# Devoir 8

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Let $c(x,t) \in C^0(\overline{\Omega} \times \mathbb{R})$ be given, in particular,

$$\|c\|_\infty := \sup_{\Omega \times \mathbb{R}} c \quad \text{is finite.}$$

Let $\varphi > 0$ be a positive solution of

$$\begin{cases} \partial_t \varphi - \Delta \varphi = c(x,t)\varphi & \text{in } \Omega \times [-3,2], \\ \mathbf{n} \cdot \nabla \varphi = 0 & \text{on } \Omega \times [-3,2]. \end{cases} \tag{0.1}$$

Show that there exists $C_0 = C_0(\Omega, \|c\|_\infty)$ (but independent of $\varphi$ such that

$$\sup_{\Omega \times [-2,2]} \leq e^{4\|c\|_\infty} \sup_{\Omega} \varphi(\cdot, -2), \tag{0.2}$$

$$\inf_{\Omega \times [-2,2]} \geq e^{-4\|c\|_\infty} \inf_{\Omega} \varphi(\cdot, -2). \tag{0.3}$$

Combining with the Harnack principle (due to J. Húska) which says that $\sup_\Omega \varphi(\cdot, -2) \leq C\varphi(\cdot, -2)$, we obtain $C_1$ independent of $\varphi$ such that

$$\sup_{\Omega \times [-2,2]} \varphi \leq C_1 \inf_{\Omega \times [-2,2]} \varphi.$$

Next, let $\varphi > 0$ be a positive solution of

$$\begin{cases} \partial_t \varphi - \Delta \varphi = c(x,t)\varphi & \text{in } \Omega \times [-k-1,k], \\ \mathbf{n} \cdot \nabla \varphi = 0 & \text{on } \Omega \times [-k-1,k]. \end{cases} \tag{0.4}$$

Show that there exists $C_2, \gamma > 0$ independent of $\varphi$ such that

$$\frac{1}{C_2} e^{-\gamma|t|} \leq \frac{\varphi(x,t)}{\sup_\Omega \varphi(\cdot, 0)} \leq C_0 e^{\gamma|t|} \quad \text{for } x \in \Omega,\ |t| \leq k.$$