Devoir 2

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1. Let $u(x,t) \in C^{2,1}(\mathbb{R} \times (0,\infty))$ be a non-negative, non-trivial, and bounded solution of the heterogeneous Fisher-KPP equation

$$u_t - u_{xx} = u(r(x,t) - u) \quad \text{in } \mathbb{R} \times (0,\infty). \tag{0.1}$$

Suppose $\sup_{\mathbb{R}\times(0,\infty)} |u| < +\infty$, and there exists $0 < a < b$ such that

$$\liminf_{t\to\infty} \inf_{a+\eta<\frac{x}{t}<b-\eta} u(x,t) > 0 \quad \text{for each } 0 < \eta < \frac{b-a}{2},$$

and that $r(x,t) \equiv 1$ for $at < x < bt$ and $t > 0$. Shpw that

$$\lim_{t\to\infty} \sup_{a+\eta<\frac{x}{t}<b-\eta} |u(x,t) - 1| = 0 \quad \text{for each } 0 < \eta < \frac{b-a}{2}.$$

[Hint: Show the following classification result: Let U be an entire solution $U \in C^{2,1}(\mathbb{R}\times\mathbb{R})$ of

$$\begin{cases} U_t - U_{xx} = U(1-U) & \text{for } (x,t) \in \mathbb{R}\times\mathbb{R}, \\ 0 < \inf_{\mathbb{R}^2} U \leq \sup_{\mathbb{R}^2} U < +\infty \end{cases}$$

Then necessarily $U(x,t) \equiv 1$ for $(x,t) \in \mathbb{R}^2$.]