



# **MECHANISMS FOR COEXISTENCE AND COMPETITIVE EXCLUSION AMONG MUTUALISTS**

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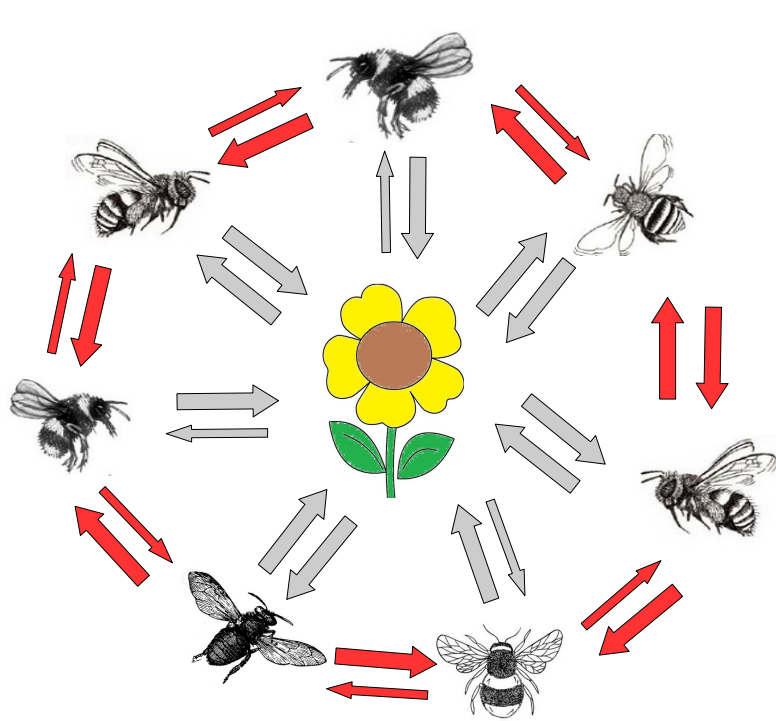
**Prof. Rebecca Tyson (UBC Okanagan)**

**Dr. Jimmy Garnier (Universite' Savoie Mont-Blanc)**

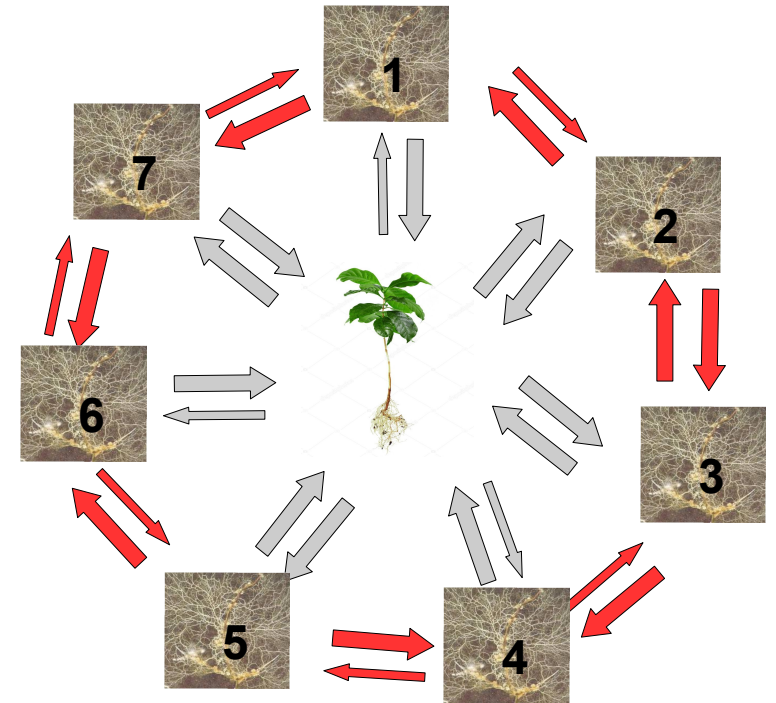
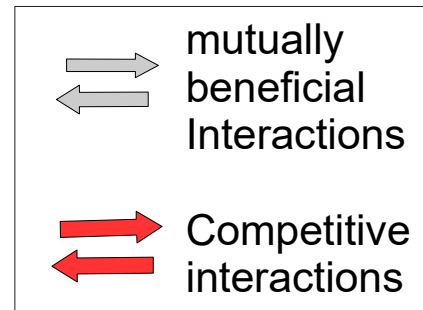
**Prof. Miranda Hart (UBC Okanagan)**

**Winter Workshop on Competition  
Dynamics in Biology  
Dec 15-17, 2021  
Ohio State University**

# Multiple mutualists compete for a common resource supplied by the same host/partner species



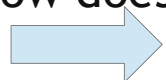
POLLINATION



PLANT-MICROBE SYMBIOSIS  
(ex. mycorrhiza: plant-fungi symbiosis)

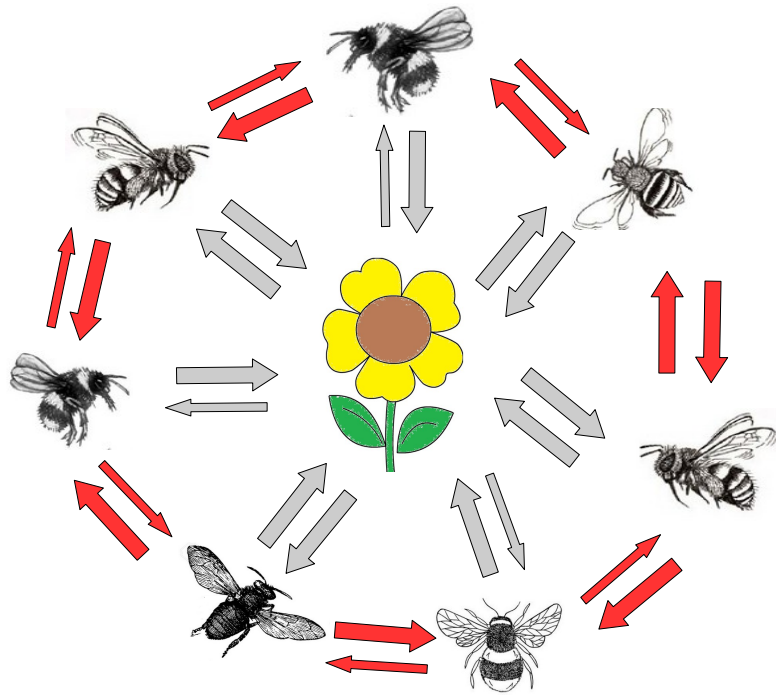
Mechanisms leading to coexistence and competitive exclusion among mutualists are poorly understood:

- What happens if we add/remove a mutualist from the system?  
(Survival and coexistence? Competitive exclusion?)
- How does the guild composition affects plant growth (productivity)?

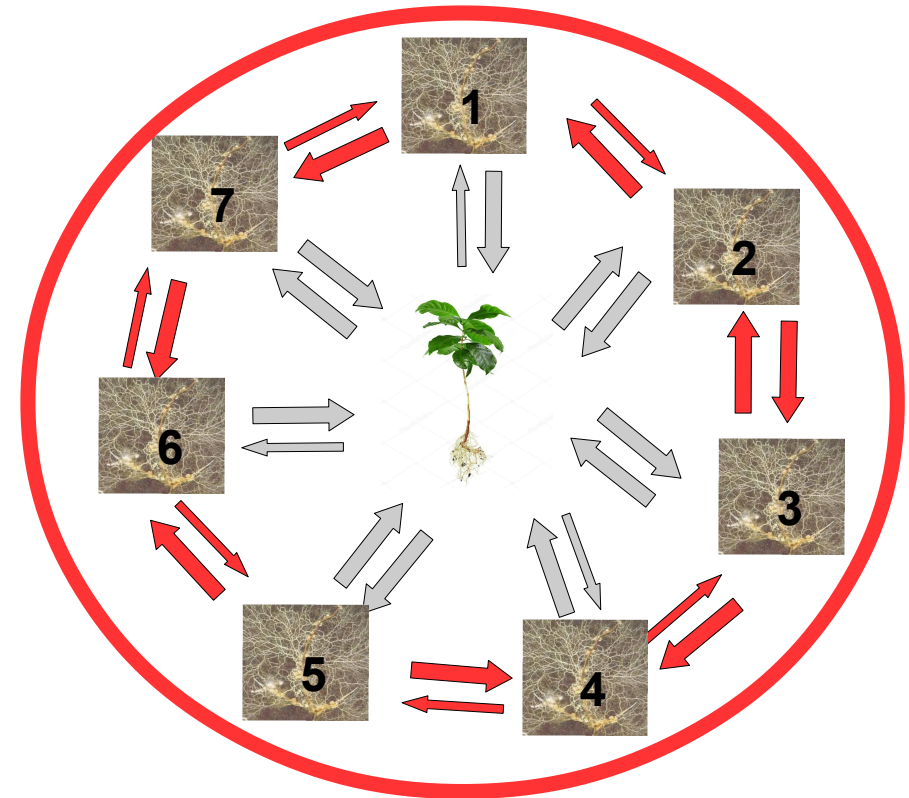
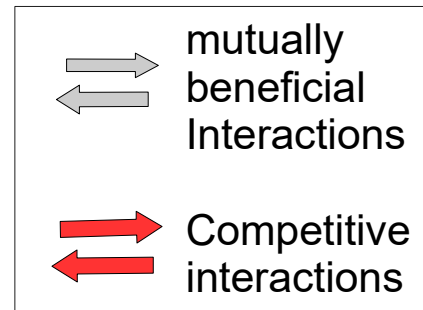


Crucial for agricultural management

# Multiple mutualists compete for a common resource supplied by the same host/partner species



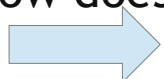
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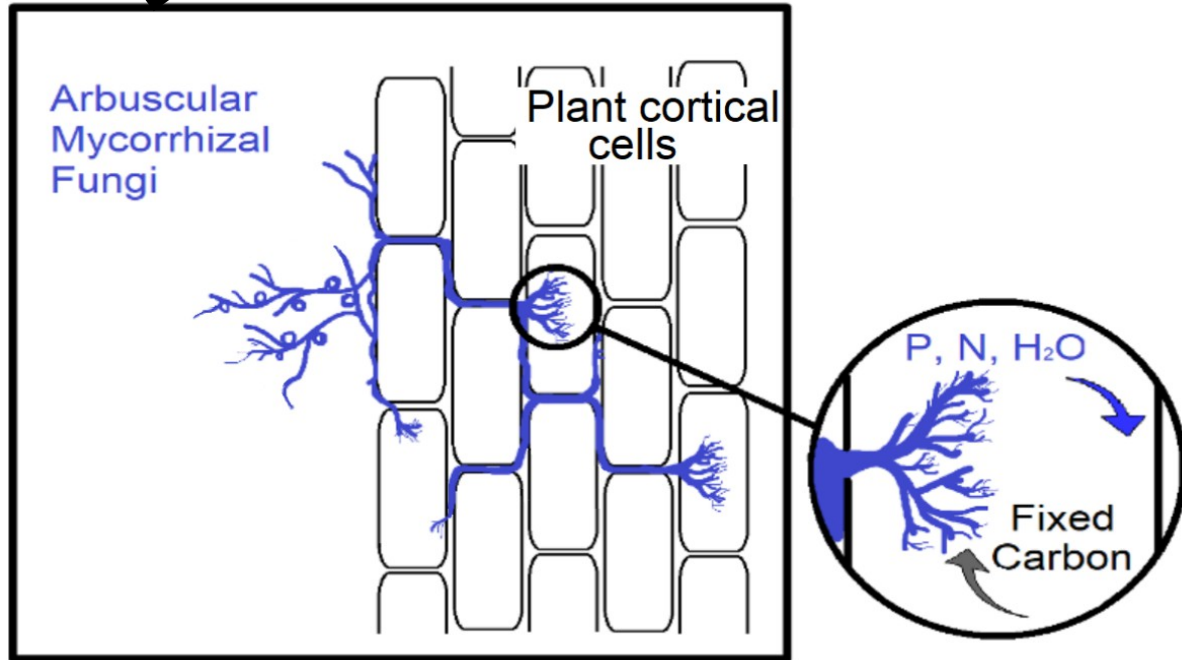
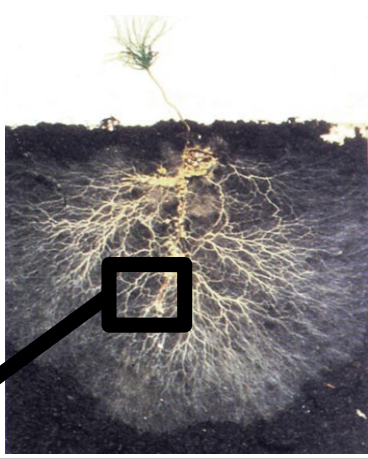
**MYCORRHIZA**  
**Plant-fungi symbiosis**

Mechanisms leading to coexistence and competitive exclusion among mutualists are poorly understood:

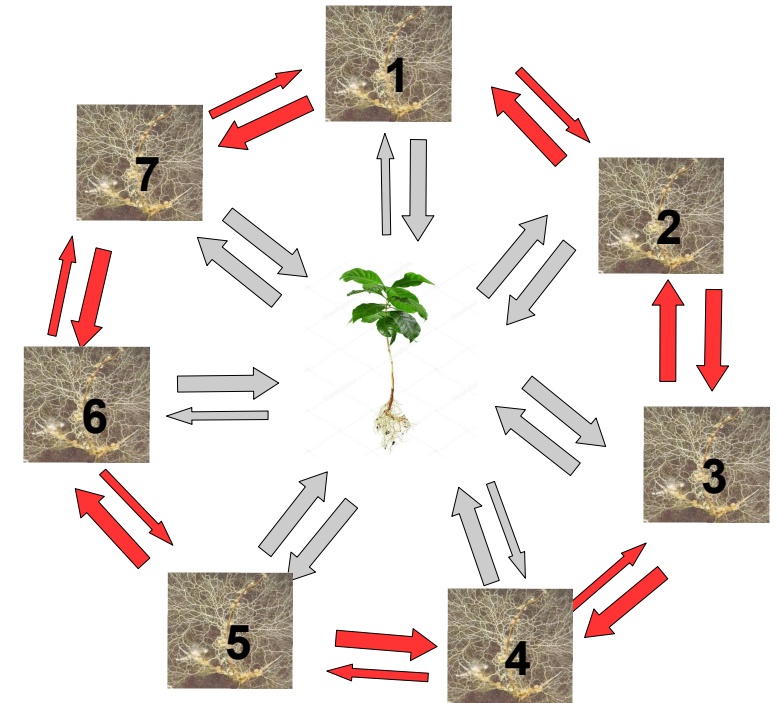
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Crucial for agricultural management



## MYCORRHIZA: plant-fungi symbiosis



### ARBUSCULAR MYCORRHIZAL FUNGI (AMF):

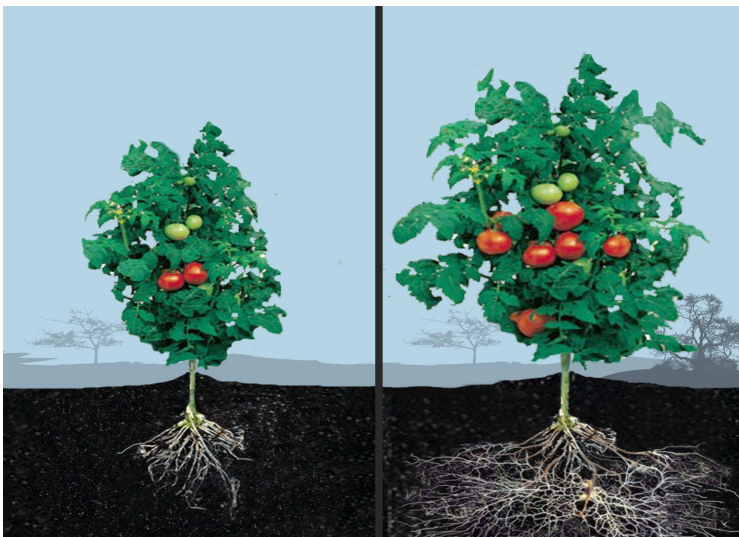
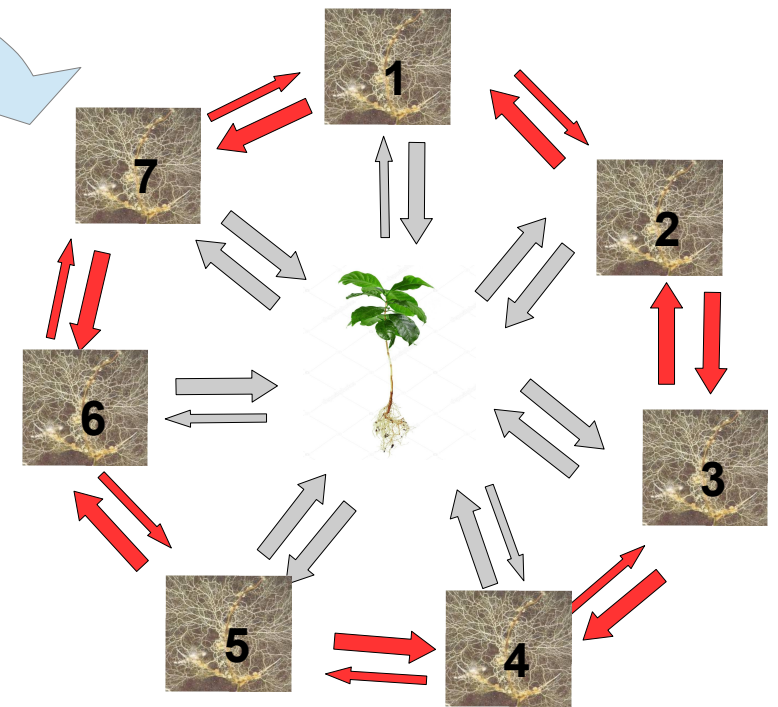
- 90% of terrestrial plants (e.g. maize, wheat, soybeans, tomatoes, strawberries,...)
- Increase in root system surface area of up to 1000 times!
- Extremely beneficial for plant fitness and health

AMF are used as commercial fertilizer !



\*\*Contains approximately 100,000 endomycorrhizal spores/lb and 110,000,000 ectomycorrhizal spores/lb.

## MYCORRHIZA: plant-fungi symbiosis

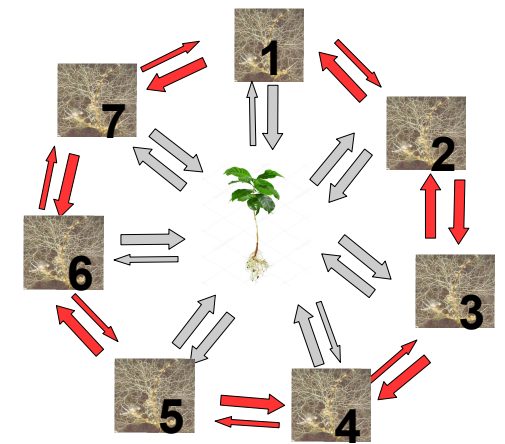


### Questions:

- What happens if we add/remove a mutualist from the system?
- How does the guild composition affects plant growth (productivity)?

Can the inoculated fungal species become invasive?

# Mathematical model: 1 plant and multiple fungi (ODE)



$$\frac{dp}{dt} = \underbrace{q_{hp} r_p p}_{\text{Intrinsic growth}} + \underbrace{q_{hp} \left( \sum_j \alpha_j m_j \right) \frac{p}{d+p}}_{\text{phosphorus received}} - \underbrace{q_{cp} \left( \sum_j \beta_j m_j \left[ 1 - \frac{\sum_{i \neq j} m_i}{a_j + \sum_{i \neq j} m_i} \right] \right) p}_{\text{carbon supplied}} - \underbrace{\mu_p p^2}_{\text{maintenance}}$$

$$\frac{dm_j}{dt} = \underbrace{q_{cm_j} \beta_j p m_j \left[ 1 - \frac{\sum_{i \neq j} m_i}{a_j + \sum_{i \neq j} m_i} \right]}_{\text{carbon received}} - \underbrace{q_{hm_j} \alpha_j \frac{p}{d+p} m_j}_{\text{phosphorus supplied}} - \underbrace{\mu_{m_j} m_j^2}_{\text{maintenance}}$$

Reduction in carbon uptake due to competition

**Competition** depends on species identity and abundance

where

$$a_j = \sum_{i \neq j} a_{ij} \frac{m_i}{\sum_{i \neq j} m_i}$$


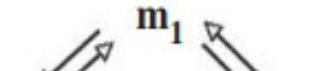



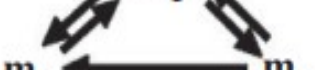
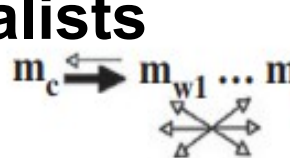


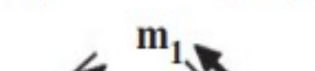
**p** : plant biomass  
**m<sub>j</sub>** : fungal biomass

**α<sub>i</sub>/ β<sub>j</sub>** : Mutualist quality of a fungal species  
**a<sub>ij</sub>** : Competitive effect of fungal species j on i

## 2 mutualists

## 3 mutualists

## N mutualists


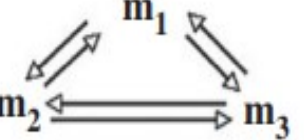



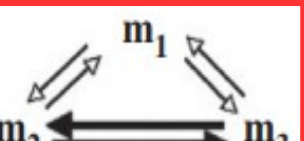
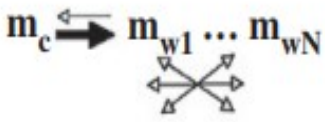
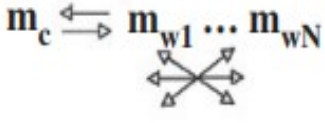
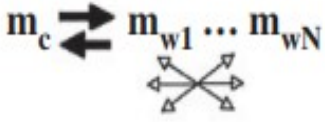
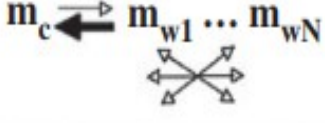
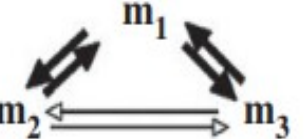
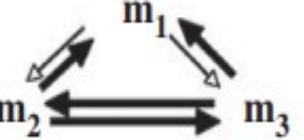
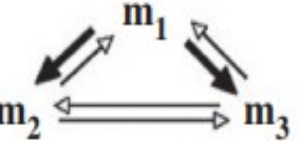
A (i)	 $(m_1^*, m_2^*)$	B (i)	 $(m_1^*, m_2^*, m_3^*)$		
(ii)	 $(m_1^*, 0), (0, m_2^*)$	(ii)	 $(m_1^*, 0, 0), (0, m_2^*, 0), (0, 0, m_3^*)$		
(iii)	 $(m_1^*, 0)$	(iii)	 $(m_1^*, m_2^*, m_3^*)$		
		(iv)	 $(m_1^*, 0, 0), (0, m_2^*, m_3^*)$		
		(i)	$(m_c^*, 0, \dots, 0)$	(v)	 $(0, m_2^*, 0), (0, 0, m_3^*)$
		(ii)	$(m_c^*, m_{w1}^*, \dots, m_{wN}^*)$	(vi)	 $(m_1^*, 0)$
		(iii)	$(m_c^*, 0, \dots, 0), (0, m_{w1}^*, \dots, m_{wN}^*)$		

**Figure 1.** Representation of the direct interactions between fungal species ( $m_i$ ) sharing a resource supplied by the same host plant (not included in the figure), and corresponding steady state stability (presented in the electronic supplementary material). Arrows indicate competition between mutualists where competition can be weak (thin arrows) or strong (thick arrows).

## 2 mutualists

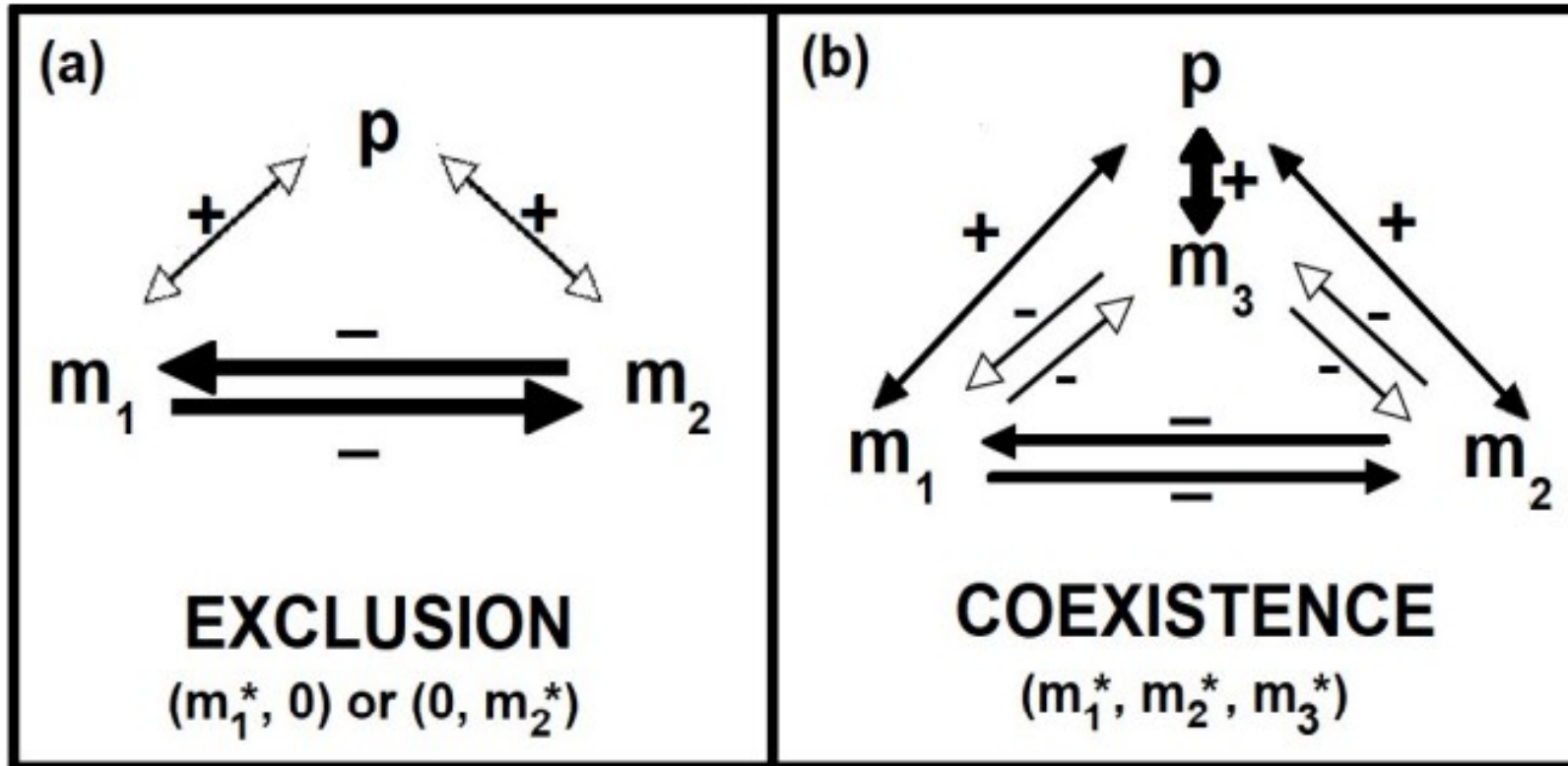
## 3 mutualists

## N mutualists

A (i)	 $(m_1^*, m_2^*)$	B (i)	 $(m_1^*, m_2^*, m_3^*)$
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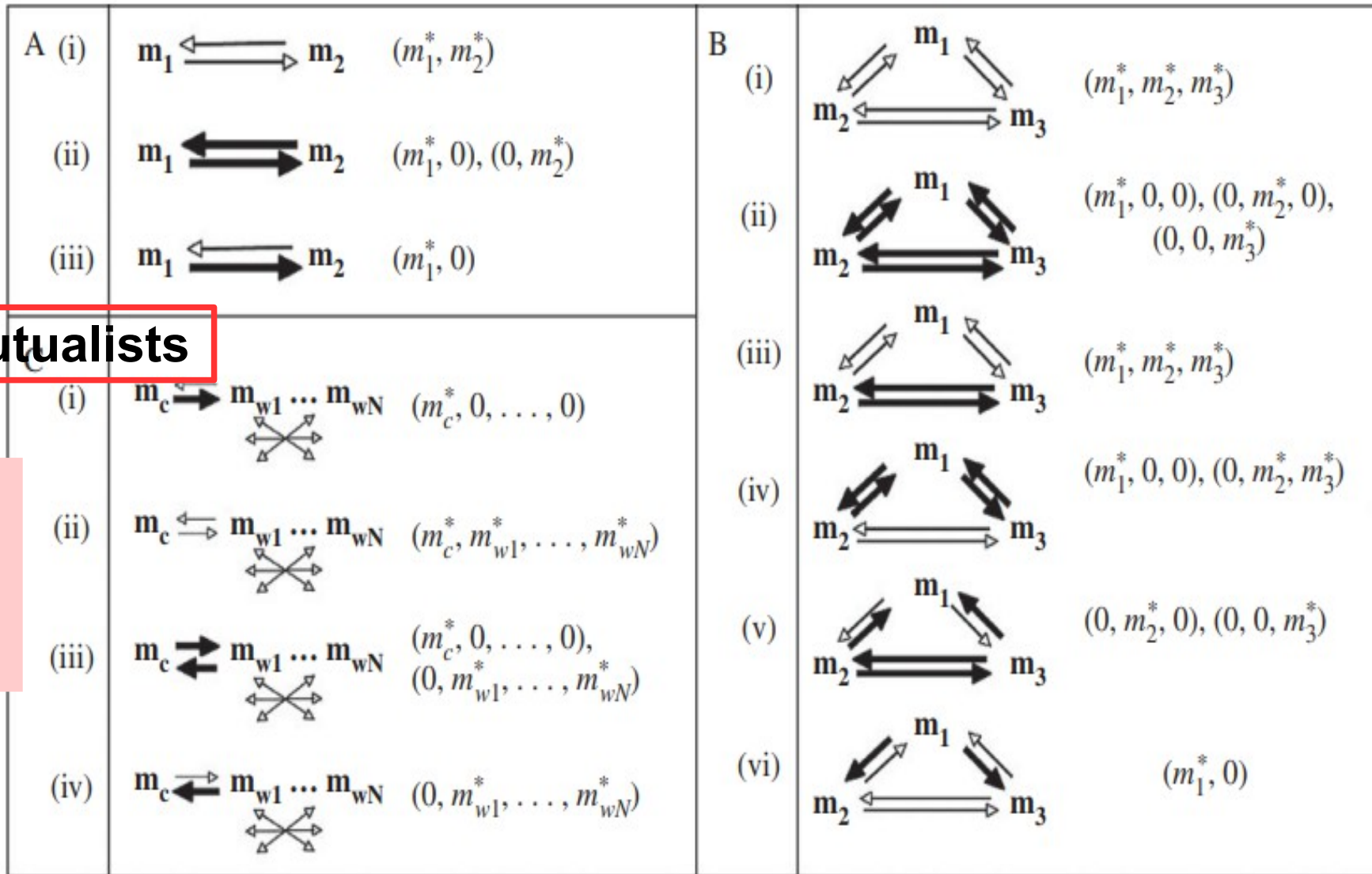


Removal of a weak competitor ( $m_3$ ) in a community of strongly competing mutualists ( $m_1$  and  $m_2$ ), may cause the extinction of other mutualists in the guild (either  $m_1$  or  $m_2$ ).

**--> Diversity promotes coexistence of species that would otherwise competitively exclude each other!**

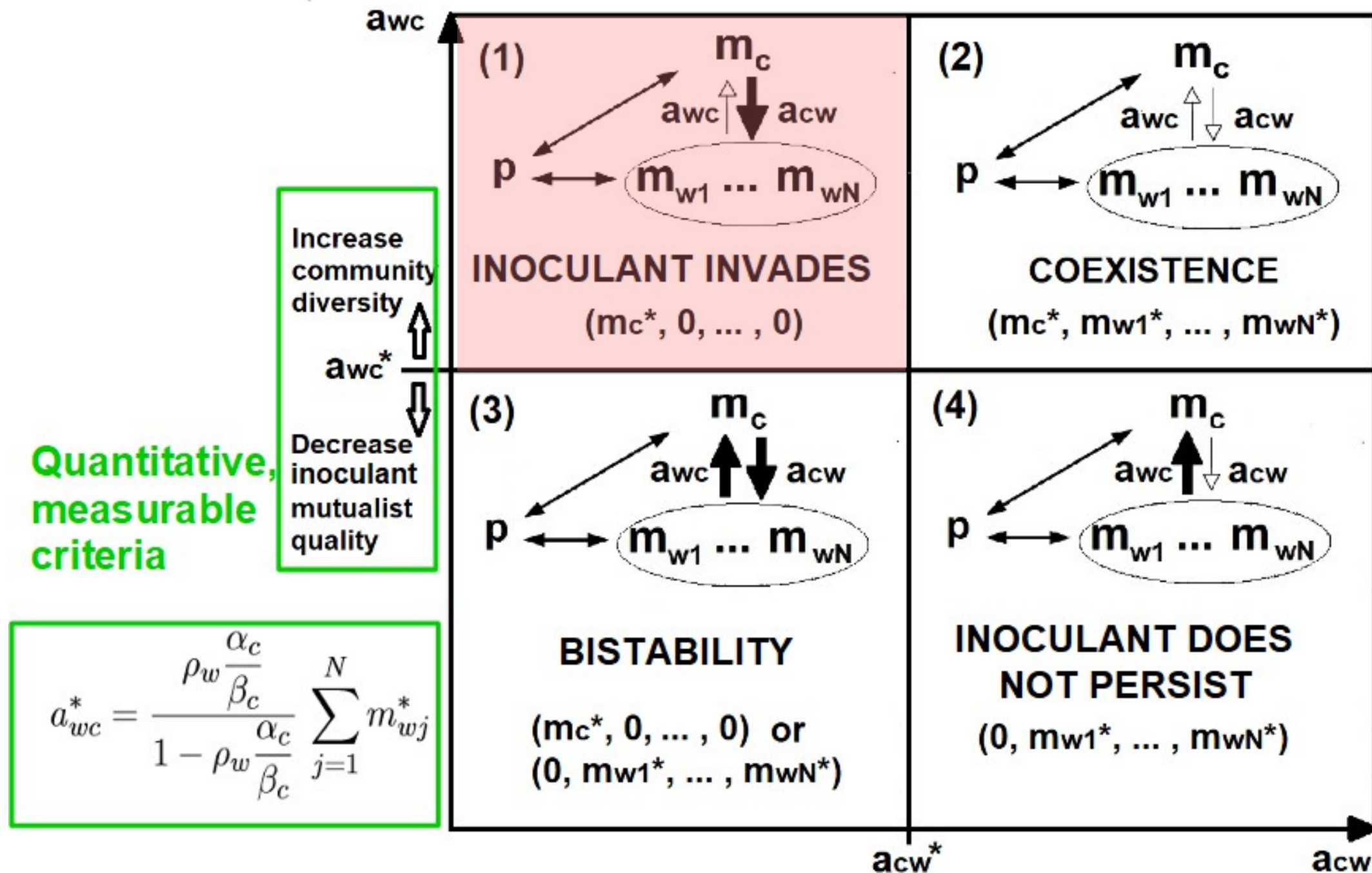
## 2 mutualists

## 3 mutualists



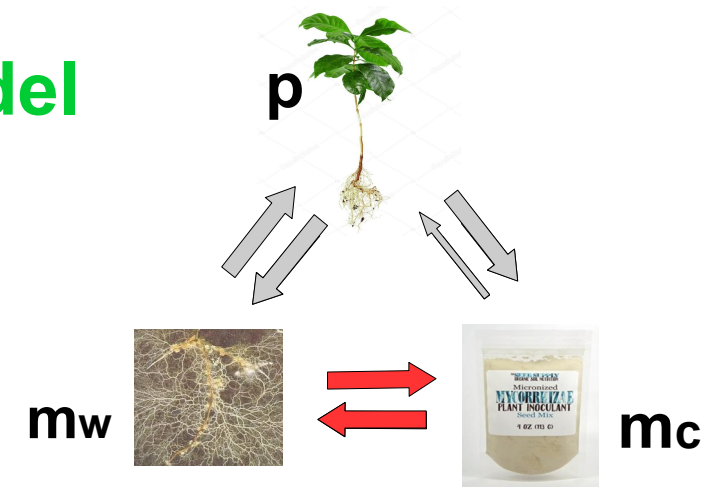
Group of N native fungi + 1 introduced species

**Figure 1.** Representation of the direct interactions between fungal species ( $m_i$ ) sharing a resource supplied by the same host plant (not included in the figure), and corresponding steady state stability (presented in the electronic supplementary material). Arrows indicate competition between mutualists where competition can be weak (thin arrows) or strong (thick arrows).



M.M. Martignoni, R.C. Tyson, M.M. Hart, J. Garnier (2020), Diversity within mutualist guilds promotes coexistence and reduces the risk of invasion from an alien mutualist, PRSB

# Impact on a landscape scale? --> PDE model



$$\partial_t p(x, t) = f_p(p, m_c, m_{w_j}),$$

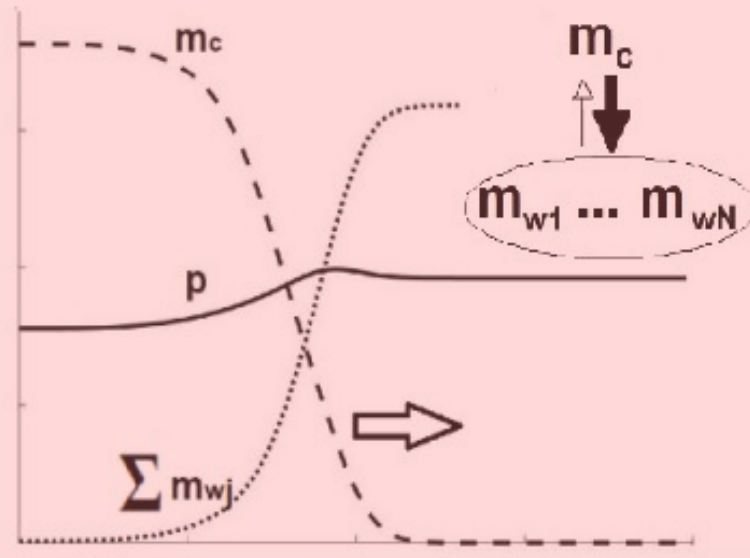
$$\partial_t m_c(x, t) = \boxed{D_c \partial_x^2 m_c} + f_{m_c}(p, m_c, m_{w_j})$$

$$\partial_t m_{w_j}(x, t) = \boxed{D_{w_j} \partial_x^2 m_{w_j}} + f_{m_j}(p, m_c, m_{w_j})$$

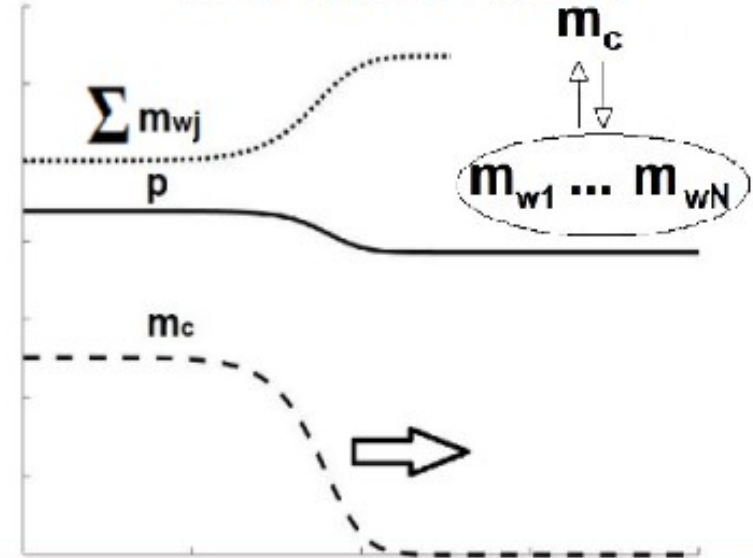
**Spread of fungi**

**Mathematically interesting: The fungi compete between each others and interact with the medium in which they disperse (i.e., the plant)**

# INOCULANT INVADES



# COEXISTENCE



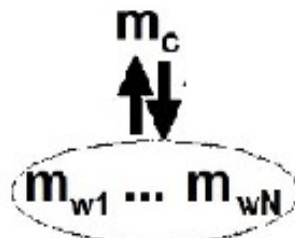
Increase community diversity



Decrease mutualist quality of the inoculants

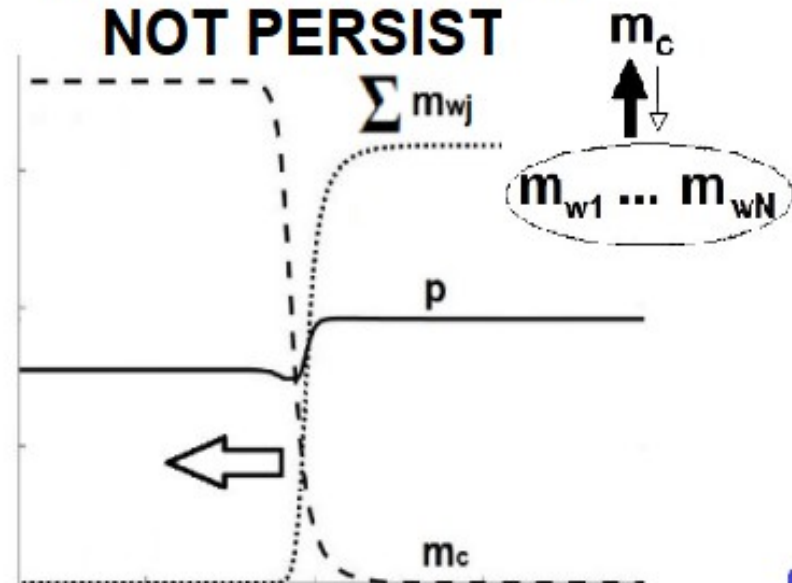


# BISTABILITY



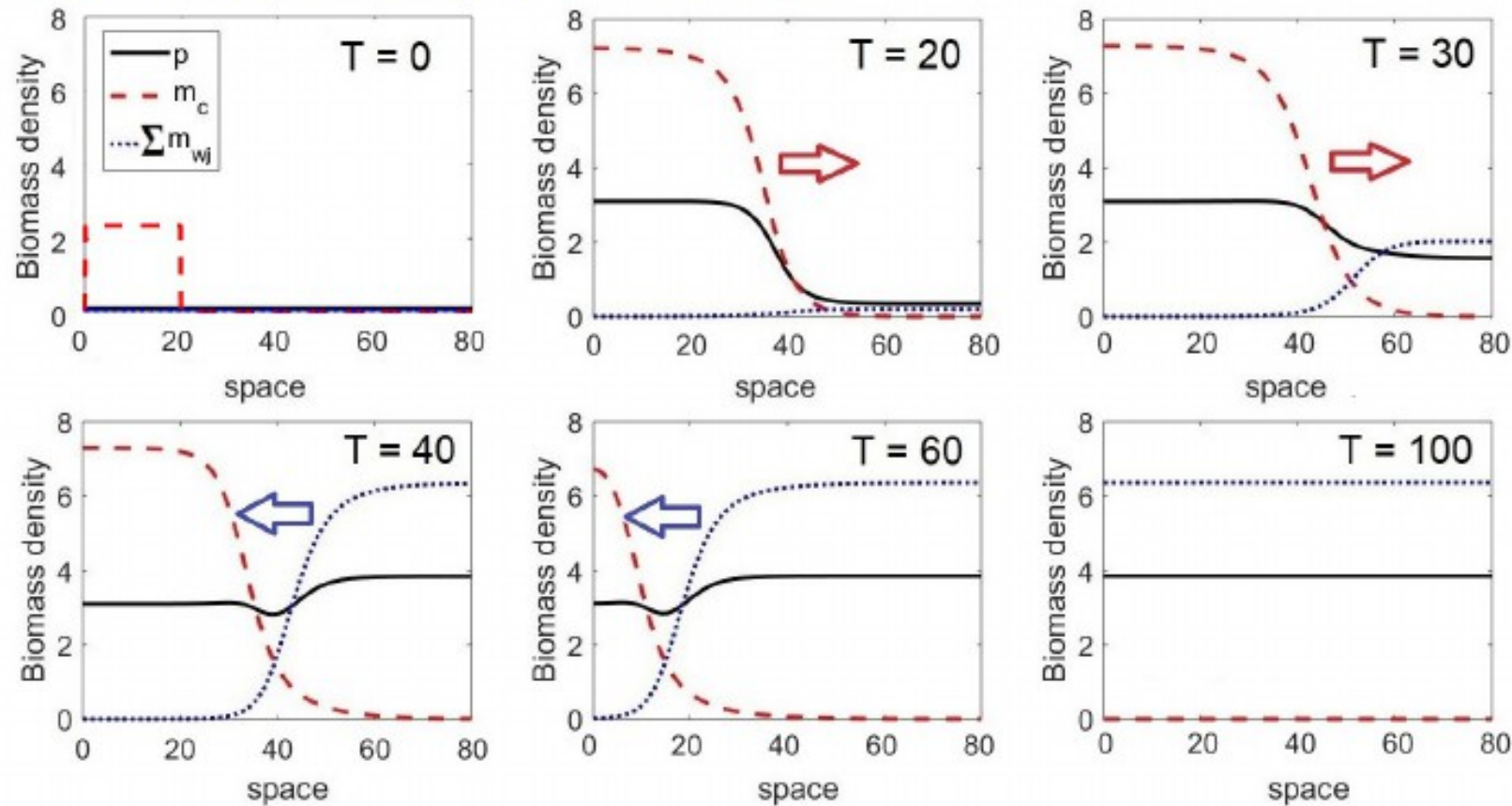
Depends on the inoculated area ...

# INOCULANT DOES NOT PERSIST



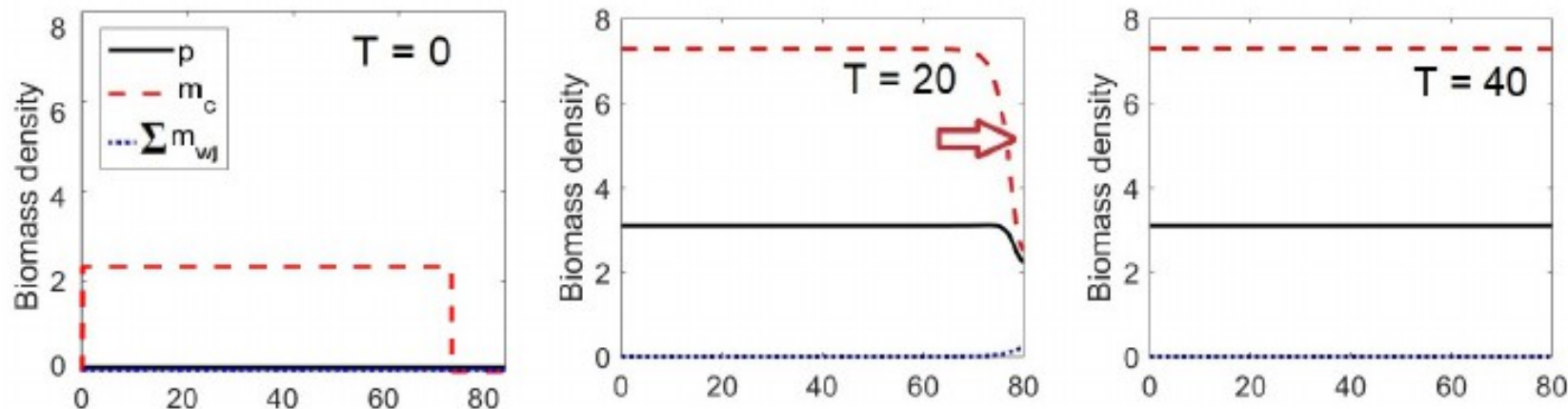
# BISTABLE TRAVELING WAVE

## INOCULANT DOES NOT PERSIST



*Displacement of the inoculant occurs through improved mutualistic relationship with the plant*

## INOCULANT INVADDES



*Invasion of the inoculant occurs if inoculation occurs over a large area*

# Can the inoculated fungal species become invasive?



**YES!**

## ***If the inoculated fungi:***

- Are strong competitors
- Have low mutualist quality
- Are inoculated over a large area

**High breeding pressure  
& large-scale fertilization**

**+**

## ***And if native fungi:***

- Are weak competitors
- Present low diversity

**Intense agricultural practises**

## **Spread speed**

$$c_c^* = 2\sqrt{D_c P_w^* \left( \frac{q_{cm} \beta_c a_{wc}}{a_{wc} + N M_w^*} - \frac{q_{hm} \alpha_c}{P_w^* + d} \right)}$$

**Dispersal ability of the inoculants**

**Mutualist quality of the inoculants**

**Native community diversity**

## Should we inoculate or not?



**YES!**

... But with **weakly**  
competing inoculants

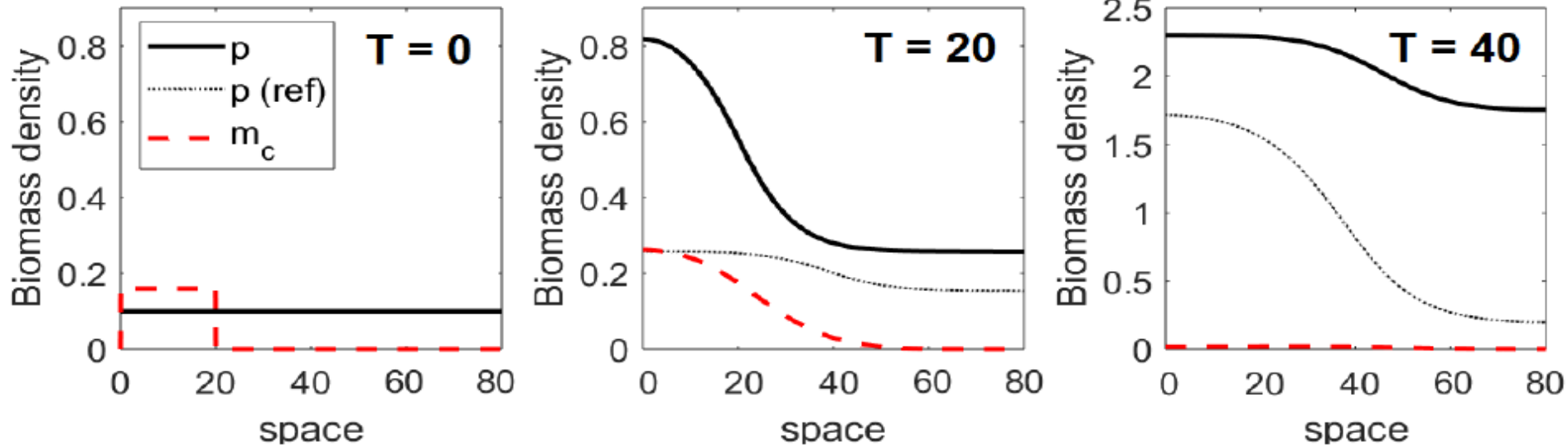
- ✓ No biodiversity risk!
- ✓ Boost in plant growth (due to an increase in fungal abundance)
- ✓ Restoration of a native community



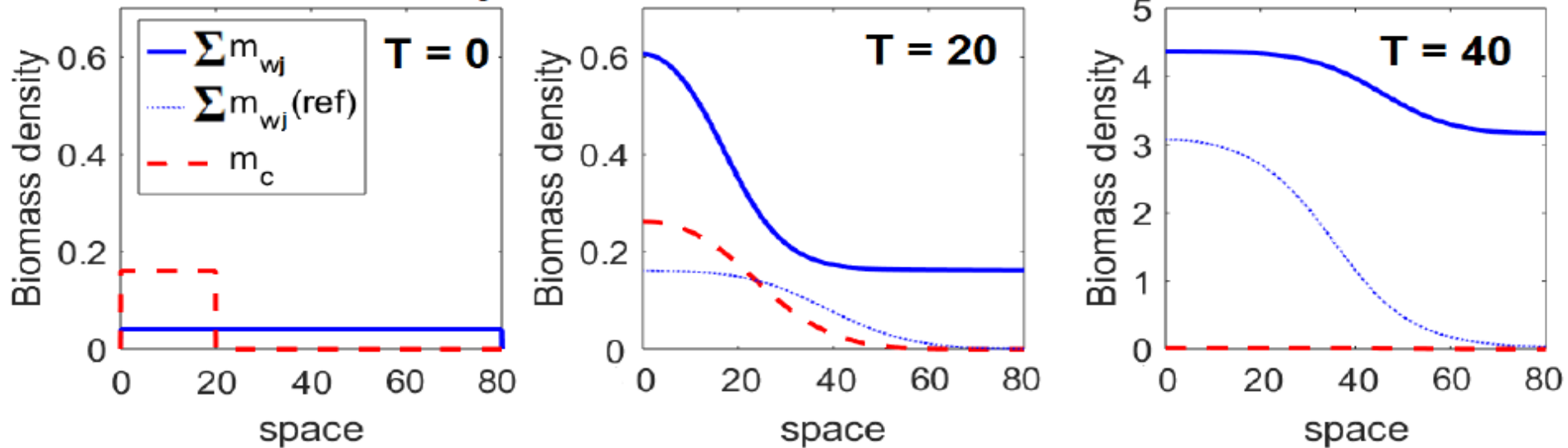


# Use weakly competing inoculants !

## Boost plant productivity



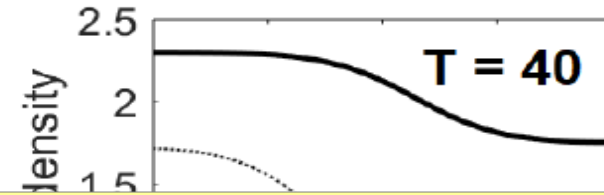
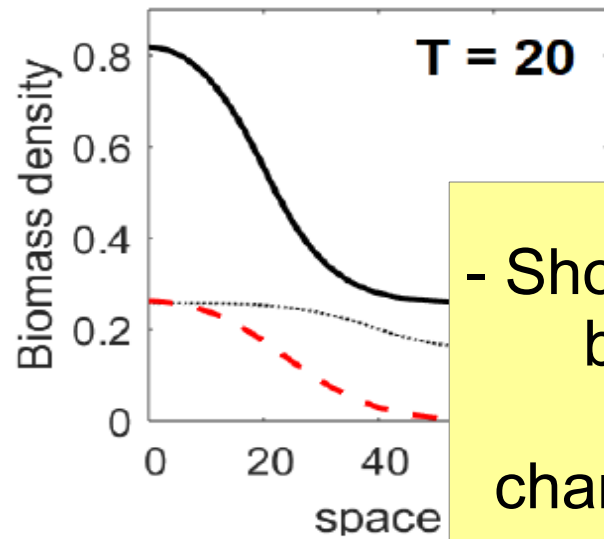
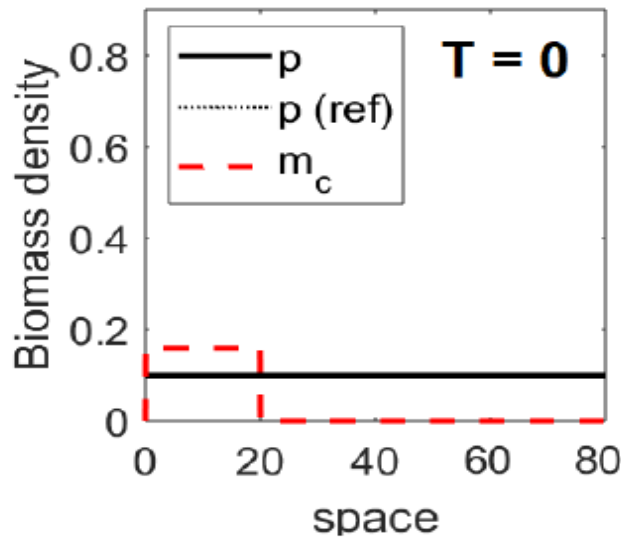
## Native community restoration



... even if the inoculated fungi do not establish !

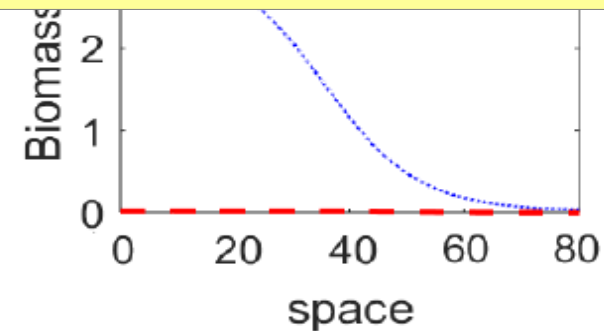
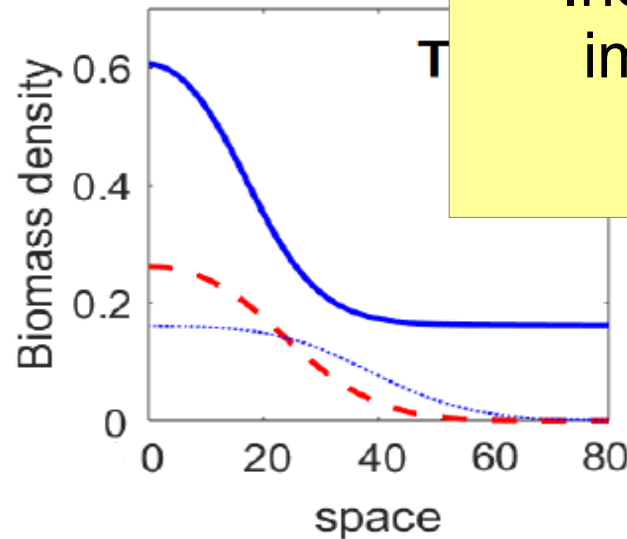
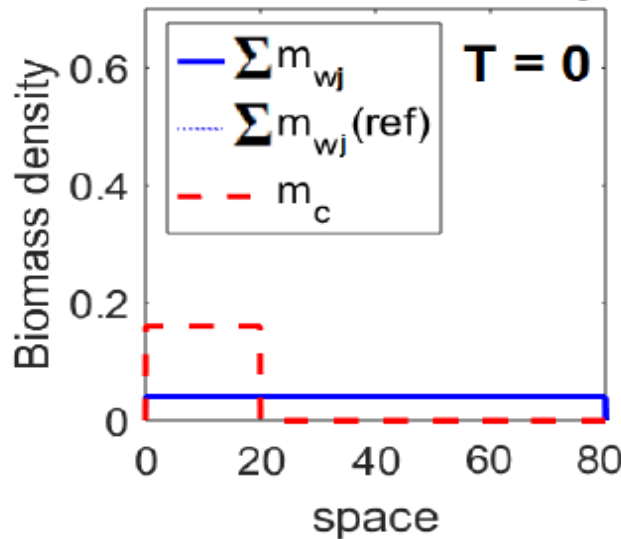
# Use weakly competing inoculants !

## Boost plant productivity



- Short-term measurements of plant biomass are not indicative!  
(long term monitoring + changes in the native community)

## Native community restoration



- Inoculants can have a positive impact on productivity even if they don't establish

... even if the inoculated fungi do not establish !

# CONCLUSION

- **We do have a theoretical framework to help the management of communities of mutualists (e.g., AM fungi), and measurable quantitative criteria to predict resilience, invasibility, spread, and productivity of these communities.**
- The models developed can be used as building blocks in larger community models (examples of future directions: include multiple partners/hosts, environmental limitations,...) .
- In our model growth rates are related to nutrients transfer, what leads to a possible interplay between theory and experiments.

**M.M. Martignoni, J. Garnier, X. Zhang, D. Rosa, V. Kokkoris, R.C. Tyson, M.M. Hart (2021), Co-inoculation with arbuscular mycorrhizal fungi differing in carbon sink strength induces a synergistic effect in plant growth, *Journal of Theoretical Biology***



## Acknowledgment

Prof. Rebecca Tyson

Dr. Jimmy Garnier

Prof. Miranda Hart

UBC-O

NSERC

MOPGA

**QUESTIONS ?**