Devoir 1

Instructeur: King-Yeung Lam

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Prove the following.

Corollary 0.1. *Suppose* $u, v \in C(\overline{\Omega}_T)$ *satisfy*

$$\begin{cases} u_t + \mathcal{L}u \leq f(x,t,u,Du) \quad \text{and} \quad v_t + Lv \geq f(x,t,v,Dv) & \text{in } \Omega_T, \\ B(u-v) \leq 0 & \text{in } S\Omega_T, \end{cases} \tag{0.1}$$

in the generalized sense. If $u(x,0) \leq v(x,0)$ *in* Ω*, then* $u \leq v$ *in* Ω_T*.*

Here's an outline of the proof.

(a) One may assume without loss of generality that f is decreasing in the third variable u. (Hint: By finding a PDE for which $\tilde{u}(x,t) = e^{Mt}u(x,t)$ and $\tilde{v}(x,t) = e^{Mt}v(x,t)$ form a pair of sub- and supersolutions, where some $M \gg 1$.)

(b) Prove that if $u(x,0) < v(x,0)$ in $\overline{\Omega}$, then $u(x,t) < v(x,t)$ in $\overline{\Omega}_T$. (Hint: Suppose to the contrary, then there exists $(x_0,t_0) \in \overline{\Omega} \times (0,T]$ (which means $t_0 > 0$) such that $u < v$ in $\overline{\Omega} \times [0,t_0)$ and $u(x_0,t_0) = v(x_0,t_0)$. Derive a contradiction with definition of generalized super/subsoltions.

(c) Show that $u(x,t) \leq v(x,t) + \epsilon$ in Ω_T, for each $\epsilon > 0$.