Title and abstracts for talks on December 17, 2021

Avner Friedman (Ohio State)

Title: Open problems from math biology : competition between two types of cells

Abstract: Competition is pervasive at every level of life, in particular in biology. At the cellular level, competition occurs between different populations of cells during development, between different types of immune cells during a disease, between normal healthy cells and cancer cells, between different types of cancer cells , and between different populations of bacteria. In this talk I will some examples that are represented by PDE models, which have not yet been considered in the literature.

Olga Vasilyeva (Memorial University of New Foundland, Grenfell Campus)

Title: Logistic reaction-diffusion-advection river network model: phase plane approach

Abstract: Spatial population dynamics of aquatic organisms in a river network can be modeled using a metric graph with the population density satisfying a reaction-diffusion-advection (RDA) equation on each edge, along with appropriate junction and boundary conditions. We focus on the case of a nonlinear (logistic) reaction term. A steady state of such RDA model can be viewed as a collection of solutions of systems of two first order ODEs. Furthermore, the steady state can be identified with a configuration of orbits in the corresponding phase plane satisfying geometric constraints induced by junction and boundary conditions. Proving a certain concavity preservation property of the flow of the corresponding system of ODEs allows us to establish uniqueness of the positive steady state solution. We obtain explicit conditions for its existence in a simple Y-shaped network with constant hydrological and biological parameters. Our analysis and results can be generalized to the case of an arbitrary binary network where parameters vary from one edge to another.

Yixiang Wu (Middle Tennessee)

Title: Global dynamics of a Lotka-Volterra competition patch model

Abstract: The global dynamics of the two-species Lotka-Volterra competition patch model with asymmetric dispersal is classified under the assumptions of weak competition and the weighted digraph of the connection matrix is strongly connected and cycle-balanced. It is shown that in the long time, either the competition exclusion holds that one species becomes extinct, or the two species reach a coexistence equilibrium, and the outcome of the competition is determined by the strength of the inter-specific competition and the dispersal rates. Our main techniques in the proofs use the theory of monotone dynamical system and a graph-theoretic approach based on the Tree-Cycle identity. This is a joint work with Shanshan Chen, Junping Shi, and Zhisheng Shuai.

King-Yeung Lam (Ohio State)

Title: Competition for light in phytoplankton populations

Abstract: The paradox of the plankton highlights the apparent contradiction between Gause's law of competitive exclusion and the observed diversity of phytoplankton. We are interested in the dynamics phytoplankton populations in competition for light in a eutrophic water column. In the first part of the talk, we analyze a reaction-diffusion model introduced by J. Huismann and collaborators. By classifying the dynamics of two-species competition, our results suggest that competition exclusion is prevalent. In the second part of the talk, we extend the model by treating light as a continuum of resources rather than a single resource. This is done by considering the visible light spectrum. We explore under what circumstance coexistence is possible from mathematical and biological perspectives. Furthermore, we provide biological context as to when coexistence is expected based on the degree of niche differentiation within the light spectrum and overall turbidity of the water.