88TH MIDWEST PDE SEMINAR

Location: The Ohio State University

Date: April 26 - 28, 2024

Main Speakers:

Jerry Bona	(University of Illinois, Chicago)
Jeff Calder	(University of Minnesota)
Alice Chang	(Princeton University)
Hongqiu Chen	(University of Memphis)
Avner Friedman	(Ohio State University)
Alex Himonas	(University of Notre Dame)
Bei Hu	(University of Notre Dame)
Chun Liu	(Illinois Institute of Technology)
Yoichiro Mori	(University of Pennsylvania)
Benoit Perthame	(Sorbonne Université)
Maria Han Veiga	(Ohio State University)
Jiaping Wang	(University of Minnesota)

To register, please visit our homepage:

https://www.asc.ohio-state.edu/lam.184/2024MWPDE/

Local Organizers:

Bo Guan
John Holmes
King-Yeung Lam
Yulong Xing

THE OHIO STATE UNIVERSITY

INSTITUTE FOR MATHEMATICS AND ITS APPLICATIONS

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About

The 88th Midwest PDE Seminar

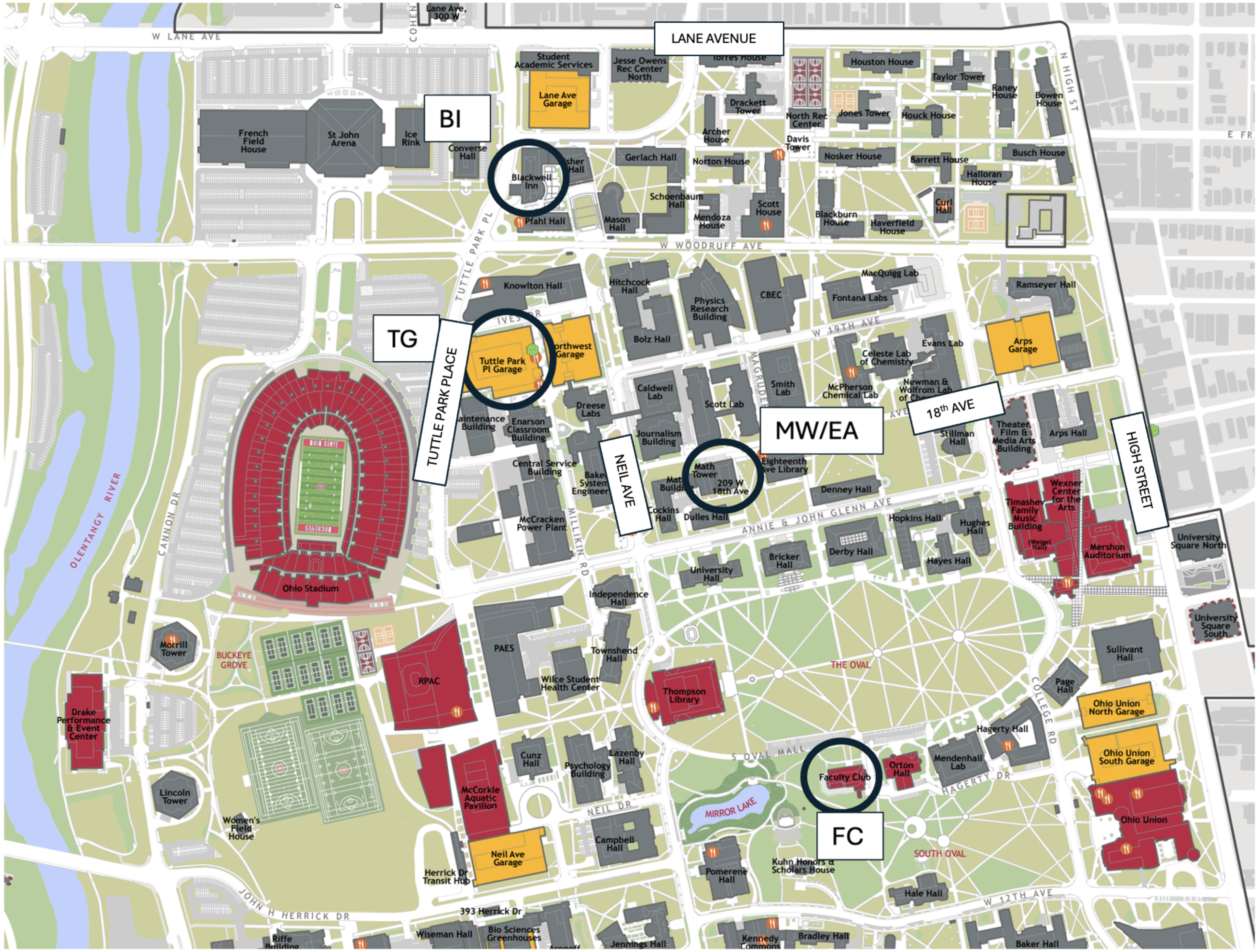
The Midwest PDE Seminar has a long and distinguished tradition. The idea, first conceived by Avner Friedman and others at Northwester University in 1977, was to bring in distinguished speakers from midwest and beyond to present the state-of-art of the field of partial differential equations. Since the first meeting in May 5-7, 1977, it is held every year and being rotated among various research universities in the midwest. It is a high level event devoted to the theory of partial differential equations and their applications to a wide range of areas including geometry, physics, biology, engineering, game theory, and many others.

We are happy to welcome you to the 88th Midwest PDE seminar (MWPDE) at the Ohio State University, in Columbus, OH. The main topics of this conference comprise of

- PDEs from geometry
- PDEs inspired by mathematical biology
- Hyperbolic problems and mathematical physics
- Numerical methods and PDE approach to data science

Organizing committee

Bo Guan   John Holmes   Adrian Lam   Yulong Xing



BI:	Blackwell Inn
TG:	Tuttle Garage
MW:	Math Tower
EA:	EA Building
FC:	Faculty Club

- Location for the talks and coffee break: EA 160 and EA 170 (EA)
- Location for the lunch on April 27: Math Tower 724 (MW)
- Location for the banquet on April 27: The Faculty Club (FC)

Dinner option for April 26

Dinner is not provided on April 26. Here are some recommendations for casual dining nearby:

Options walkable from Blackwell Inn and Math Department

- Blackwell Inn's lounge (American style burger and fries)
- Moy's Kitchen (Chinese, Hong Kong style) 1994 N High St, Columbus, OH 43201
- Diaspora (Korean) 2118 N High St, Columbus, OH 43201
- Roots Natural Kitchen (Good healthy place with lots of veggie options) 10 E 15th Ave, Columbus, OH 43201
- Buckeye Wok Express (Chinese, with some spicy option) 1610 N High St, Columbus, OH 43201
- Blaze Pizza 1708 N High St, Columbus, OH 43201

Options walkable from Aloft

- CoCo's Grill (Chinese with spicy options) 845 W 5th Ave, Columbus, OH 43212
- Columbus Fish Market ($$) 1245 Olentangy River Rd, Columbus, OH 43212
- Potbelly, 1171 Olentangy River Rd, Columbus, OH 43212

Options requiring driving

- Schmidts Sausage Haus (German in German Village) 240 E Kossuth St, Columbus, OH 43206 (could be long wait)
- Lindey's (Convivial American Bistro) ($$$) 169 E Beck St, Columbus, OH 43206 (reserve on google maps or call 6142284343) • Barcelona (Spanish) ($$$) 263 E Whittier St, Columbus, OH 43206 (reserve on google maps or call 6144433699)
- Ty Ginger (Chinese Cantonese with dim sum options) ($$) 5689 Woerner Temple Rd, Dublin, OH 43016
- Akai Hana (Japanese) ($$) 1173 Old Henderson Rd, Columbus, OH 43220 (6144515411)

Timetable

All talks are in EA 160 and EA 170.(See the map for EA building.) Coffee break is located outside EA 160 and EA 170.

Friday, April 26

13:00–13:30	**Registration & Welcome (EA Building)**		
13:30–14:20	MT	**Avner Friedman** Ohio State	Free boundary problems in bio-medicine
14:30–15:20	MT	**Bei Hu** Notre Dame	Periodic solution for a free-boundary tumor model - recent progress
15:20–15:50	**Coffee**		
15:50–16:40	MT	**Hongqiu Chen** U. Memphis	Water Waves: Bore Propagation

20-min Talks in Parallel Sessions

		EA 160		EA 170
16:50–17:10	ST	**Xiaoqin Guo** U. Cincinnati Optimal rate of convergence in the stochastic homogenization of a non-divergence form operator		**Alessandro Arsie** U. Toledo Miura Invariants and Integrable Systems of conservation laws
17:15–17:35	ST	**Son Tu** Michigan State Properties of the effective Hamiltonian and homogenization of the Hamilton-Jacobi equation		**Liding Yao** Ohio State On Seeley-type universal extension operators for the upper half space
17:40–18:00	ST	**Nicollò Forcillo** Michigan State Lipschitz continuity for almost minimizers of a degenerate Bernoulli-type functional		**Ryan Thompson** U. North Georgia Continuity Properties of a Generalized Camassa-Holm System

Saturday, April 27

8:15–9:00	**Breakfast provided at the EA building outside EA 160**		
9:00–9:50	MT	**Alice Chang** Princeton	On a conformal Einstein fill in problem
10:00–10:50	MT	**Jiaping Wang** Minnesota	A volumetric Minkowski inequality and applications
15:20–15:50	**Coffee**		
11:00–11:50	MT	**Benoit Perthame** Sorbonne	Structured equations in biology; relative entropy, Monge-Kantorovich distance
12:00–13:30	**Lunch**		
13:30–14:20	MT	**Maria Han Veiga** Ohio State	On connections and efficiency of the (modern) arbitrary derivative (ADER) approach
14:30–15:20	MT	**Jeff Calder** Miinnestoa	PDEs and graph-based semi-supervised learning
15:20–15:40	**Coffee**		
15:40–16:30	MT	**Yoichiro Mori** U. Pennsylvania	Inextensible interface problem in 2D Stokes fluid
16:30–16:35		**EA 160**	**EA 170**
16:35–16:55	ST	**Charis Tsikkou** West Virginia One and Multi-Dimensional Systems of Conservation Laws with Unbounded Solutions	**Wenrui Hao** Penn. State Exploring Solution Structures in Nonlinear Differential Equations
17:00–17:20	ST	**Prerona Dutta** Ohio State Lagrangian Transformations for Non Convex Scalar Conservation Laws	**German Mora Saenz** Ohio State TBD
17:30–	**Banquet at the Faculty Club**		

All talks are in EA 160 and EA 170; Breakfast and Coffee break is located outside EA 160 and EA 170; Lunch is provided at Math Tower MW724; Banquet is provided at the Faculty Club.

Sunday, April 28

8:15–9:00	**Breakfast (outside of EA 160)**		
9:00–9:50	MT	**Jerry Bona** U. Illinois Chicago	Dissipation versus finite time blow-up
10:00–10:50	MT	**Chun Liu** Illionois Inst. Tech	Energetic Variational Approaches for Phase Field Models: Boundary Conditions and Temperature Effects
15:20–15:50	**Coffee**		
11:00–11:50	MT	**Alex Himonas** Notre Dame	Analysis of IBVP via the Fokas Method: The KdV and NLS models
12:00–12:30	**Farewell**		

All talks are in EA 160 and EA 170. See the map for EA building.
Breakfast and Coffee break is located outside EA 160 and EA 170.

List of Abstracts – Talks

Friday, April 26

Free boundary problems in bio-medicine

Avner Friedman

Ohio State University

A free boundary problem (FBP) consists of a system of PDEs which needs to be solved simultaneously with the unknown boundary of the domain. Such problems arise in models of bio-medical processes, for example: Cancer growth with treatment aimed at decreasing the growing unknown boundary; a growing plaque in cardiac artery which by blocking the artery will result in heart attack; chronic or diabetic dermal wound which, if not healed in proper time, may require amputation; cartilage shrinkage in rheumatoid arthritis; fungal skin infection which, if not treated, may spread over the whole body. Each if these diseases was modeled as a FBP, and numerical simulations of the model were performed and used to gain understanding, and to make recommendations for effective treatments in experimental studies or in clinical trials. But what about rigorous analysis, e.g. theorems and proofs? In this talk I will briefly review such models and then proceed to describe mathematical results for simplified version of the models, showing that these results actually capture, in some "generalized" sense, those derived by simulations. I will also mention some open questions.

Periodic solution for a free-boundary tumor model - recent progress

Bei Hu

University of Notre Dame

A large number of free boundary models describing solid tumor growth were proposed and analyzed in recent decades; they involve mass conservation laws and reaction-diffusion processes for cell densities; nutrient concentrations are also incorporated into these models. Many models assume tumor cells are immersed in a constant supply of nutrients for simplicity. In reality, nutrient concentration varies with the intake of food. In this talk, we study the periodic solution and stability for the radially symmetric case. In particular, we shall establish the existence and uniqueness of the periodic solution in the biologically reasonable case and establish a global attractor in the class of radially symmetric initial data. This is a joint work with Dr Yaodan Huang.

Water Waves: Bore Propagation

Hongqiu Chen

University of Memphis

Discussed here is a regularized version of the classical Gardner equation

$$u_t + u_x + uu_x + Au^2u_x - u_{xxt} \, = \, 0,$$

that arises in hydrodynamics and plasma physics. This initial-value problem posed on all of $\mathbb{R}$ will be considered with bore-like initial data. That is, the initial wave configuration will consist of a moderately smooth function that asymptotes to zero as the spatial variable $x \to +\infty$, but converges to $r > 0$ as $x \to -\infty$. Such initial profiles can arise in internal wave propagation, for example. In their idealized versions set on all of $\mathbb{R}$, they possess an infinite amount of potential energy. This makes the analysis of the initial-value problem a little more subtle than the common situation where the initial profile is assumed to be localized, so being modeled by Sobolev-class initial data.

Optimal rate of convergence in the stochastic homogenization of a non-divergence form operator

Xiaoqin Guo

University of Cincinnati

We consider the rate of convergence of a random non-divergence form difference equation on $\mathbb{Z}$ to its "effective" differential equation on $\mathbb{R}$. When the coefficient field has a finite range of dependence, it is expected that the optimal rate is $O(\epsilon)$. In this talk, we will show that if the distribution of the coefficient field has isotropic symmetry, then a better rate $O(\epsilon^2)$ can be achieved in high dimensions. Joint work with Hung V. Tran and Timo Sprekler.

On Seeley-type universal extension operators for the upper half space

Liding Yao

Ohio State University

The Seeley's extension operator is the simplest model of linear extension operator on domains. It extends function on the upper half space $\mathbb{R}^n_+ := \{x = (x', x_n) \in \mathbb{R}^n | x_n > 0\}$ to $\mathbb{R}^n$ that has the form $Ef(x) = \sum_j a_j f(x', -b_j x_n)$ for $x_n < 0$, where a_j, b_j are numbers. We prove the existence of $(a_j, b_j)_j$ such that $\sum_j a_j (-b_j)^k = 1$ for all integer k, and as a result the corresponding operator is bounded on all the nice function spaces, e.g. C^k, Sobolev $W^{k,p}$. This is joint with Haowen Lu.

Properties of the effective Hamiltonian and homogenization of the Hamilton-Jacobi equation

Son Tu

Michigan State University

The homogenization of Hamilton-Jacobi equations in the periodic setting was pioneered by Lions, Papanicolaou, and Varadhan in the 1980s. However, understanding the intricate properties of the effective Hamiltonian and its impact on the homogenization limit across various settings remains a substantial open problem. In this presentation, we will discuss recent developments that highlight a connection between this phenomenon and weak KAM theory on a qualitative level, as well as present some quantitative results.

Miura Invariants and Integrable Systems of conservation laws

Alessandro Arsie

University of Toledo

I will describe invariants (Miura invariants) under Miura transformations associated to systems of conservation laws and how they are related to dispersion relations. Furthermore, focusing on one-parameter families of dispersionless systems of integrable conservation laws associated to the Coxeter groups of rank 2, we study the corresponding integrable deformations up to order 2 in the deformation parameter.

Each family contains both bi-Hamiltonian and non-Hamiltonian systems of conservation laws and therefore we use it to probe to which extent the properties of the dispersionless limit impact the nature and the existence of integrable deformations. I will also briefly comment on the existence of integrable deformations to all order (in the non-Hamiltonian case), using recent results obtained via cohomological field theory. In the Hamiltonian case, the existence of integrable deformations to all order was already known.

These results have been obtained in collaboration with Paolo Lorenzoni and for the part concerning cohomological field theory also with Alexander Buryak and Paolo Rossi.

Continuity Properties of a Generalized Camassa-Holm System

Ryan Thompson

University of North Georgia

In this talk, we consider a generalized two-component Camassa-Holm system. Based on local well-posedness results and lifespan estimates, we establish sharpness of continuity on the data-to-solution map. Our proof is based upon the use of approximate solutions.

Lipschitz continuity for almost minimizers of a degenerate Bernoulli-type functional

Nicolò Forcillo

Michigan State University

In this talk, we deal with almost minimizers of the energy functional

$$J_p(u,\Omega) := \int_\Omega \Big(|\nabla u(x)|^p + \chi_{\{u>0\}}(x) \Big)\, dx, \quad p > 1, \tag{0.1}$$

where Ω is a bounded domain in $\mathbb{R}^n$ and $u \geq 0$. The functional J_p is a generalization to each $p > 1$ of the classical one-phase (Bernoulli) energy functional ($p = 2$ in (0.1)).
Almost minimizers of J_2 were investigated in [David and Toro, 2015, David et al., 2019]. However, D. De Silva and O. Savin provided in [De Silva and Savin, 2020] a different approach than [David and Toro, 2015, David et al., 2019], based on nonvariational techniques, to study almost minimizers of J_2 and their free boundaries. Precisely, inspired by [De Silva and Savin, 2021], they showed that almost minimizers of J_2 are "viscosity solutions" in a more general sense. This property roughly means that almost minimizers satisfy comparison in a neighborhood of a touching point whose size depends on the properties of the test functions. Once this fact was established, the regularity of the free boundary for almost minimizers followed via the techniques developed by De Silva in [De Silva, 2011].
In this talk, we present an optimal Lipschitz continuity result for almost minimizers of J_p, with $p > \max\left\{\frac{2n}{n+2}, 1\right\}$. Our approach is inspired by [De Silva and Savin, 2020]. Our method mostly relies on using p-harmonic replacements as competitors. The regularity properties of these replacements indeed allow us to infer the Lipschitz continuity of almost minimizers. In particular, we first prove a dichotomy-type result, and next, we improve and iterate one of the two alternatives of the dichotomy. The talk is based on joint work with S. Dipierro, F. Ferrari, and E. Valdinoci, see [Dipierro et al.].

Saturday, April 27

On a conformal Einstein fill in problem

Alice Chang

Princeton University

Given a manifold (M^n, h), when is it the boundary of a conformally compact Einstein manifold (X^{n+1}, g), in the sense that there exists some defining function r on X so that $r^2\, g$ is compact on the closure of X and $r^2\, g$ restricted to M is the given metric h? The model example is the n-sphere as the conformal infinity of the hyperbolic $(n+1)$ ball.

In the special case when $n = 3$, one can formulate the problem as an Dirichlet to Neumann type inverse problem. In the talk, I will report on some progress made with Yuxin Ge on the issues of the “compactness", and as an application, the “existence" and “uniqueness" of the fill in problem for a class of metrics defined on the 3-sphere with positive scalar curvature.

A volumetric Minkowski inequality and applications

Jiaping Wang

University of Minnesota

The classical Minkowski inequality states that the surface area of a convex compact domain in the Euclidean space is bounded from above by an integral of its mean curvature with equality if and only if the domain is a ball. Combining with the isoperimetric inequality, one concludes that the volume of the domain is bounded by the integral of the mean curvature as well. In this talk, we will discuss a generalization of the latter inequality to complete manifolds with Ricci curvature bounded from below. As an application, it implies that a complete manifold with sufficiently large bottom spectrum relative to its Ricci curvature lower bound admits no compact minimal hypersurfaces. In particular, this is the case for conformally compact Einstein manifolds whose boundaries at infinity have nonnegative scalar curvature.

Structured equations in biology; relative entropy, Monge-Kantorovich distance

Benoit Perthame

Sorbonne Université

Models arising in biology are often written in terms of Ordinary Differential Equations. The celebrated paper of Kermack-McKendrick (1927), founding mathematical epidemiology, showed the necessity to include parameters in order to describe the state of the individuals, as time elapsed after infection. During the 70s, many mathematical studies were developed when equations are structured by age, size, more generally a physiological trait. The renewal, growth-fragmentation are the more standard equations.

The talk will present structured equations, show that a universal generalized relative entropy property is available in the linear case, which imposes relaxation to a steady state under non-degeneracy conditions. In the nonlinear cases, it might be that periodic solutions occur, which can be interpreted in biological terms, e.g., as network activity in the neuroscience.

When the equations are conservation laws, a variant of the Monge-Kantorovich distance (called Fortet-Mourier distance) also gives a general non-expansion property of solutions.

On connections and efficiency of the (modern) arbitrary derivative (ADER) approach

Maria Han Veiga

Ohio State University

The (modern) arbitrary derivative (ADER) approach is a popular technique for the numerical solution of differential problems based on iteratively solving an implicit discretization of their weak formulation. We first establish the connection between the ADER method and the deferred correction (DeC) formalism to determine the formal order of the accuracy of the method. Then, focusing on an ODE context, we investigate several strategies to improve the ADER approach: first, we emphasise on the order of accuracy of the method in connection with the polynomial discretization of the weak formulation, demonstrating that precise choices lead to higher-order convergences in comparison to the existing literature. Then, through the Deferred Correction (DeC) formalism, we introduce efficient p-adaptive modifications. These are defined by matching the order of accuracy achieved and the degree of the polynomial reconstruction at each iteration.

In the end I will provide analytical and numerical results, including the stability analysis of the new modified methods, the investigation of the computational efficiency, an application to adaptivity and an application to hyperbolic PDEs with a Spectral Difference (SD) space discretization.

PDEs and graph-based semi-supervised learning

Jeff Calder

University of Minnesota

Graph-based semi-supervised learning is a field within machine learning that uses both labeled and unlabeled data with an underlying graph structure for classification and regression tasks. In problems where very little labeled data is available, the classical Laplacian regularization gives very poor results. This can be explained through its PDE continuum limit, which is an ill-posed elliptic equation. Much work recently has been focused on designing graph-based learning methods with well-posed continuum limits, including the p-Laplacian, higher order Laplacians, re-weighted Laplacians, and Poisson equations.

In this talk, we will survey this literature, and present our recent work on using Poisson equations for semi-supervised learning. We will present theoretical results which establish that learning with Poisson equations is provably well-posed at arbitrarily low label rates, and experimental results showing that it outperforms existing graph-based semi-supervised learning methods on challenging data sets. We will also present some recent work on applications of Poisson learning to graph-based active learning, where the goal is to select a training set with the most informative examples, often in a sequential online setting starting at extremely low label rates.

Inextensible interface problem in 2D Stokes fluid

Yoichiro Mori

University of Pennsylvania

We consider the dynamics of a closed inextensible interface immersed in a 2D Stokes fluid, a model that has been used for 2D simulations of vesicle dynamics. In this model, a 1D closed interface exerts a bending force and the interface is subject to an inextensibility constraint. As part of the problem, one must solve for the unknown tension that ensures membrane inextensibility. Given a force exerted on the interface, we first show that the problem of determining the tension is solvable if and only if the interface is not a circle. Using this result, we prove local-in-time well-posedness for this problem. We will finally discuss open questions and future directions.

One and Multi-Dimensional Systems of Conservation Laws with Unbounded Solutions

Charis Tsikkou

University of West Virginia

We consider one and multidimensional systems of two and three conservation laws, which are routinely used in simulations, but we lack a proper mathematical understanding. For example, in compressible fluid flow, radial similarity flows with focusing shocks can be studied in detail and are of theoretical interest as they involve unbounded amplitudes. We produce a coherent explanation and description of these blowup solutions.

Exploring Solution Structures in Nonlinear Differential Equations

Wenrui Hao

Pennsylvania State University

In this talk, I will survey the numerical methods based on the homotopy continuation technique to solve nonlinear differential equations with multiple solutions. I will explore this approach with several benchmark problems in pattern formation. Additionally, I will demonstrate the utility of neural networks in overcoming the curse of dimensionality introduced by the homotopy method when solving large-scale systems, thereby enhancing the efficiency and effectiveness of this computational approach.

Lagrangian Transformations for Non Convex Scalar Conservation Laws

Prerona Dutta

Ohio State University

Reformulating systems of hyperbolic conservation laws in a Lagrangian form provides an equivalent description of the problem by means of particle paths. Research in this direction was pioneered by Arnold in 1966, who described the dynamics of an ideal fluid as a geodesic flow on the group of volume-preserving diffeomorphisms of a fixed domain equipped with the metric defined by kinetic energy. For systems which describe fluid flow, construction of particle paths follows naturally from the associated velocity field. However, the idea of a particle path can be extended to scalar equations and to systems which do not include velocity fields explicitly. In this talk, we demonstrate how to find Lagrangian transformations in the form of particle paths, for all scalar conservation laws having a smooth flux not necessarily convex, using the notion of weak diffeomorphisms.

Sunday, April 28

Dissipation versus finite time blow-up

Jerry Bona

University of Illinois, Chicago

The question under investigation is what kinds of dissipation can win the battle over possible finite-time blow-up of solutions of certain classes of partial differential equations.

Energetic Variational Approaches for Phase Field Models: Boundary Conditions and Temperature Effects

Chun Liu

Illinois Institute of Technology

The boundary effects and thermal effects play crucial roles in real life applications. These problem give formidable challenges to the mathematical analysis and numerical simulations. I will demonstrate the difficulties and subtleties by looking at these effects in various general diffusion dynamics, especially those related to phase field models.

Analysis of IBVP via the Fokas Method: The KdV and NLS models

Alex Himonas

University of Notre Dame

We discuss the progress made during the last few years in solving initial-boundary value problems for evolution equations. Our focus is on dispersive equations and systems and in particular the Korteweg-de Vries and Nonlinear Schrödinger equations with data in low regularity Sobolev spaces. Using the Fokas solution formula for the forced linear problem we derive appropriate linear estimates, which combined with the multilinear estimates suggested by the nonlinearity, make the well-posedness study of initial-boundary value problems analogous to that of initial value problems. Thus, in the framework of Bourgain spaces, we obtain similar optimal well-posedness results. The talk is based on work with A. Fokas, D. Mantzavinos and F. Yan.

Partner Institutions and Sponsors

The MWPDE conference is funded by the Mathematical Research Institute at Ohio State University and the Institute for Mathematics and its Applications at University of Minnesota.

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