Devoir 6

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le 21 fevrier, 2022

1. Let $\mu > 0$ and $P, m \in C^2(\overline{\Omega})$ be given. Let $\theta_P(x)$ be the unique positive equilibrium of

$$\begin{cases} \mu\nabla \cdot [\nabla u - u\nabla P] + u(m(x) - u) = 0 & \text{in } \Omega, \\ \mathbf{n} \cdot [\nabla u - u\nabla P] = 0 & \text{on } \partial\Omega. \end{cases}$$

   Show that $\theta_P(x) \equiv m(x)$ if and only if $P(x) - \log m(x)$ is a constant.

2. Let $F \in C(\overline{\Omega} \times [0, 1])$ be given. Show that $\int_0^1 \sup_{x\in\Omega} F(x, t)\, dt > 0$ if and only if

$$\int_0^1 F(\gamma(t), t)\, dt > 0$$

   for some $\gamma \in C^1(\mathbb{R}; \operatorname{Int}\Omega)$ that is 1-periodic in t.