Time Dilation, the Twin Paradox and all that

The Michelson-Morely experiment showed that the light moves with the same speed in any inertial reference frame (reference frame=coordinate system). Note: an inertial frame has no acceleration (you can feel acceleration – you can’t feel the velocity of your motion). Different inertial frames have a constant relative velocity. The Michelson-Morely experiment has two immediate implications:

1. We can use light to make a clock. By measuring how far light has traveled we can keep track of time. For example, light travels about one foot per nanosecond (more accurately it travels 30 cm/nanosecond). The RULE FOR MEASURING TIME:

   \[ \Delta t = \Delta x / c \]  

   (the distance traveled in feet is approximately the time interval in nanoseconds.)

   Essentially, this reduces to measuring time with a ruler. Note: we can do this only because the speed of light is independent of frame --- otherwise you’d have to make corrections to “light clocks” according to observing frame.

2. Time dilation is an immediate consequence of the fact that a light has the same speed in all frames. To see this we do a “thought experiment” using a “light clock” in which a pulse of light bounces back and forth between two mirrors. We’ll space the mirrors such that the round trip travel time is 1 ns --- i.e. the pulse travels about a foot. Note: the light clock keeps the same time as any other clock in it’s reference frame. A billion cycles will correspond to a watch tick or a heart beat --- for a heart beating once/second. Time dilation refers to the observation that clocks in a moving frame appear to be running slow. For example, if we put a light clock in a very fast rocket --- a rocket whose velocity is a significant fraction of \( c \) ---- and describe the light pulse’s path, as viewed from earth, we’ll see the clock moving during a pulse’s round trip. See Fig. 2. Using the RULE FOR MEASURING TIME, because the distance traveled by the light pulse is more than 1 foot/round trip, we’ll conclude that a round trip takes more than a nanosecond. In comparing

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Fig. 1: Light Clock in its rest frame. One roundtrip of the light pulse is about 1 foot and measures 1 ns.

Fig. 2: View from earth of a light pulse clock in a rocket moving with velocity \( v \): the light pulse is traveling more than a foot, roundtrip, in the earth’s frame and therefore each roundtrip is longer than a nanosecond. From earth, this clock and all other activities on the rocket appear to be running slow.
to clocks on earth, we conclude the clock on the rocket is running slow. And since all other measures of time on the rocketed are “in synch” with the light clock, we observe that time appears to be passing more slowly on the rocket.

Quantitatively, if the light clock on the rocket measures that the time interval for some activity (e.g. a heart beat) \( \Delta t_0 \), then on earth, we’d measure the activity takes time:

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\Delta t = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} \Delta t_0 = \gamma \Delta t_0
\]

(1.1)

Where the time dilation factor, \( \gamma \), is a number greater than 1. Similarly (as discussed in the text), lengths and distances along the direction of motion, that are observed in a moving frame will appeared shorter (contracted) by the factor \( 1/\gamma \). For example, a meter stick in the rocket oriented parallel to its velocity will appear \( 1/\gamma \) meters long from earth. Note: time dilation (and length contraction) is “symmetric.” An observer on the rocket, using the rocket’s frame, will see the earth rapidly moving at a constant velocity. Light clocks on earth will move during the round trips of their pulses, and the observer on the rocket will judge that time appears to be running slowly for those on earth. And distances/objects in the rest frame of the earth are contracted.

The twin paradox challenges time dilation by suggesting that a contradiction follows from the symmetry of the effect. As an example of the paradox, consider the case of two twins, Roy and Ernie. Roy takes a relativistically-fast rocket ride to a nearby star and back. For the quantitative case we consider shortly, Roy’s rocket travels \( V = 0.86c \). Ernie stays at how on Earth. Note, the twins may be kept straight by the first letters of their names: Ernie \( \equiv \) Earth; Roy \( \equiv \) Rocket. For \( V = 0.86c \), the “time dilation factor,” \( \gamma \), is very close to 2 (more accurately, \( \gamma = 1.96 \), but we will ignore the difference in what follows). The star is 12 light-years away. For this distance, in the earth’s frame the one-way trip to the star should take \( \Delta t = \frac{L_0}{V} = \frac{12 \text{lyr}}{0.86c} = 14 \text{yr} \) (more accurately, 13.95 yr --- note: these differences will cause some round-off errors below which we will ignore) and the round trip takes 28 years. Ernie argues:

Roy in his rocket experiences time dilation. His clocks will have ticked off only \( \frac{1}{2} \) as much time and Roy will have aged only \( \frac{1}{2} \times 28 \) years during the trip. When we meet, Roy will be younger.

Roy claims:

In my frame, the earth, and with it Ernie and his light clock have been moving at 0.86c. I know, from the argument above, that time dilation occurs for his frame. When we meet, Ernie will be younger.

Who is right? The short, correct, answer (see your textbook) is that Ernie is right. The reason is that Ernie’s frame, the Earth, is an inertial frame for the entire trip and special relativity requires an inertial frame. It is true that Roy’s frame, for the trip out, is
an inertial frame and his observation of time dilation of Ernie is valid. Moreover, Roy’s frame for the trip back is also an inertial frame (though a different one than his frame on the way out) and time dilation of Ernie is then, too, a valid observation. At the turn around, however, Roy’s rocket must decelerate and accelerate back in the opposite direction. During the turn around, Roy’s frame is NOT an inertial frame and because of this, Roy’s observations are unreliable and wrong.

While correct, this answer seems unsatisfying. Couldn’t you make the turn around time so short that it just doesn’t enter in? It turns out that Roy’s turning around --- no matter how short it is --- and changing inertial frames is the crucial difference and it does make his perspective very different than Ernie’s.

To get a feel for this, we’ll work through this thought experiment, quantitatively. Roy and Ernie agree to signal to each other every time a year passes. Specifically, imagine that Roy begins his journey from Earth on January 1st (both agree on this). Both have very powerful telescopes to observe the other and they agree that after every year ($\approx 3.15 \times 10^6$ ticks of their respective 1 ns light clocks), they will send a flash of light for the other to observe. Remember: since a light clock is in synch with all other activities in its frame, receiving light flashes will let a twin know how his counterpart appears to be aging. Figure 3 is a sketch of the journey. The Earth and the star are shown. The orange marks are a scale showing distances in light years (in the earth’s frame). Note: the star is 12 lyr distant. Below, the blue marks are the locations of Roy’s rocket for each year of the journey. Roy takes 14 years on the way out and then retraces his steps on the way back. For example, his position at year 27 --- one year before the end --- is the same as after the first year of the trip. What do the twins see in their telescopes?

First, consider Ernie’s view from the Earth’s frame. He sees Roy’s time clock dilated by a factor of two. Thus, according to Ernie’s calendar, Roy’s first flash will be sent after two years. At this time, Roy has traveled $2 \times 0.86 = 1.7$ lyr. The flash will therefore need to travel another 1.7 years before it gets to Ernie’s telescope and so Ernie actually sees the first pulse arrive about 2 years + 1.7 years $\approx 3.7$ years after the trip began.¹ The next flash is sent 4 years into the journey and arrives $\approx 3.7$ years later, and so

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¹ Note: in carrying out this thought experiment, actually two effects are getting mixed up: time dilation and the Doppler effect for light. Recall, the Doppler effect, is a lowering in frequency (in this case, the frequency is that of the flashes once/year) when the source is moving away from the receiver. But this is what Ernie would actually see in his telescope.
on. So for the trip out, Ernie receives pulses every 3.7 years. At year 14, when Roy arrives at the star, Ernie will have received a total of three pulses.

Fig. 4. In Ernie's frame, Roy sends out a flash of light every ~3.7 years. In this figure we show Roy’s locations from which he has sent a light flash. According to Ernie, Roy arrives at the star after 14 years of travel. We see Roy has sent 7 flashes during the trip out, but because it takes time for flashes to travel from their points of origin back to Earth, after 14 years, Ernie has only seen the first three flashes in his telescope.

What does Roy see? In his own rest frame, on the outward part of the trip, Roy sees the Earth and the star flying past him (to the left in the figure above) at a speed, $v = 0.86c$. The distance between the Earth and the star, $L$, --- like any other length or distance in the frame of the Earth (from Roy’s perspective, a moving frame) --- appears contracted by a factor of $\gamma \approx 2$ down to 6 light years. At the beginning of his trip, Roy sees Earth flying past him at a speed $0.86c$. An interval, $\Delta t = \frac{L}{v} = \frac{6 \text{ lyr}}{0.86c} = 7 \text{ years}$ later, Roy sees the star passing him (i.e. he has arrived at the star). How many flashes has he received from Ernie? Ernie sends out flashes once/year but Roy is running away from them. Roy will arrive at the star at almost exactly the same time as Ernie’s 2nd year flash arrives (he sends it out at year 2 and, traveling at the speed of light, it arrives at the star at year 14 --- the same time as Roy). Therefore Roy has received 2 flashes. For the outgoing part of the trip, he receives a flash every $\frac{7 \text{ years}}{2 \text{ flashes}} = 3.5 \text{ years}$. This is actually the same period, $\approx 3.7 \text{ years}$, seen by Ernie --- the difference is in the round-off errors mentioned.
at the beginning. I.e., for the trip out, Roy receives pulses every 3.7 years, the same as Ernie --- the situation is symmetric.

Immediately after the turn around, and for sometime thereafter, Ernie’s observations are unchanged. Indeed, he doesn’t even receive the light signaling the turn around until 12 years after the event (year 14+12 years light travel time = year 26 of the journey). Ernie receives a light flash every 3.7 years and if he could see the image of his twin, it would be aging accordingly. On the other hand, immediately after the turn around, the asymmetry of the twins’ situations becomes apparent. Now Roy is running into Ernie’s flashes and over the course of the return, he will receive them every

\[
\frac{7 \text{ years}}{26 \text{ flashes}} = 0.27 \text{ years}^2.
\]

If he could actually see his twin, Roy would view Ernie through his telescope as aging 1 year every 3 months! For the final two years of the trip Ernie, too, receive flashes from Roy at an increased rate --- a flash every 0.27 years. But in the end, it is Ernie who has it right. He has aged 28 years during Roy’s trip where Roy is only 14 years older.

In concluding, it must be emphasized once again that, while the above is an accurate account of what would be seen through telescopes if such a hypothetical journey were undertaken, the understanding is complicated in that what is seen through the telescope includes effects both from time dilation and from the Doppler effect.

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2 7 years = time of the return journey in Roy’s frame. Ernie sends out a total of 28 flashes, Roy received 2 on the outward part, and will receive 26 on the homeward leg.