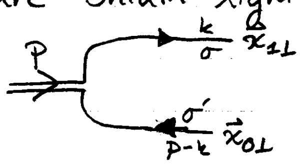
Mueller's dipole model

meson of heavy quark and anti-quark (onium)

"bare" onium light cone wavefunction $\Psi^{(0)}_{\sigma\sigma'}(\vec{k}_\perp, z)$ $\quad z = k^+/p^+$



$\vec{x}_{10} = \vec{x}_{1\perp} - \vec{x}_{0\perp}$ (tranverse size of the dipole)

the onium is moving in the light cone plus direction.

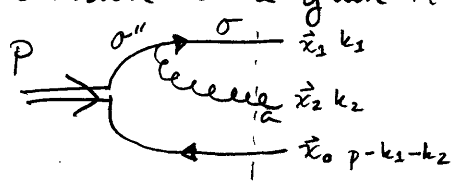
$$\Psi^{(0)}_{\sigma\sigma'}(\vec{x}_{10}, z) = \int \frac{d^2k_\perp}{(2\pi)^2} e^{i\vec{k}_\perp\cdot\vec{x}_{10}} \Psi^{(0)}_{\sigma\sigma'}(\vec{k}_\perp, z) \quad (F.T.)$$

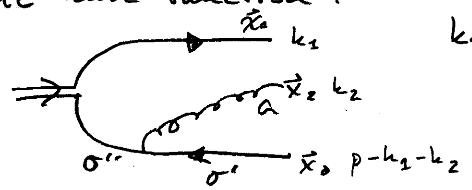
normalization: $$\int_0^1 \frac{dz}{z(1-z)} \underbrace{\int \frac{d^2k_\perp}{2(2\pi)^3} \sum_{\sigma\sigma'} |\Psi^{(0)}_{\sigma\sigma'}(\vec{k}_\perp, z)|^2}_{\int \frac{d^2x_{10}}{4\pi} \sum_{\sigma\sigma'} |\Psi^{0}_{\sigma\sigma'}(\vec{x}_{10}, z)|^2} = 1$$

$A^+ = 0$

- Goal: interested in mod. this wave-function under small-x evol. in the LLA approx. → resum $\alpha_s \ln \frac{1}{x}$ terms
  leading logarithmic approx.

→ emission of a gluon in the wave-function.





$k_2^+ \ll k_1^+, p^+ - k_1^+$

$z_i = \frac{k_i^+}{p}$

→ note: longitudinal mom ordered while DGLAP transverse momenta ordered

$$\Psi^{(1)}_{\sigma\sigma'}(\vec{k}_{1\perp}, \vec{k}_{2\perp}, z_1, z_2) = \frac{g t^a \Theta(k_2^+)}{k_2^- + k_2^- + (p-k_1-k_2)^- - p^-} \Bigg[$$

$$\times \sum_{\sigma''=\pm 1} \frac{u_\sigma(k_1)\gamma\cdot\epsilon^*_\lambda(k_2) u_{\sigma''}(k_1+k_2)}{k_1^+ + k_2^+} \Psi^0_{\sigma''\sigma'}(\vec{k}_{1\perp}+\vec{k}_{2\perp}, z_1+z_2) - \frac{\quad}{p^+ - k_1^+} \Psi^{(0)}_{\sigma\sigma'}(\vec{k}_{1\perp}, z_1)\Bigg]$$

$K_{3,3}$

K_5

since $k_2^+ \ll k_1^+, p^+ - k_1^+ \rightarrow z_2 \ll z_1, 1-z_1$ $\qquad k_2^- = \dfrac{k_{2\perp}^2}{k_2^+}$

$$\frac{1}{k_2^- + k_1^- + (p-k_1-k_2)^- - p^-} \approx \frac{1}{k_2^-} = \frac{k_2^+}{k_{2\perp}^2}$$

Simplify dirac matrix $u_\sigma(k_1)\gamma \cdot \epsilon_\lambda^*(k_2) u_{\sigma''}(k_1+k_2) \approx 2\delta_{\sigma\sigma''} k_1^+ \dfrac{\vec{\epsilon}_\perp^{\lambda *} \cdot \vec{k}_{2\perp}}{k_2^+}$

new w-f $\longrightarrow$ $\Psi_{\sigma\sigma'}(\vec{k}_{1\perp}, \vec{k}_{2\perp}, z_1, z_2) \approx 2g t^a \theta(z_2) \dfrac{\vec{\epsilon}_\perp^{\lambda *} \cdot \vec{k}_{2\perp}}{k_{2\perp}^2} \Big[\Psi^{(0)}_{\sigma\sigma'}(\vec{k}_{1\perp} + \vec{k}_{2\perp}, z_1) - \Psi^{(0)}_{\sigma\sigma'}(\vec{k}_{1\perp}, z_1) \Big]$

F.T. $\rightarrow$ $\Psi^{(1)}_{\sigma\sigma'}(\vec{x}_{10}, \vec{x}_{20}, z_1, z_2) = i \dfrac{g t^a}{\pi} \Psi^{(0)}_{\sigma\sigma'}(\vec{x}_{10}, z_1) \vec{\epsilon}_\perp^{\lambda *} \left(\dfrac{\vec{x}_{21}}{x_{21}^2} - \dfrac{\vec{x}_{20}}{x_{20}^2} \right)$

$\vec{x}_{20} = \vec{x}_{2\perp} - \vec{x}_{0\perp} \qquad \vec{x}_{21} = \vec{x}_{2\perp} - \vec{x}_{1\perp} \qquad x_{ij} = |\vec{x}_{ij}|$

$$\sum_{\sigma\sigma',\lambda a} \left| \Psi^{(1)}_{\sigma\sigma'} \right|^2 = \frac{4\alpha_s C_F}{\pi} \frac{x_{10}^2}{x_{20}^2 x_{21}^2} \sum \left| \Psi^{(0)}_{\sigma\sigma'} \right|^2$$

to find the prob. of finding one gluon in the onium w-f we have to integrate over phase space. $\displaystyle\int_{z_0}^{\min\{z_1, 1-z_1\}} \frac{dz_2}{z_2} \int \frac{d^2x_2}{4\pi}$

z_0: IR cutoff

$$\int \frac{dz_2}{z_2} \rightarrow \log 1/x$$

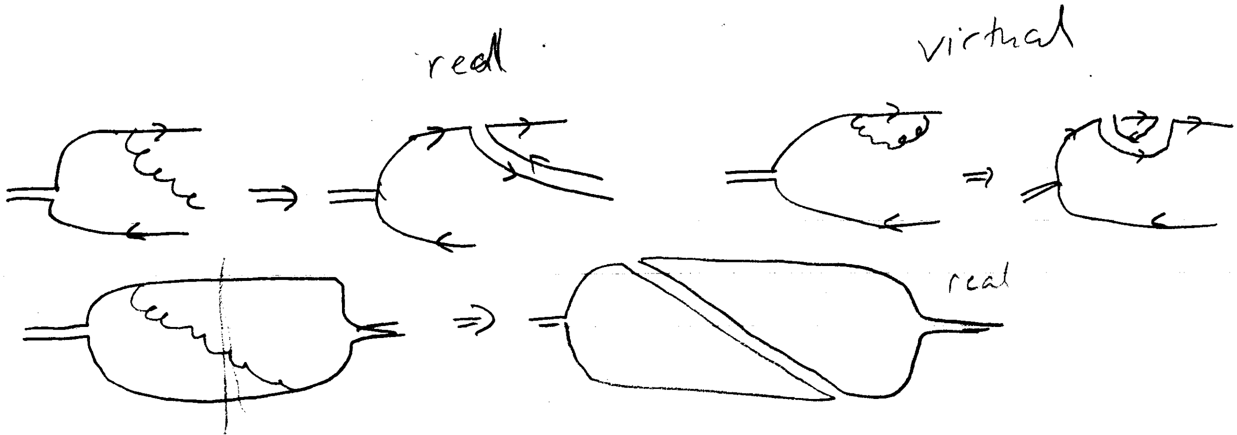
notice UV divergence at $x_{20} \approx 0$, $x_{21} \approx 0$ $\qquad$ UV cutoff $\rho < x_{20}, x_{21}$

$$\underbrace{\Psi^{(0)}_{\sigma\sigma'}(\vec{x}_{10}, z_1)\Big|_{\mathcal{O}(\alpha_s)}}_{\text{virtual correction}} = -\frac{1}{2} \int_{z_0}^{\min(z_1, 1-z_1)} \frac{dz_2}{z_2} \int d^2x_2 \frac{\alpha_s C_F}{\pi^2} \frac{x_{10}^2}{x_{20}^2 x_{21}^2} \Psi_{\sigma\sigma'}(\vec{x}_{10}, z_1)\Big|_{\mathcal{O}(\alpha_s^0)}$$

$$= -\frac{2\alpha_s C_F}{\pi} \ln\left(\frac{x_{01}}{\rho}\right) \int_{z_0}^{\min(z_1, 1-z_1)} \frac{dz_2}{z_2} \Psi^{(0)}_{\sigma\sigma'}(\vec{x}_{20}, z_1)\Big|_{\mathcal{O}(\alpha_s^0)}$$

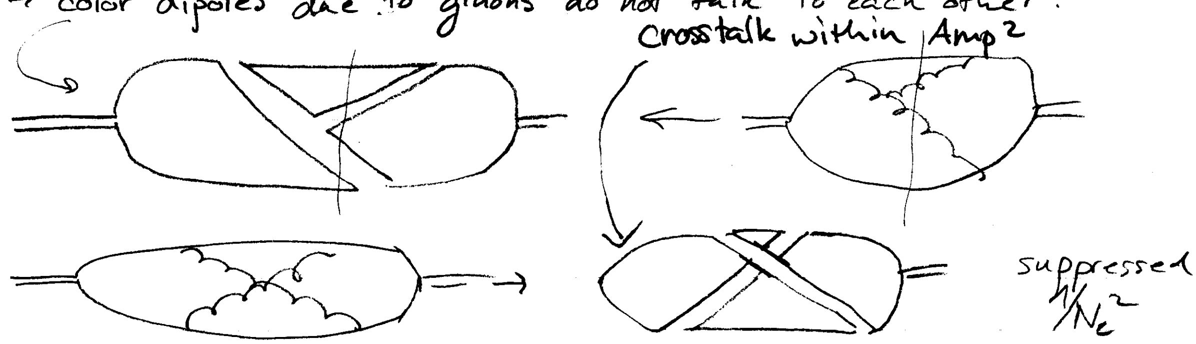
While keeping $\underline{\alpha_s N_c \text{ const.}}$ take the large N_c limit to obtain higher-order gluon emissions

single gluon line $\longrightarrow$ double line $q\bar{q}$ in a color octet config.

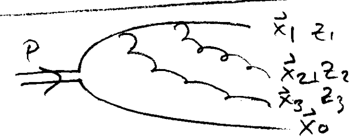


In this limit it is useful to talk about color dipoles instead of gluons. The original color dipole being $\vec{x}_{1\perp}$ + $\vec{x}_{0\perp}$.

→ only planar diagrams contribute
  → non-planar diagrams are suppressed by powers of N_c
  → color dipoles due to gluons do not talk to each other.



<u>In order to obtain leading order in $\ln 1/x$, soft gluons (small z) have to be emitted ~~later than hard gluons~~.</u>

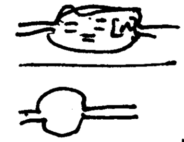


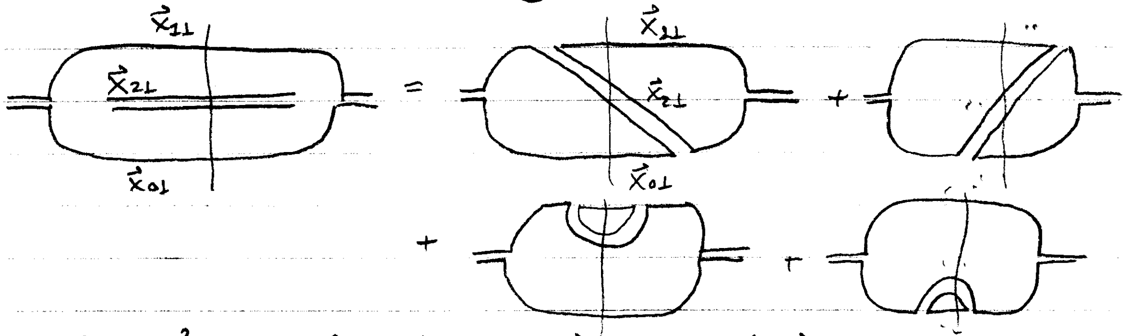
$z_i = \frac{k_i^+}{p^+}$ if $z_0 \ll z_3 \ll z_2 \ll z_1$

Square of w-f yields $\rightarrow \alpha_s^2 \int_{z_0}^{z_1} \frac{dz_2}{z_2} \int_{z_0}^{z_2} \frac{dz_3}{z_3} \frac{z_3^2}{z_2^2} \approx \frac{1}{2}\alpha_s \ln\left(\frac{z_1}{z_0}\right)$ } Ask Yuri

doesn't contribute to LO
(it does in NLO though)

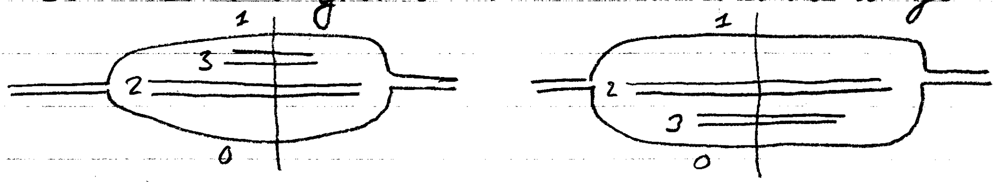
important conc. $z_2 \gg z_3 \gg \ldots \gg z_n$

$z =$ 



$$\frac{\alpha_s C_F}{\pi^2}\frac{x_{10}^2}{x_{20}^2 x_{21}^2} = \frac{\alpha_s C_F}{\pi^2}\left(\frac{1}{x_{21}^2} - 2\frac{\vec{x}_{21}\cdot\vec{x}_{20}}{x_{21}^2 x_{20}^2} + \frac{1}{x_{20}^2}\right)$$

So then two real gluons in the LLA and the large-N_c



factor in the w-f $\Longrightarrow \displaystyle\int_{z_0}^{z_1}\frac{dz_2}{z_2}\int_{z_0}^{z_2}\frac{dz_3}{z_3}\int d^2x_2\, d^2x_3 \left(\frac{\alpha C_F}{\pi^2}\right)^2 \frac{x_{10}^2}{x_{20}^2 x_{21}^2}\left(\frac{x_{12}^2}{x_{31}^2 x_{32}^2} + \frac{x_{20}^2}{x_{32}^2 x_{30}^2}\right)$

Describe the onium w-f including $\alpha_s \ln(1/x)$

dipole generating functional $Z(\vec{x}_{10}, \vec{b}_{0\perp}, Y; u)$

$$Z(\vec{x}_{10}, \vec{b}_{0\perp}, Y; u) \sum \left|\Psi^{(0)}_{\alpha\alpha'}(\vec{x}_{10}, z_1)\right|^2\Big|_{O(\alpha_s^0)}$$

$$= \int d^2r_1\, d^2b_1 \left|\Psi^{(1)}(\vec{r}_{1\perp}, \vec{b}_{1\perp}, Y)\right|^2 u(\vec{r}_{1\perp}, \vec{b}_{1\perp})$$

$$+ \frac{1}{2!}\int d^2r_1\, d^2b_1\, d^2r_2\, d^2b_2 \left|\Psi^{[2]}(\vec{r}_{1\perp}, b_{1\perp}, \vec{r}_{2\perp}, b_{2\perp}, Y)\right|^2 \times u(\vec{r}_{1\perp}, b_{1\perp})\, u(\vec{r}_{2\perp}, b_{2\perp})$$

$$+ \dots$$

$$= \sum_{n=1}^{\infty}\frac{1}{n!}\int d^2r_1\, d^2b_1 \dots d^2r_n\, d^2b_n \left|\Psi^{[n]}(\vec{r}_{1\perp}, b_{1\perp}, \dots, \vec{r}_{n\perp}, b_{n\perp}, Y)\right|^2$$

$$\times\, u(\vec{r}_{1\perp}, \vec{b}_{1\perp}) \dots u(\vec{r}_{n\perp}, b_{n\perp})$$

rapidity $Y = \ln\left(\frac{z_1}{z_0}\right)$ z_0 is the smallest fraction of mom that the gluon carries

$\vec{r}$ = dipole size $\vec{b}$ = impact parameter $\vec{b}_{01} = \frac{1}{2}(\vec{x}_{1\perp} + \vec{x}_{0\perp})$

$[n]$ = # of dipoles $u(\vec{r}_{n\perp}, \vec{b}_{n\perp})$ dummy functions

(n) = # of gluons

$$|\Psi^{[n]}(\vec{r}_1, b_{1\perp}, \ldots, \vec{r}_{n\perp}, b_{n\perp}, Y)|^2 = \sum_{\sigma\sigma'} |\Psi^{(0)}_{\sigma\sigma'}(\vec{x}_{10}, z_1)|^2$$

$$\times \frac{\delta^n}{\delta u(\vec{r}_{1\perp}, b_{1\perp}) \ldots \delta u(\vec{r}_{n\perp}, b_{n\perp})} Z(\vec{x}_{10}, b_{01}, Y; u)\Big|_{u=0}$$

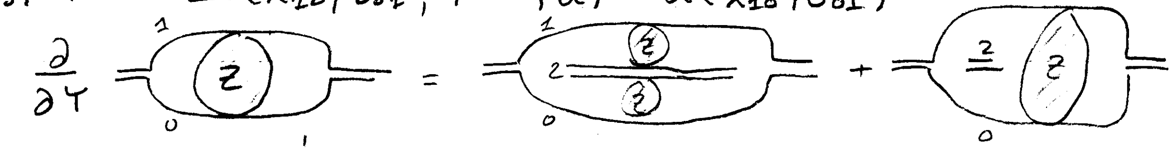
Since $|\Psi^{[n]}|^2$ gives prob. of n dipoles in onium w-f in a given transverse space config. the sum over prob. in all trausverse config. is 1. $\Rightarrow Z(\vec{x}_{10}, \vec{b}_{01}, Y, u=1) = 1$

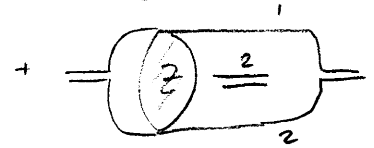
Want the evolution equation for generating functional Z summing all powers of $\alpha_s Y$.

initial cond. : $Y=0 \rightarrow$ no evolution, no gluon emissions.

$$|\Psi^{[n>1]}(Y=0)|^2 = 0 \quad + \quad |\Psi^{[1]}(r_{1\perp}, b_{1\perp}, Y=0)|^2 = \delta^2\left(\vec{b}_{1\perp} + \frac{\vec{r}_{1\perp}}{2} - \vec{x}_{1\perp}\right)\delta^2\left(b_{1\perp} - \frac{\vec{r}_{1\perp}}{2} - \vec{x}_{0\perp}\right)$$

s.t. $Z(\vec{x}_{10}, \vec{b}_{01}, Y=0; u) = u(\vec{x}_{10}, \vec{b}_{01})$





replacing C_F by $N_c/2$

$$\frac{\partial}{\partial Y} Z(\vec{x}_{10}, \vec{b}_{01}, Y; u)$$

— P(Y | gluon emitted)

$$= \frac{\alpha_s N_c}{2\pi^2} \int d^2x_2 \frac{d^2x_{10}}{x_{20}^2 x_{21}^2}$$

where does this come from?

$$\left[Z\left(\vec{x}_{12}, \vec{b}_{01} + \frac{\vec{x}_{20}}{2}, Y; u\right) Z\left(\vec{x}_{20}, \vec{b}_{01} + \frac{\vec{x}_{21}}{2}, Y; u\right) - Z(\vec{x}_{10}, b_{01}, Y; u)\right]$$