

## Title and abstracts for talks on December 15, 2021

Sebastian Schreiber (UC Davis)

Title: Coevolution of Patch Selection in Stochastic Environments

Abstract: Species live and interact in patchy landscapes where environmental conditions vary both in time and space. In the face of this spatial-temporal heterogeneity, species may co-evolve strategies of patch-selection which determine their spatial distributions. Under equilibrium conditions and no costs to dispersal, coevolution of patch-selection leads to an ideal-free distribution for each species: the per-capita growth rates are zero in the patches occupied by the species and negative in the unoccupied patches. These ideal-free distributions explain some observed empirical patterns including enemy-free space and complete spatial segregation of competing species i.e. the ghost of competition past. However, they do not explain why some species live in sink patches where their per-capita growth rate is negative, why competing species aren't always fully spatially segregated, or why predators make use of prey-free patches. To understand how accounting for temporal fluctuations may explain these latter patterns, we analyze a system of  $n$  species, stochastic Lotka-Volterra models that account for patchy space implicitly. To identify potential coevolutionary endpoints, we introduce a definition of a stochastic coevolutionarily stable strategy (coESS): a collection of patch-selection strategies for each species that resist invasion attempts from mutant subpopulations of any species utilizing other patch-selection strategies. We show that the coESS can be characterized via a system of second-order equations of the parameters. This characterization shows, surprisingly, that the stochastic per-capita growth rates of species are negative in all occupied patches despite the species persisting. Applying this characterization to the coevolution of habitat-choice of competitors and predator-prey systems identifies under what environmental conditions, natural selection exorcizes the ghost of competition past and banishes some predators to prey-free patches. Collectively, our results highlight the importance of temporal fluctuations, spatial heterogeneity and species interactions on the evolution of species' spatial distributions. This work is in collaboration with Alex Hening (College Station) and Dang Nguyen (Tuscaloosa).

Yu Jin (Nebraska-Lincoln)

Title: The dynamics of a two host-two virus system in a chemostat environment.

Abstract. The coevolution or coexistence of multiple viruses with multiple hosts has been an important issue in viral ecology. This work is to study the mathematical properties of the solutions of a chemostat model for two host species and two virus species. By virtue of the global dynamics of its submodels and the theories of uniform persistence and Hopf bifurcation, we derive sufficient conditions for the coexistence of two hosts with two viruses and coexistence of two hosts with one virus, as well as occurrence of Hopf bifurcation.

Chris Cosner (Miami)

Title: Reaction-diffusion models that are cooperative at low densities and competitive at high densities.

Abstract: Methods based on monotone iteration or the theory of monotone dynamical systems have been widely used in the study of mathematical models in biology, and in the study of reaction-diffusion-advection systems more generally. Many models are either cooperative or competitive or involve competing coalitions. All of those can be viewed as monotone semi-dynamical systems with respect to some ordering. However, including more effects found in nature in models related to the dispersal of organisms can lead to systems that are cooperative at low densities but competitive at high densities. This makes their analysis more challenging. In this talk I will discuss some examples of such systems and the phenomena that arise in them.

One type of system that is cooperative at low densities but competitive at high densities arises in modeling a single population where individuals can switch between different movement modes. Actual animals are often observed to switch between two or more different movement modes for large scale search to locate resources and for small scale search and exploitation once those are located. At longer timescales mutations that increase or decrease movement rates can spread through populations. This has been observed in the invasion of cane toads in Australia. Combining switching in movement modes with logistic self-limitation of populations leads to models that are cooperative at low densities and competitive at high densities.

Another place where such systems arise is in models for stage structured populations where adults and juveniles compete with each other for resources. It is well known that in bounded domains logistic reaction-diffusion models predict that slower diffusion rates are advantageous relative to faster diffusion, but in stage structured models that is not necessarily true.

A third context where systems that are cooperative at low densities but competitive at high densities arise is in models for the evolution of dispersal in a population with an Allee effect. In that modeling context, ecologically identical subpopulations with different dispersal rates or modes compete with each other. That leads to models whose dynamical terms have the forms  $f(x,u+v)u$ ,  $f(x,x,u+v)v$ , where  $u$  and  $v$  are population densities. In the case of an Allee effect,  $f(x,u)$  is increasing when  $u$  is small but decreasing when  $u$  is large, which leads to a system that again is cooperative at low densities and competitive at high densities.

The details of problems and properties differ among the various types of models that will be described, but together they suggest that trying to understand the dynamics of systems that can change from competitive to cooperative is a challenging but worthwhile area of research.

Rebecca Tyson (British Columbia)

Title: Indirect competitors in a spatial landscape: Mutualist hosts

Abstract: Plant-fungal and plant-pollinator mutualisms are ubiquitous and of fundamental importance to agricultural and ecological sustainability. In both cases, the plants play the role of host, vying to attract the attention of their mutualist partner. In this talk, we look at the interaction between the spatial arrangement of competing hosts, and the memory-guided behaviour of the mutualist foragers. This interaction leads to particular patterns of the benefits accrued to the host population. Our results provide further insights into the role of memory in foraging patterns, and suggest approaches for increasing pollination services to crops in agricultural landscapes.

Maria Martignoni (New Foundland)

Title: Mechanisms for coexistence and competitive exclusion among mutualists

Abstract: In the last few decades, microbial inoculants have been used as organic fertilizers worldwide. Among the most widely used commercial products are arbuscular mycorrhizal (AM) fungi, as these fungi can associate with a variety of crops. Despite the potential benefits for soil quality and crop yield associated with AM fungal colonization, experiments assessing the persistence of the fungi in the field have yielded inconsistent results. Additionally, it is not yet clear whether or not the introduction of commercial inoculants could lead to changes to the resident fungal community, and eventually to invasion of the commercial products with a possible displacement of resident species. Here we use a partial differential equation model to assess the potential biodiversity risks and benefits for plant productivity deriving from inoculation. We study the impact of AM fungal inoculation on the resident fungal community and on plant growth at a landscape scale. We determine how inoculant persistence and spread is affected by its competition with resident fungal species, by its mutualist quality, and by fungal dispersal. We suggest that the increase in fungal abundance due to inoculation always leads to a short-term increase in host productivity, regardless of inoculant identity. However, the use of strongly competing inoculants constitutes a biodiversity risk, and may result in the invasion of low quality mutualists.

Xueying Wang (Washington State)

Title: Target reproduction numbers for reaction-diffusion population models

Abstract: A very important population threshold quantity is the target reproduction number, which is a measure of control effort required for a target prevention, intervention, or control. This concept, as a generalization of type reproduction number, was first introduced in Shuai et al. (J Math Biol 67:1067-1082, 2013) for nonnegative matrices with immediate applications to compartmental population models of ordinary differential equations. The current paper is devoted to the study of all target reproduction numbers for reaction-diffusion population models with compartmental structure. It turns out that the target reproduction number can be regarded as the basic reproduction number of a modified system, where the state of newborn individuals is limited to the target control set and the offspring from the non-target set is regarded as a part of the transition. In other words, the target reproduction number can be interpreted as the expected

number of offspring in a specific target set that a primary newborn individual of the same set would produce during its lifetime. We also characterize the target reproduction number so that it can be easily computed numerically for reaction-diffusion models. At the end, we demonstrate our theoretical observations using two examples. This is joint work with Xiao-Qiang Zhao.

Matt Holzer (George Mason)

Title: Epidemic spreading in complex networks as front propagation into unstable states

Abstract: We study epidemic arrival times in meta-population disease models through the lens of front propagation into unstable states. We demonstrate that several features of invasion fronts in the PDE context are also relevant to the network case. The susceptible-infected-recovered model on a network is linearly determined in the sense that the arrival times in the nonlinear system are well approximated by the arrival times of the instability in the system linearized near the disease free state. Extensions to other epidemic models, nonlinear invasion modes (pushed fronts) and the role of inhomogeneous infection rates are studied via the analogy to traveling waves. Most of this work applies to complex networks in the limit of small diffusion – we also will briefly address larger rates of diffusion by restricting to the case of homogeneous trees.

Qiliang Wu (Ohio Univ.)

Title: Pearling and Localized Undulation of Bilayers in Amphiphilic Morphology

Abstract: Amphiphiles, such as lipids and functionalized polymers, plays a central role in the self-assembly of solvent accessible, intricately structured nano-scaled network structures, which are vital in cell functionality and offer wide applications to drug delivery, detergent production, emulsion stabilization and energy conversion devices. We study amphiphilic morphology in the framework of the functionalized Cahn-Hilliard (FCH) energy. The FCH is a continuum model accommodating various co-dimensional structures such as bilayers (co-dim 1), filaments (co-dim 2) and micelles (co-dim 3). We focus on defect structures that break the dimensional reduction and include endcaps that terminate filaments or bilayers and Y junctions. More specifically, we show the existence of pearled bilayer solutions via a spatial dynamics formulation, in combination with center manifold reduction and a fixed point argument. In addition, we also show via a functional analytic framework that in the presence of spatial inhomogeneity, localized undulation appears under proper functionalization terms. More interestingly, both the pearling and localized undulation are shown to be a manifestation of a degenerate 1:1 resonance Hopf bifurcation encoded in a reduced ODE system from the FCH energy.