Let

$$A = \begin{pmatrix} 0 & -3 & -2 & -4 \\ 2 & 0 & 2 & -2 \\ 1 & 2 & 3 & 3 \\ -1 & 1 & -1 & 3 \end{pmatrix}.$$

Then

$$A = P^{-1}JP,$$

where

$$P = \begin{pmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 2 & 0 \\ 0 & 1 & 0 & 2 \end{pmatrix}, \quad P^{-1} = \begin{pmatrix} 2 & 0 & -1 & 0 \\ 0 & 2 & 0 & -1 \\ -1 & 0 & 1 & 0 \\ 0 & -1 & 0 & 1 \end{pmatrix}.$$

$$J = \begin{pmatrix} 1 & -1 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 2 & 1 \\ 0 & 0 & 0 & 2 \end{pmatrix}.$$

Step 1. Exponential of the Jordan blocks

For the first 2×2 block:

$$\begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix} = I + K, \qquad K = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}, \quad K^2 = -I.$$

Thus

$$e^{t\begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix}} = e^t e^{tK} = e^t \begin{pmatrix} \cos t & -\sin t \\ \sin t & \cos t \end{pmatrix}.$$

For the Jordan block at $\lambda = 2$:

$$\begin{pmatrix} 2 & 1 \\ 0 & 2 \end{pmatrix} = 2I + N, \qquad N = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}, \quad N^2 = 0.$$

So

$$e^{t\begin{pmatrix} 2 & 1 \\ 0 & 2 \end{pmatrix}} = e^{2t}(I + tN) = e^{2t} \begin{pmatrix} 1 & t \\ 0 & 1 \end{pmatrix}.$$

Step 2. Exponential of J

Putting the two blocks together:

$$e^{Jt} = \begin{pmatrix} e^t \cos t & -e^t \sin t & 0 & 0 \\ e^t \sin t & e^t \cos t & 0 & 0 \\ 0 & 0 & e^{2t} & te^{2t} \\ 0 & 0 & 0 & e^{2t} \end{pmatrix}.$$

Step 3. Solve with initial condition

If

$$z(0) = \begin{pmatrix} 1 \\ 0 \\ 1 \\ 0 \end{pmatrix},$$

then

$$z(t) = e^{Jt} z(0) = \begin{pmatrix} e^t \cos t \\ e^t \sin t \\ e^{2t} \\ 0 \end{pmatrix}.$$

Step 4. Relation to the original system

If $A = P^{-1} J P$, then

$$x(t) = P^{-1} z(t), \qquad z(t) = e^{Jt} P x(0).$$

For the specific case $x(0) = (1, 0, 0, 0)^\top$, one obtains

$$\boxed{\begin{aligned} &x_1(t) = 2e^t \cos t - e^{2t}, \\ &x_2(t) = 2e^t \sin t, \\ &x_3(t) = -\, e^t \cos t + e^{2t}, \\ &x_4(t) = -\, e^t \sin t. \end{aligned}}$$