Devoir 3

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1. Let X be a Banach space ordered by a solid cone K, and let $T : X \to X$ be a compact, linear and strongly monotone operator. Use the strong form of the classical Krein-Rutman Theorem to prove that for $\lambda = r(T)$ and $h \in K \setminus \{0\}$, the inhomogeneous equation

$$\lambda x - Tx = h$$

has no solution in X. Then use this fact to show that the principal eigenvalue $r(T)$ is algebraically simple.

2. Let μ_1 and ϕ_1 be the principal eigenvalue and positive eigenfunction of

$$-\Delta\phi = f(x,0)\phi + \mu\phi \quad \text{in } \Omega, \quad \mathbf{n}\cdot\nabla\phi = 0 \quad \text{on } \partial\Omega.$$

If $\mu_1 < 0$, show that the the single population is strongly persistent, i.e. for each nonnegative, nontrivial solution $u(x,t)$ of

$$\begin{cases} u_t - \Delta u = f(x,u)u & \text{in } \Omega \times (0,\infty), \\ \mathbf{n}\cdot\nabla u & \text{on } \partial\Omega \times (0,\infty), \\ u(x,0) = u_0(x) & \text{in } \Omega, \end{cases}$$

there exists a positive constant $\epsilon_0 > 0$ such that

$$\liminf_{t\to\infty} \left[\inf_{x\in\Omega} u(x,t)\right] \geq \epsilon_0.$$

[Optional: Show that the population is uniformly strongly persistent, i.e. ϵ_0 can be chosen independently of all nonnegative, nontrivial initial data u_0. As a first step, show that if $\mu_1 < 0$ and has eigenfunction $\phi_1 > 0$, there exists $\epsilon_0 > 0$ such that for any $t_0 \in \mathbb{R}$,

$$\underline{u}(x,t) = \epsilon_0 e^{-\mu_1(t-t_0)}\phi_1(x) \quad \text{is a subsolution in } \Omega \times (-\infty, t_0].$$

The show that every nonnegative, nontrivial solution satisfies $\liminf\limits_{t\to\infty}(u(x,t) - \epsilon_0\phi_1(x)) \geq 0$.]