Devoir 9

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In the lecture, it is show that the positive equilibirum of the selection-mutation model

$$\begin{cases} \partial_t u = \epsilon^2 \Delta u + (m(z) - \int_\Omega m(y)u(y,t)\,dy)u & \text{in } \Omega \times (0,\infty), \\ \mathbf{n}\cdot\nabla u = 0 & \text{on } \partial\Omega \times (0,\infty), \\ u(z,0) = u_0(z) & \text{in } \Omega, \end{cases}$$

is (i) globally attracting and (ii) a scalar multiple of the principal eigenfunction ϕ_ϵ of

$$\begin{cases} \epsilon^2 \Delta\phi + m(z)\phi + \mu\phi = 0 & \text{in } \Omega, \\ \mathbf{n}\cdot\nabla\phi = 0 & \text{on } \partial\Omega, \\ \int_\Omega \phi\,dz = 1, \end{cases}$$

When $m(z)$ attains global maximum at a unique point $\bar{z}$, it was proved that $\phi_\epsilon \to \delta(z - \hat{z})$.

Suppose now that m is C^2 and attains its global maximum value at exact two points $z_i \in \Omega$. Show that, up to a subsequence,

$$\phi_\epsilon(z) \to a\delta(z - z_1) + (1-a)\delta(z - z_2) \quad \text{for some } a \in [0,1].$$

Suppose, in addition, that

$$\xi^T D^2 m(z_1)\xi < \xi^2 D^2 m(z_2)\xi, \quad \text{for all } \xi \in \mathbb{R}^n$$

show that $\phi_\epsilon(z) \to \delta(z - z_2)$.