Devoir 4

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1. Let ϕ_1 be the positive eigenfunction of

$$-\epsilon^2 \Delta\phi_1 - c(x)\phi_1 = \mu_1\phi_1 \quad \text{in } \Omega, \quad \mathbf{n}\cdot\nabla\phi_1 + p^0\phi_1 = 0 \quad \text{on } \partial\Omega,$$

where Ω is a bounded domain with smooth boundary $\partial\Omega$; $\mathbf{n}$ is the outer unit normal vector on $\partial\Omega$; $c, p^0 \in C^\infty(\overline{\Omega})$ such that $p^0 \geq 0$. Show, without assuming convexity of Ω, that

$$\phi_1 \to 0 \quad \text{locally uniformlly in } \left\{ x \in \overline{\Omega} : \; c(x) < \sup_{\Omega} c \right\}.$$

[Hint: The case $x_0 \in \Omega$ is the same as in convex case. For each $x_0 \in \partial\Omega$, construct a sequence of smooth functions $\rho_k(x)$ such that

$$n \cdot \nabla\rho_k\big|_{\partial\Omega} > 0$$

and ρ_k attains the unique global minimum at x_k for some sequence $x_k \in \Omega$ such that $x_k \to x_0$. The consider the mininum point of $w_\epsilon(x) + \rho_k(x)$.]