Gapping as Like-Category Coordination

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Abstract. We propose a version of Type-Logical Categorial Grammar (TLCG) which combines the insights of standard TLCG (Morrill 1994, Moortgat 1997) in which directionality is handled in terms of forward and backward slashes, and more recent approaches in the CG literature which separate directionality-related reasoning from syntactic combinatorics by means of λ-binding in the phonological component (Oehrle 1994, de Groote 2001, Muskens 2003). The proposed calculus recognizes both the directionality-sensitive modes of implication ($/$ and $\backslash$) of the former and the directionality-insensitive mode of implication tied to phonological λ-binding in the latter (which we notate here as $|$).

Empirical support for the proposed system comes from the fact that it enables a straightforward treatment of Gapping, a phenomenon that has turned out to be extremely problematic in the syntactic literature including CG-based approaches.

Keywords: Gapping, coordination, Type-Logical Categorial Grammar, Lambda Grammar, scope, phenogrammar, tectogrammar.

1 (Apparent) Anomalies of Gapping

The examples in (1) are instances of *Gapping*:

(1) a. Leslie bought a CD, and Robin, a book.
b. I gave Leslie a book, and she a CD.
c. Terry can go there with me, and Pat with you.

Gapping is a type of non-canonical coordination, but what distinguishes it from other kinds of non-canonical coordinations is that the strings which appear to be coordinated do not look very much like each other. For example, in cases of nonconstituent coordination such as *I told the same joke to Robin on Friday and (to) Leslie on Sunday*, or in examples of Right-Node Raising (RNR), it is possible to identify two coordinated substrings which are parallel up to the point where they combine with the rest of the sentence in which they appear; the problem is only that expressions such as *to Robin on Friday and (to) Leslie on Sunday* are not phrase structure constituents, nor are the partial clauses in RNR. But in the case of Gapping, we seem to be coordinating a whole clause with a sequence of

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words which would be a clause if a copy of the verb in the first conjunct were introduced into the second conjunct. As they stand, however, *Leslie bought a CD* has a completely different status from *Robin a book*.

For this reason, Gapping has continued to pose a difficult challenge in the tradition of both phrase structure grammar and categorial grammar. In categorial grammar (CG), there have been proposals both in the tradition of CCG (Steedman 1990) and TLCG (Morrill and Solias 1993, Morrill 1994, Hendriks 1995, Morrill and Merenciano 1996, Morrill et al. 2011). These proposals share an important key analytic intuition (which we take to be basically correct) which views Gapping as a case of discontinuous constituency: in Gapping, if the shared verb is stripped off from the left conjunct, then it has the same combinatorial property and semantic type as the right conjunct, which therefore supports coordination under the law of coordination of likes. The challenge essentially lies in characterizing precisely the status of the two coordinated conjuncts manifesting (in this view) discontinuity. The most recent proposal by Morrill et al. (2011) improves on previous related approaches in this respect, but it still suffers from empirical shortcomings in not straightforwardly extending to cases in which Gapping interacts with other phenomena which themselves manifest discontinuous constituency, as we will discuss in section 3.

A further challenge for any analysis of Gapping comes from the scopal properties of modal and negative auxiliaries, in examples like the following (Oehrle 1987):

(2) a. Kim didn't play bingo or Sandy sit at home all evening.
 b. Mrs. J can't live in Boston and Mr. J in LA.

The only available interpretation of (2a) is $\neg\varphi \wedge \neg\psi$ ($\equiv \neg(\varphi \vee \psi)$), where φ is the proposition expressed by *Kim played bingo* and ψ the proposition expressed by *Sandy sat at home all evening*. (2b) is—at least for some speakers including one of the authors—ambiguous between the reading in which the modal *can't* scopes over the conjunction and one in which it scopes below it; the latter reading can be made prominent by having an intonational break between the first and second conjuncts. The only explicit analysis of data like (2) in CG to date is Oehrle (1987). Oehrle's very insightful analysis, which alone among prior treatments of Gapping provides a starting point for an explanation for the apparent scope anomaly of such examples, unfortunately falls short of a general treatment of Gapping given the several non-standard assumptions about syntax that he crucially exploits in formulating his semantic analysis (see section 3).

In short, there is as yet no analysis of Gapping in CG that captures both the range of syntactic patterns and semantic interpretations associated with this construction. In the next section, we propose a version of Type-Logical Categorial Grammar utilizing a typed λ-calculus for notating the phonologies of linguistic signs. The novelty of the proposed system consists in recognizing both the directionality-sensitive modes of implication ($/$ and $\backslash$) of the standard TLCG and the directionality-insensitive mode of implication tied to phonological λ-binding in more recent versions of CG (Oehrle 1994, de Groote 2001, Muskens 2003). In the framework we propose below, λ-binding in phonology provides a simple and explicit mechanism for representing constituents with missing objects in the medial position. This plays a key role in enabling an analysis of Gapping that overcomes the inadequacies of the previous approaches. It will be shown that once a proper analysis of the apparent asymmetry between the two conjuncts is formulated, the apparent anomalies related to scopal interactions with auxiliaries in examples like (2) above immediately disappear.

2 λTLCG and Gapping

We assume a version of Type-Logical Categorial Grammar (TLCG) in the labelled deduction format utilizing a typed λ-calculus for notating the phonologies of linguistic expressions, called λTLCG. We write linguistic expressions as tuples of phonological representation, semantic interpretation and syntactic category (written in that order). The full set of inference rules are given in (3).

(3)

Connective	Introduction	Elimination
$/$	$\begin{array}{c} \vdots \quad \underline{[\pi; x; A]^n} \quad \vdots \\ \vdots \qquad \vdots \qquad \vdots \\ \hline \mathsf{b} \circ \pi; \varphi; B \\ \hline \mathsf{b}; \lambda x.\varphi; B/A \end{array} /\mathrm{I}^n$	$\dfrac{\mathsf{a}; \varphi; A/B \quad \mathsf{b}; \psi; B}{\mathsf{a} \circ \mathsf{b}; \varphi(\psi); A} /\mathrm{E}$
$\backslash$	$\begin{array}{c} \vdots \quad \underline{[\pi; x; A]^n} \quad \vdots \\ \vdots \qquad \vdots \qquad \vdots \\ \hline \pi \circ \mathsf{b}; \varphi; B \\ \hline \mathsf{b}; \lambda x.\varphi; A\backslash B \end{array} \backslash\mathrm{I}^n$	$\dfrac{\mathsf{b}; \psi; B \quad \mathsf{a}; \varphi; B\backslash A}{\mathsf{b} \circ \mathsf{a}; \varphi(\psi); A} \backslash\mathrm{E}$
$\vert$	$\begin{array}{c} \vdots \quad \underline{[\pi; x; A]^n} \quad \vdots \\ \vdots \qquad \vdots \qquad \vdots \\ \hline \mathsf{b}; \varphi; B \\ \hline \lambda\pi.\mathsf{b}; \lambda x.\varphi; B\vert A \end{array} \vert\mathrm{I}^n$	$\dfrac{\mathsf{a}; \psi; A \quad \mathsf{b}; \varphi; B\vert A}{\mathsf{b}(\mathsf{a}); \varphi(\psi); B} \vert\mathrm{E}$

The key difference between /,\ and | is that while the Introduction and Elimination rules for /,\ refer to the phonological forms of the input and output strings (so that, for example, the applicability of the /I rule is conditioned on the presence of the phonology of the hypothesis p on the right periphery of the phonology of the input $\mathsf{b} \circ p$),[1] the rules for | are not constrained that way. For reasoning involving |, the phonological terms themselves fully specify the ways in which the output phonology is constructed from the input phonologies. Specifically, for

[1] In this respect, the present calculus follows most closely Morrill and Solias (1993) and Morrill (1994); see Moortgat (1997) and Bernardi (2002) for an alternative formulation where sensitivity to directionality is mediated through a presumed correspondence between surface string and the form of structured antecedents in the sequent-style notation of natural deduction.

|, the phonological operations associated with the Introduction and Elimination rules mirror exactly the semantic operations for these rules: function application and λ-abstraction, respectively. We assume that the binary connective $\circ$ in the phonological term calculus represents the string concatenation operation and that $\circ$ is associative in both directions. For notational convenience, we implicitly assume the axiom $(\pi_1 \circ \pi_2) \circ \pi_3 \equiv \pi_1 \circ (\pi_2 \circ \pi_3)$ and leave out all the brackets indicating the internal constituency of complex phonological terms.[2] Thus, the present system without the rules for | is equivalent to the Lambek calculus (Lambek 1958), while the system with only the rules for | is essentially equivalent to the term-labelled calculus of Oehrle (1994), λ-Grammar (Muskens 2003) and Abstract Categorial Grammar (de Groote 2001), with some implementational details aside, which are irrelevant for the following discussion.

λ-binding in the phonological component provides a simple and explicit way of modelling expressions with medial gaps. Oehrle (1994) originally showed this point by formalizing an explicit and straightforward implementation of Montague's (1973) quantifying-in; Muskens (2003) discusses how the same technique can be employed to solve the problem of medial extraction—a perennial problem in TLCG, where the directionality-sensitive modes of implication / and \ are inherently not suited for that purpose. The analysis of Gapping we propose below builds crucially on this analytic technique. Specifically, we treat Gapping to be a case of coordination of like-category constituents (with standard generalized conjunction for its semantics), where the coordinated constituents have medial gaps of the verbal category created via phonological (and semantic) variable binding. After the coordinate structure is built, the verb is 'lowered' into the medial position (just like the treatment of quantifiers by Oehrle (1994)), but the extra phonological property of the Gapping construction (which we encode in the Gapping-specific lexical entry for the conjunction) dictates that it be realized only once, in the initial conjunct of the whole coordinate structure.

In the present system, such constituents with medial verbal category gaps can be obtained by simply hypothesizing a variable for the main verb of the sentence and binding it by | after the whole sentence is built up:

(4)

$$
\dfrac{\lambda\sigma_1.\sigma_1(\mathsf{a} \circ \mathsf{book});\ \exists_{\mathbf{book}};\ \mathrm{S}|(\mathrm{S}|\mathrm{NP}) \qquad \dfrac{\dfrac{\mathsf{robin};\ \mathbf{r};\ \mathrm{NP} \qquad \dfrac{[\pi_1; P; \mathrm{VP}/\mathrm{NP}]^1\ [\pi_2; x; \mathrm{NP}]^2}{\pi_1 \circ \pi_2;\ P(x);\ \mathrm{VP}}}{\mathsf{robin} \circ \pi_1 \circ \pi_2;\ P(x)(\mathbf{r});\ \mathrm{S}}}{\lambda\pi_2.\mathsf{robin} \circ \pi_1 \circ \pi_2;\ \lambda x.P(x)(\mathbf{r});\ \mathrm{S}|\mathrm{NP}}}{\dfrac{\mathsf{robin} \circ \pi_1 \circ \mathsf{a} \circ \mathsf{book};\ \exists_{\mathbf{book}}(\lambda x.P(x)(\mathbf{r}));\ \mathrm{S}}{\lambda\pi_1.\mathsf{robin} \circ \pi_1 \circ \mathsf{a} \circ \mathsf{book};\ \lambda P.\exists_{\mathbf{book}}(\lambda x.P(x)(\mathbf{r}));\ \mathrm{S}|(\mathrm{VP}/\mathrm{NP})}}
$$

[2] For a more fine-grained control of surface morpho-phonological constituency, see Kubota and Pollard (2010) (and also Muskens (2007) for a related approach), which formalizes the notion of multi-modality from the earlier TLCG literature (Moortgat and Oehrle 1994, Morrill 1994) by modelling the mapping from syntax to phonology by means of an interpretation of (phonological) λ-terms into preorders.

For coordinating such $st \to st$ functions (phonologically), we introduce the following Gapping-specific lexical entry for the conjunction:

(5) $\lambda\sigma_2\lambda\sigma_1\lambda\pi_0[\sigma_1(\pi_0) \circ \mathsf{and} \circ \sigma_2(\varepsilon)]; \lambda\mathscr{W}\lambda\mathscr{V}.\mathscr{V} \sqcap \mathscr{W};$
$((\mathrm{S}|(\mathrm{VP/NP}))|(\mathrm{S}|(\mathrm{VP/NP})))|(\mathrm{S}|(\mathrm{VP/NP}))$

where ε is the empty string and $\mathscr{V}$ and $\mathscr{W}$ are variables over terms of type $\langle\langle e, \langle e, t\rangle\rangle, t\rangle$. Note here that syntactically the conjunction takes two arguments of the same category S|(VP/NP), and returns an expression of the same category, and the semantics is nothing other than the standard generalized conjunction. This is fully consistent with the general treatment of coordination in CG in terms of like category coordination. The only slight complication is in the phonology. The output phonological term is of the same phonological type $st \to st$ as the input phonologies, but instead of binding the variables in each conjunct by the same λ-operator, the gap in the second conjunct is filled by an empty string ε, since the verb is pronounced only once in the first conjunct in Gapping. This is an idiosyncrasy of the construction that needs to be stipulated in any account, and in the present approach it is achieved by a lexical specification of the phonological interpretation of the conjunction, without invoking any extra rule, empty operator or null lexical item.

With this entry for the conjunction, the simple Gapping sentence (6) can be derived as in (7) (with TV an abbreviation for VP/NP).

(6) Leslie bought a CD, and Robin, a book.

(7)

$\lambda\sigma_2\lambda\sigma_1\lambda\pi_0.\sigma_1(\pi_0) \circ \mathsf{and} \circ \sigma_2(\varepsilon);$ $\lambda\mathscr{W}\lambda\mathscr{V}.\mathscr{V} \sqcap \mathscr{W};$ $(\mathrm{S}|\mathrm{TV})|(\mathrm{S}|\mathrm{TV})|(\mathrm{S}|\mathrm{TV})$ $\lambda\pi_1.\mathsf{robin} \circ \pi_1 \circ \mathsf{a} \circ \mathsf{book};$ $\lambda P.\exists_{\mathbf{book}}(\lambda x.P(x)(\mathbf{r}));$ $\mathrm{S}|\mathrm{TV}$

$\lambda\pi_1.\mathsf{leslie} \circ \pi_1 \circ \mathsf{a} \circ \mathsf{CD};$ $\lambda Q.\exists_{\mathbf{CD}}(\lambda y.Q(y)(\mathbf{l}));$ $\mathrm{S}|\mathrm{TV}$ $\lambda\sigma_1\lambda\pi_0.\sigma_1(\pi_0) \circ \mathsf{and} \circ \mathsf{robin} \circ \varepsilon \circ \mathsf{a} \circ \mathsf{book};$ $\lambda\mathscr{V}.\mathscr{V} \sqcap \lambda P.\exists_{\mathbf{book}}(\lambda x.P(x)(\mathbf{r}));$ $(\mathrm{S}|\mathrm{TV})|(\mathrm{S}|\mathrm{TV})$

$\mathsf{bought};$ $\mathbf{buy};$ TV $\lambda\pi_0[\mathsf{leslie} \circ \pi_0 \circ \mathsf{a} \circ \mathsf{CD} \circ \mathsf{and} \circ \mathsf{robin} \circ \varepsilon \circ \mathsf{a} \circ \mathsf{book}];$ $\lambda Q.\exists_{\mathbf{CD}}(\lambda y.Q(y)(\mathbf{l})) \sqcap \lambda P.\exists_{\mathbf{book}}(\lambda x.P(x)(\mathbf{r}));$ $\mathrm{S}|\mathrm{TV}$

$\mathsf{leslie} \circ \mathsf{bought} \circ \mathsf{a} \circ \mathsf{CD} \circ \mathsf{and} \circ \mathsf{robin} \circ \varepsilon \circ \mathsf{a} \circ \mathsf{book};$
$\exists_{\mathbf{CD}}(\lambda y.\mathbf{buy}(y)(\mathbf{l})) \wedge \exists_{\mathbf{book}}(\lambda x.\mathbf{buy}(x)(\mathbf{r})); \mathrm{S}$

What is crucial in the above analysis is that two conjoined gapped sentences form a tectogrammatical constituent, to which the verb lowers into. This enables a treatment of Gapping without any surface deletion operation or phonologically inaudible verbal pro-form of any sort. We will see below that this is also what enables a straightforward analysis of the scopal interactions between negative and modal auxiliaries and Gapping.

For the analysis of cases involving modal and negative auxiliaries, we assume an analysis of auxiliaries that treats them as quantifier-like scope-taking expressions. Morpho-phonologically, auxiliaries have a distributional property of a VP modifier of category VP/VP (which differs from VP adverbs VP\VP only in the direction in which the argument is sought). But semantically, modals and negation are sentential operators μ which take some proposition φ as an argument and return another proposition $\mu(\varphi)$. In the present approach, this syntax-semantics mismatch can be straightforwardly captured by assigning lexical entries of the following form to auxiliaries:

(8) $\lambda\sigma.\sigma(\mathsf{must})$; $\lambda\mathscr{F}.\Box\mathscr{F}(\vartheta)$; S|(S|VP/VP) (where $\vartheta =_{def} \lambda f_{\langle e,t\rangle}.f$)

That is, the auxiliary verb binds a VP/VP (i.e. forward-looking VP modifier) gap in a sentence to return a fully saturated S. The VP modifier gap is vacuously bound by supplying an identify function in its place, and the real semantic contribution of the auxiliary corresponds to the modal operator that takes as its scope the entire proposition obtained by binding that VP modifier gap.[3]

As an illustration, *Robin must discover a solution* is derived as:

(9)

$$\frac{\mathsf{discover};\ \mathbf{discover};\ \mathrm{VP/NP} \qquad \pi_1;\ x;\ \mathrm{NP}}{\mathsf{discover} \circ \pi_1;\ \mathbf{discover}(x);\ \mathrm{VP}}$$

$$\frac{\pi_2;\ f;\ \mathrm{VP/VP} \qquad \mathsf{discover} \circ \pi_1;\ \mathbf{discover}(x);\ \mathrm{VP}}{\pi_2 \circ \mathsf{discover} \circ \pi_1;\ f(\mathbf{discover}(x));\ \mathrm{VP}}$$

$$\frac{\mathsf{robin};\ \mathbf{r};\ \mathrm{NP} \qquad \pi_2 \circ \mathsf{discover} \circ \pi_1;\ f(\mathbf{discover}(x));\ \mathrm{VP}}{\mathsf{robin} \circ \pi_2 \circ \mathsf{discover} \circ \pi_1;\ f(\mathbf{discover}(x))(\mathbf{r});\ \mathrm{S}}$$

$$\frac{\mathsf{robin} \circ \pi_2 \circ \mathsf{discover} \circ \pi_1;\ f(\mathbf{discover}(x))(\mathbf{r});\ \mathrm{S}}{\lambda\pi_1.\mathsf{robin} \circ \pi_2 \circ \mathsf{discover} \circ \pi_1;\ \lambda x.f(\mathbf{discover}(x))(\mathbf{r});\ \mathrm{S|NP}}$$

$$\frac{\lambda\sigma.\sigma(\mathsf{a} \circ \mathsf{solution});\ \exists_{\mathbf{solution}};\ \mathrm{S|(S|NP)} \qquad \lambda\pi_1.\mathsf{robin} \circ \pi_2 \circ \mathsf{discover} \circ \pi_1;\ \lambda x.f(\mathbf{discover}(x))(\mathbf{r});\ \mathrm{S|NP}}{\mathsf{robin} \circ \pi_2 \circ \mathsf{discover} \circ \mathsf{a} \circ \mathsf{solution};\ \exists_{\mathbf{solution}}(\lambda x.f(\mathbf{discover}(x))(\mathbf{r}));\ \mathrm{S}}$$

$$\frac{\mathsf{robin} \circ \pi_2 \circ \mathsf{discover} \circ \mathsf{a} \circ \mathsf{solution};\ \exists_{\mathbf{solution}}(\lambda x.f(\mathbf{discover}(x))(\mathbf{r}));\ \mathrm{S}}{\lambda\pi_2.\mathsf{robin} \circ \pi_2 \circ \mathsf{discover} \circ \mathsf{a} \circ \mathsf{solution};\ \lambda f.\exists_{\mathbf{solution}}(\lambda x.f(\mathbf{discover}(x))(\mathbf{r}));\ \mathrm{S|(VP/VP)}}$$

$$\frac{\lambda\sigma_0.\sigma_0(\mathsf{must});\ \lambda\mathscr{F}.\Box\mathscr{F}(\vartheta);\ \mathrm{S|(S|(VP/VP))} \qquad \lambda\pi_2.\mathsf{robin} \circ \pi_2 \circ \mathsf{discover} \circ \mathsf{a} \circ \mathsf{solution};\ \lambda f.\exists_{\mathbf{solution}}(\lambda x.f(\mathbf{discover}(x))(\mathbf{r}));\ \mathrm{S|(VP/VP)}}{\mathsf{robin} \circ \mathsf{must} \circ \mathsf{discover} \circ \mathsf{a} \circ \mathsf{solution};\ \Box\exists_{\mathbf{solution}}(\lambda x.\mathbf{discover}(x)(\mathbf{r}));\ \mathrm{S}}$$

We are now ready to illustrate how the auxiliary wide-scope readings are obtained for Gapping sentences. We start with a variant in (10a) in which only the auxiliary is gapped (for which the derivation is a bit simpler), and then move on to the case of (10b) where the whole auxiliary + verb combination is gapped.

(10) a. John can't eat steak and Mary eat pizza.

b. John can't eat steak and Mary pizza.

[3] As it is, the analysis of auxiliaries here overgenerates. To capture the clause-boundedness of the scope of auxiliaries, we can employ the technique proposed by Pogodalla and Pompigne (2011) which enables formulating constraints on scope islands at the tectogrammatical level with the notion of dependent types (dependent types are used in certain logic-based grammars for the purpose of implementing syntactic features; see de Groote and Maarek (2007)).

As in the basic-case analysis given in (7) above, the overall strategy is straightforward: we coordinate two categories which are in effect clauses missing VP/VP functors in each conjunct, forming a larger sign of the same category:

(11)

$$\dfrac{\dfrac{\dfrac{\mathsf{john};\ \mathbf{j};\ \mathrm{NP} \qquad \dfrac{\pi_1;\ f;\ \mathrm{VP/VP} \qquad \mathsf{eat} \circ \mathsf{steak};\ \mathbf{eat}(\mathbf{s});\ \mathrm{VP}}{\pi_1 \circ \mathsf{eat} \circ \mathsf{steak};\ f(\mathbf{eat}(\mathbf{s}));\ \mathrm{VP}}}{\mathsf{john} \circ \pi_1 \circ \mathsf{eat} \circ \mathsf{steak};\ f(\mathbf{eat}(\mathbf{s}))(\mathbf{j});\ \mathrm{S}}}{\lambda\pi_1.\mathsf{john} \circ \pi_1 \circ \mathsf{eat} \circ \mathsf{steak};\ \lambda f.f(\mathbf{eat}(\mathbf{s}))(\mathbf{j});\ \mathrm{S|(VP/VP)}} \qquad \dfrac{\lambda\sigma_2\lambda\sigma_1\lambda\pi_0.\sigma_1(\pi_0) \circ \mathsf{and} \circ \sigma_2(\varepsilon);\ \lambda\mathscr{F}_2\lambda\mathscr{F}_1.\mathscr{F}_1 \sqcap \mathscr{F}_2;\ \mathrm{(S|X)|(S|X)|(S|X)} \qquad \lambda\pi_2.\mathsf{mary} \circ \pi_2 \circ \mathsf{eat} \circ \mathsf{pizza};\ \lambda g.g(\mathbf{eat}(\mathbf{p}))(\mathbf{m});\ \mathrm{S|(VP/VP)}}{\lambda\sigma_1\lambda\pi_0.\sigma_1(\pi_0) \circ \mathsf{and} \circ \mathsf{mary} \circ \varepsilon \circ \mathsf{eat} \circ \mathsf{pizza};\ \lambda\mathscr{F}_1.\mathscr{F}_1 \sqcap \lambda g.g(\mathbf{eat}(\mathbf{p}))(\mathbf{m});\ \mathrm{(S|(VP/VP))|(S|(VP/VP))}}}{\lambda\pi_0.\mathsf{john} \circ \pi_0 \circ \mathsf{eat} \circ \mathsf{steak} \circ \mathsf{and} \circ \mathsf{mary} \circ \varepsilon \circ \mathsf{eat} \circ \mathsf{pizza};\ \lambda f.f(\mathbf{eat}(\mathbf{s}))(\mathbf{j}) \sqcap \lambda g.g(\mathbf{eat}(\mathbf{p}))(\mathbf{m});\ \mathrm{S|(VP/VP)}}$$

This coordinated 'gapped' constituent is then given as an argument to the auxiliary to complete the derivation, just as in the simpler example in (9) above.

(12)

$$\dfrac{\lambda\sigma_0.\sigma_0(\mathsf{can't});\ \lambda\mathscr{F}.\neg\Diamond\mathscr{F}(\vartheta);\ \mathrm{S|(S|(VP/VP))} \qquad \lambda\pi_0.\mathsf{john} \circ \pi_0 \circ \mathsf{eat} \circ \mathsf{steak} \circ \mathsf{and} \circ \mathsf{mary} \circ \varepsilon \circ \mathsf{eat} \circ \mathsf{pizza};\ \lambda f.f(\mathbf{eat}(\mathbf{s}))(\mathbf{j}) \sqcap \lambda g.g(\mathbf{eat}(\mathbf{p}))(\mathbf{m});\ \mathrm{S|(VP/VP)}}{\mathsf{john} \circ \mathsf{can't} \circ \mathsf{eat} \circ \mathsf{steak} \circ \mathsf{and} \circ \mathsf{mary} \circ \varepsilon \circ \mathsf{eat} \circ \mathsf{pizza};\ \neg\Diamond[\mathbf{eat}(\mathbf{s})(\mathbf{j}) \wedge \mathbf{eat}(\mathbf{p})(\mathbf{m})];\ \mathrm{S}}$$

Here, crucially, due to generalized conjunction, the proposition that the modal scopes over is the conjunction of the propositions expressed by the first conjunct (without the modal) and the second conjunct. Thus, we get an interpretation in which the modal scopes over the conjunction, as desired. For the phonology, just as in the simpler Gapping example, due to the lexical definition of the Gapping-type conjunction, the modal auxiliary is pronounced only in the first conjunct, resulting in the surface string corresponding to (10a).[4]

We now show how this same approach yields a wide scope reading for the auxiliary where both the auxiliary and the main verb are missing in the second conjunct, as in (10b). The derivation goes as in (13). The extra complexity involved in this case is that we need to fill in both the verb and the auxiliary in the first conjunct to obtain the surface form of the sentence. This is done in a stepwise manner. First, the verb and a hypothesized forward-looking VP modifier (to be bound by the auxiliary) form an expression of the VP/NP category via

[4] See Siegel (1987) for a closely related approach in terms of wrapping in the framework of Montague Grammar. For the auxiliary-gapping example like (10a), our analysis can be thought of as a formally precise rendition of the basic analytic idea prefigured in Siegel's analysis. However, the presence vs. absence of an explicit prosodic calculus that interacts with the combinatoric component of syntax becomes crucial in the more complex case in (10b), where both the verb and the auxiliary are gapped. It is not at all clear how the right pairing of meaning and surface string can be derived for such examples in Siegel's setup, which assumes a rather primitive and unformalized infixation operation within Montague Grammar for dealing with discontinuous constituency.

hypothetical reasoning. This is then given as an argument to a coordinated gapped sentence of type S|(VP/NP). Finally, by binding the VP modifier of type VP/VP, the sentence has the right syntactic (and phonological and semantic, as well) type to be given as an argument to the auxiliary *can't*.

(13)

$$\dfrac{\dfrac{\dfrac{\begin{array}{l}\pi_0;\\ f;\mathrm{VP/VP}\end{array} \quad \dfrac{\begin{array}{l}\mathsf{eat};\\ \mathbf{eat};\mathrm{VP/NP}\end{array} \quad \begin{array}{l}\pi_1;\\ x;\mathrm{NP}\end{array}}{\mathsf{eat}\circ\pi_1;\ \mathbf{eat}(x);\ \mathrm{VP}}}{\pi_0\circ\mathsf{eat}\circ\pi_1;\ f(\mathbf{eat}(x));\ \mathrm{VP}}}{\pi_0\circ\mathsf{eat};\ \lambda x.f(\mathbf{eat}(x));\ \mathrm{VP/NP}} \quad \begin{array}{l}\lambda\pi_2.\mathsf{john}\circ\pi_2\circ\mathsf{steak}\circ\mathsf{and}\circ\mathsf{mary}\circ\varepsilon\circ\mathsf{pizza};\\ \lambda Q.[Q(\mathbf{s})(\mathbf{j})]\sqcap\lambda P.[P(\mathbf{p})(\mathbf{m})];\\ \mathrm{S}|(\mathrm{VP/NP})\end{array}}{\dfrac{\mathsf{john}\circ\pi_0\circ\mathsf{eat}\circ\mathsf{steak}\circ\mathsf{and}\circ\mathsf{mary}\circ\varepsilon\circ\mathsf{pizza};\ f(\mathbf{eat}(\mathbf{s}))(\mathbf{j})\wedge f(\mathbf{eat}(\mathbf{p}))(\mathbf{m});\ \mathrm{S}}{\begin{array}{c}\lambda\pi_0.\mathsf{john}\circ\pi_0\circ\mathsf{eat}\circ\mathsf{steak}\circ\mathsf{and}\circ\mathsf{mary}\circ\varepsilon\circ\mathsf{pizza};\\ \lambda f.[f(\mathbf{eat}(\mathbf{s}))(\mathbf{j})\wedge f(\mathbf{eat}(\mathbf{p}))(\mathbf{m})];\mathrm{S}|(\mathrm{VP/VP})\end{array}}}$$

$$\dfrac{\begin{array}{l}\lambda\sigma_0.\sigma_0(\mathsf{can't});\\ \lambda\mathscr{F}.\neg\Diamond\mathscr{F}(\vartheta);\\ \mathrm{S}|(\mathrm{S}|(\mathrm{VP/VP}))\end{array} \quad \begin{array}{l}\lambda\pi_0.\mathsf{john}\circ\pi_0\circ\mathsf{eat}\circ\mathsf{steak}\circ\mathsf{and}\circ\mathsf{mary}\circ\varepsilon\circ\mathsf{pizza};\\ \lambda f.[f(\mathbf{eat}(\mathbf{s}))(\mathbf{j})\wedge f(\mathbf{eat}(\mathbf{p}))(\mathbf{m})];\\ \mathrm{S}|(\mathrm{VP/VP})\end{array}}{\begin{array}{l}\mathsf{john}\circ\mathsf{can't}\circ\mathsf{eat}\circ\mathsf{steak}\circ\mathsf{and}\circ\mathsf{mary}\circ\varepsilon\circ\mathsf{pizza};\\ \neg\Diamond[\mathbf{eat}(\mathbf{s})(\mathbf{j})\wedge\mathbf{eat}(\mathbf{p})(\mathbf{m})];\mathrm{S}\end{array}}$$

Again, since the auxiliary takes the coordinated sentence (after the verb is fed to it) as its argument in the derivation, we obtain the auxiliary wide-scope interpretation. In the present account, the wide-scope option for the auxiliary in examples like (10a,b) transparently reflects the (tectogrammatical) syntax of Gapping where sentences with missing elements are *directly* coordinated and the missing element is supplied at a later point in the derivation. Thus, the availability of such a reading is not a surprise, but a naturally expected consequence.

The present analysis predicts the availability of conjunction wide-scope readings for sentences like those in (10) as well. The key component of the analysis involves deriving a VP/VP entry for an auxiliary from the more basic type assigned in the lexicon above in the category S|(S|(VP/VP)), which reflects their semantic property more transparently. The derivation proceeds through a couple of steps of hypothetical reasoning:

(14)

$$\dfrac{\dfrac{\dfrac{\begin{array}{l}\lambda\sigma.\sigma(\mathsf{can't});\\ \lambda\mathscr{F}.\neg\Diamond\mathscr{F}(\vartheta);\\ \mathrm{S}|(\mathrm{S}|\mathrm{VP/VP})\end{array} \quad \dfrac{\dfrac{\pi_1;\ x;\ \mathrm{NP} \quad \dfrac{\pi_2;\ g;\ \mathrm{VP/VP} \quad \pi_3;\ f;\ \mathrm{VP}}{\pi_2\circ\pi_3;\ g(f);\ \mathrm{VP}}}{\pi_1\circ\pi_2\circ\pi_3;\ g(f)(x);\ \mathrm{S}}}{\begin{array}{l}\lambda\pi_2.\pi_1\circ\pi_2\circ\pi_3;\\ \lambda g.g(f)(x);\\ \mathrm{S}|(\mathrm{VP/VP})\end{array}}}{\pi_1\circ\mathsf{can't}\circ\pi_3;\ \neg\Diamond f(x);\ \mathrm{S}}}{\mathsf{can't}\circ\pi_3;\ \lambda x.\neg\Diamond f(x);\ \mathrm{VP}}}{\mathsf{can't};\ \lambda f\lambda x.\neg\Diamond f(x);\ \mathrm{VP/VP}}$$

The derived entry in the VP/VP category is the familiar entry for auxiliaries in non-transformational approaches like G/HPSG and categorial grammar. The above result depends crucially on the property of the present system where reasoning involving the directional mode of implication can be carried out based on the results of reasoning involving |, which allows for operations that are more complex than string concatenation.[5]

With the above, derived, type assignment for the auxiliary, the conjunction wide-scope reading for (10a) is straightforward. The derivation is identical to the one for the auxiliary wide-scope reading up to the point that the coordinated gapped sentence is formed, and differs only at the final step. Instead of having the scope-taking S|(S|(VP/VP)) entry of the auxiliary take this coordinated gapped S as an argument, we simply give the lowered VP/VP entry for the auxiliary as an argument to the gapped sentence, as follows:

(15)
$$\frac{\begin{array}{ll}\mathsf{can't}; & \lambda\pi.[\mathsf{john} \circ \pi \circ \mathsf{eat} \circ \mathsf{steak} \circ \mathsf{and} \circ \mathsf{mary} \circ \varepsilon \circ \mathsf{eat} \circ \mathsf{pizza}]; \\ \lambda f \lambda x.\neg\Diamond f(x); & \lambda h.[h(\mathbf{eat}(\mathbf{s}))(\mathbf{j}) \wedge h(\mathbf{eat}(\mathbf{p}))(\mathbf{m})]; \\ \mathrm{VP/VP} & \mathrm{S}|(\mathrm{VP/VP})\end{array}}{\begin{array}{l}\mathsf{john} \circ \mathsf{can't} \circ \mathsf{eat} \circ \mathsf{steak} \circ \mathsf{and} \circ \mathsf{mary} \circ \varepsilon \circ \mathsf{eat} \circ \mathsf{pizza}; \\ \neg\Diamond\mathbf{eat}(\mathbf{s})(\mathbf{j}) \wedge \neg\Diamond\mathbf{eat}(\mathbf{p})(\mathbf{m}); \mathrm{S}\end{array}}$$

The resulting string is identical as above since the phonology of the auxiliary is embedded in the gap site in the initial conjunct only, but the semantic interpretation that is paired with it is different from the above analysis. Here crucially, the VP-modifier meaning of the auxiliary is distributed to the two conjuncts via the definition of generalized conjunction, which results in an interpretation where the auxiliary takes scope separately within each conjunct which is then conjoined by the conjunction, resulting in the conjunction wide-scope reading.

The derivation for the conjunction wide-scope reading for (10b), a sentence in which both the auxiliary and the main verb are missing, is also straightforward. In fact, like the previous case, the analysis just involves replacing the auxiliary entry at the final step of the derivation for the auxiliary wide-scope reading of the same sentence (given above in (13)) with the derived entry in (14). This yields the conjunction wide-scope reading for the sentence for exactly the same reason as in the previous example, as the reader can easily verify by themselves.

3 Comparison with Related Approaches

3.1 Steedman (1990)

Steedman (1990) proposes an insightful analysis of Gapping in CCG which can be thought of as a precursor of the present proposal. The key analytic idea of Steedman's approach is the assumption that Gapping involves coordination of like-category constituents. To reconcile the strictly surface-oriented syntax of CCG with this assumption about the 'underling' syntax of Gapping, Steedman invokes a syntactic rule called the Left Conjunct Revealing Rule (LCRR):[6]

[5] So far as we are aware, interactions between the two kinds of syntactic reasoning (or modes of composition) of this sort is a completely novel property of the present system that is not shared by any other formal theory of syntax, whether categorial, constraint-based, or transformational. This certainly opens up many questions conceptually, technically and empirically, which we will not pursue further here.

[6] We use the Lambek-style notation for slashes for consistency.

(16) **The Left Conjunct Revealing Rule**
$\mathrm{S} \Rightarrow Y\ Y\backslash\mathrm{S}$

This rule ‘decomposes’ the left conjunct into two syntactic categories, one corresponding to the right conjunct and the other corresponding to the shared verbal element. Once this decomposition is in place, the rest of the derivation is straightforward with standard generalized conjunction to form a coordinate structure which recombines with the shared verbal category as in:

(17)

$$\dfrac{(\mathrm{NP}\backslash\mathrm{S})/\mathrm{NP} \qquad \dfrac{((\mathrm{NP}\backslash\mathrm{S})/\mathrm{NP})\backslash\mathrm{S} \qquad \dfrac{\text{and Barry potatoes}}{((\mathrm{NP}\backslash\mathrm{S})/\mathrm{NP})\backslash\mathrm{S}}}{((\mathrm{NP}\backslash\mathrm{S})/\mathrm{NP})\backslash\mathrm{S}}}{\mathrm{S}}$$

where $(\mathrm{NP}\backslash\mathrm{S})/\mathrm{NP}$ and $((\mathrm{NP}\backslash\mathrm{S})/\mathrm{NP})\backslash\mathrm{S}$ are obtained from $\dfrac{\text{Harry will buy bread}}{\mathrm{S}}$ by the double-line decomposition.

Simple and elegant though it might appear, this analysis is problematic for both conceptual/technical and empirical reasons. Conceptually and technically, note that (16) constitutes a clearcut violation of the principle of compositionality. The key problem is that there is no way, on Steedman’s account, to guarantee that either the Y or the $Y\backslash\mathrm{S}$ can be independently assembled from the component of S on the basis of its subcomponents that they are supposed to correspond to, and, concomitantly, no way to ensure the existence of an actual semantic interpretation for the ‘revealed’ $Y\backslash\mathrm{S}$ pseudocategory or its complement Y.[7] Steedman’s account thus seems to ride roughshod over the fundamental motivation of the categorial approach, viz., the conception of syntactic derivations as logical proofs (whose structure is not an object that the grammar can manipulate).[8]

There is also an empirical problem. In the strictly surface-oriented syntax of CCG, there does not seem to be any straightforward account of the auxiliary wide-scope readings of sentences with modal and negation. Like in other non-transformational approaches to syntax, CCG assumes the VP/VP-type entry for auxiliaries. However, as shown in the previous section, such an entry produces only the conjunction wide-scope readings for examples like those in (2).

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[7] So far as we can see, the only way to ensure such a decomposition in the grammar is by ‘appealing to the parser, or to some reification of the derivation’ (Steedman 1990, 247), a possibility which, curiously enough, Steedman rejects flatly. He instead resorts to some vague (and what seems to us to be an ill-conceived) pragmatic strategy of recovering the *syntactic category* (as opposed to just the interpretation) of the gapped verb through the presupposition of the gapped sentence.

[8] Note in this connection that the formal status of the LCRR is quite unclear. In a labelled deduction presentation of derivations of the kind we have adopted above, there is no way to formulate such a rule, since in (16) the pieces of linguistic expression that the decomposed categories are supposed to correspond to are entirely unspecified. This alone makes Steedman’s whole approach to Gapping quite dubious.

3.2 Morrill et al. (2011)

In the TLCG literature, a series of related approaches to the analysis of Gapping have been proposed (see the references cited in section 1) that provide an explicit solution for the problem of identifying the gap constituent that is left open in Steedman's analysis. These approaches all treat Gapping as a case of discontinuous constituency, employing the various extensions to the Lambek calculus for handling discontinuity that they respectively propose. We review Morrill et al.'s proposal here since it is the most recent among these related approaches and it improves both technically and empirically on the earlier accounts.

The key analytic idea of Morrill et al.'s approach, which is due to Hendriks (1995), and which is a formalization of the underlying idea of Steedman's approach, is that Gapping can be thought of as a case of like-category coordination by allowing the conjunction to coordinate two discontinuous constituents with medial gaps of the verbal type and then infixing the missing verb in the gap position of the initial conjunct after the whole coordinate structure is built. Specifically, Morrill et al. assign the syntactic type $((S{\uparrow}TV)\backslash(S{\uparrow}TV))/\hat{}(S{\uparrow}TV)$ to the conjunction. $\uparrow$ is roughly equivalent to our $|$. Thus, $S{\uparrow}TV$ is the category for a sentence missing a TV somewhere inside it. $\hat{}$ corresponds to an operation that erases the 'insertion point' keeping track of the gap position of a discontinuous constituent. Thus, the category of the right conjunct $\hat{}(S{\uparrow}TV)$ indicates that it is a sentence with a TV gap inside it like $S{\uparrow}TV$, except that the gap is already 'closed off'. The whole coordinate structure inherits the gap position from the left conjunct alone, to which the verb is infixed after the whole coordinate structure is built.

If we limit ourselves to cases in which the gapped material is just a string, our analysis and Morrill et al.'s can be thought of as notational variants of each other.[9] However, a difference between the two emerges when we examine more complex examples where the missing material in the gapped clause is itself a discontinuous constituent, such as the following:

(18) a. John *gave* Mary *a cold shoulder*, and Bill, Sue.
b. John *called* Mary *up*, and Bill, Sue.

Idiomatic expressions like *give ... the cold shoulder* and verb-particle constructions like *call ... up* are analyzed as discontinuous constituents in CG, including Morrill et al.'s own approach. However, their analysis of Gapping does not interact properly with their analysis of these constructions to license examples like those in (18). The difficulty essentially lies in the fact that Morrill et al.'s system is set up in such a way that it only recognizes discontinuous constituents with string-type gaps (whose positions are kept track of by designated symbols

9 But note that it remains to be established that the analysis of the scope ambiguity of auxiliaries in examples like (10) in our account can be replicated in their setup—so far as we can see, the analysis of the auxiliary-wide scope reading seems to carry over to their setup straightforwardly, whereas the case of the conjunction-wide scope reading is less clear.

called separators for marking insertion points in the prosodic representations of linguistic expressions) and the only operations that one can perform on such expressions (each tied to different syntactic rules in the calculus) are to close off the gap with some string (including an empty string) or to pass it up to a larger expression. Thus, there is no direct way of representing the combinatorial property of the gapped clause *Bill, Sue* in (18a), which needs to be treated as a sentence missing a VP↑NP in order to induce a like-category analysis of Gapping along the above lines. It should be noted that this problem is by no means specific to Morrill et al.'s account but is common to all previous approaches in the TLCG literature that employ some form of wrapping operation for the treatment of discontinuity. Such approaches fall short of extending to cases like (18) essentially because the prosodic component is not fully independent of the syntactic calculus and the extension to the basic concatenative system is directly regulated by the set of additional syntactic connectives introduced in the system. (To see this, note that in these approaches, each syntactic connective for discontinuity is tied to some specific operation (such as infixation) on the propsodic form(s) of the input expressions).

Our proposal differs from these earlier approaches precisely in this respect. Indeed, with the flexible syntax-prosody interface enabled by having a full-blown λ-calculus for the prosodic component—a feature that the present system inherits from λ-Grammar/ACG—the analysis of examples like (18) turns out to be relatively straightforward. Assuming that the discontinuous constituency of idioms and verb-particle constructions is treated by assigning to the relevant expressions lexical entries of the following form (of phonological type $st \to st$):

(19) $\lambda\pi_1.\mathsf{gave} \circ \pi_1 \circ \mathsf{the} \circ \mathsf{cold} \circ \mathsf{shoulder}; \mathbf{shun}; \mathrm{VP}|\mathrm{NP}$

it only suffices to generalize the lexical entry of the Gapping-type conjunction to a higher-order (phonological) type which takes arguments of type $(st \to st) \to st$ (phonologically) as left and right conjuncts:

(20) $\lambda\rho_1\lambda\rho_2\lambda\sigma.\rho_2(\sigma) \circ \mathsf{and} \circ \rho_1(\lambda\pi.\pi); \lambda\mathscr{W}\lambda\mathscr{V}.\mathscr{V} \sqcap \mathscr{W}; (\mathrm{S}|(\mathrm{VP}|\mathrm{X}))|(\mathrm{S}|(\mathrm{VP}|\mathrm{X}))|(\mathrm{S}|(\mathrm{VP}|\mathrm{X}))$

Then, via hypothetical reasoning with a variable of (phonological) type $st \to st$, the left and right conjuncts can be treated as discontinuous constituents of type $(st \to st) \to st$ of the following form, where the gap itself is a discontinuous constituent of type $st \to st$:

(21) $\lambda\sigma_1.\mathsf{john} \circ \sigma_1(\mathsf{mary}); \lambda P.P(\mathbf{m})(\mathbf{j}); \mathrm{S}|(\mathrm{VP}|\mathrm{NP})$

It is straightforward to see that by giving such expressions to the higher-order Gapping conjunction entry (20), the right surface string in (18) is obtained.

Thus, while the proposed analysis owes much to previous approaches to Gapping in terms of discontinuous constituency in the TLCG literature in the formulation of the basic analysis, it goes beyond all previous proposals in straightforwardly generalizing to more complex cases like (18) where Gapping interacts with other phenomena exemplifying discontinuity. So far as we are aware, such a systematic interaction of complex empirical phenomena is unprecedented in any previous work.

3.3 Oehrle (1987)

Oehrle's analysis assumes a free Boolean algebra over the set of generators comprising the cartesian product NP × NP, with meet $\wedge$ and join $\vee$, which Oehrle notates $\mathbf{L}$[NP×NP]. The set NP × NP is the domain of functors corresponding to verb signs, which are taken to comprise both phonological and semantic functors. An embedding from NP × NP to an algebra with meet and join operations yields $\mathbf{L}$[NP× NP], which is the closure of its atoms in NP × NP under $\vee$ and $\wedge$.

For each verbal sign $\boldsymbol{v}$, which is a function NP×NP $\mapsto$ 2, Oehrle defines an extension of that function $\boldsymbol{v}^*$ $\mathbf{L}$[NP×NP] $\mapsto$ 2. For example, for *bakes*, we have ***bake*** NP×NP $\mapsto$ 2, comprising the phonological function $\boldsymbol{bake}_\pi$ and the semantic function $\boldsymbol{bake}_\sigma$ such that (here again, π_1 and π_2 are projection functions):

(22) a. $\boldsymbol{bake}_\pi = \lambda P.\ \pi_1(P) \circ \textsf{bakes} \circ \pi_2(P)$
b. $\boldsymbol{bake}_\sigma = \lambda X.\mathbf{bake}(\pi_1(X), \pi_2(X))$

$\boldsymbol{bake}^*$ is then defined as consisting of the phonological function $\boldsymbol{bake}^*_\pi$ and the semantic function $\boldsymbol{bake}^*_\sigma$ such that:

(23) i. For all P that are atomic, $\boldsymbol{bake}^*_\pi(P) = \boldsymbol{bake}_\pi(P)$
ii. If $P = P_1 \wedge P_2$, then $\boldsymbol{bake}^*_\pi(P) = \boldsymbol{bake}_\pi(P_1) \circ \textsf{and} \circ \pi_1(P_2) \circ \pi_2(P_2)$
iii. If $P = P_1 \vee P_2$, then $\boldsymbol{bake}^*_\pi(P) = \boldsymbol{bake}_\pi(P_1) \circ \textsf{or} \circ \pi_1(P_2) \circ \pi_2(P_2)$

(24) i. For all X that are atomic, $\boldsymbol{bake}^*_\sigma(X) = \boldsymbol{bake}_\sigma(X)$
ii. If $X = X_1 \wedge X_2$, then $\boldsymbol{bake}^*_\sigma(X) = \boldsymbol{bake}_\sigma(X_1) \wedge \boldsymbol{bake}_\sigma(X_2)$
iii. If $X = X_1 \vee X_2$, then $\boldsymbol{bake}^*_\sigma(X) = \boldsymbol{bake}_\sigma(X_1) \vee \boldsymbol{bake}_\sigma(X_2)$

The grammar thus admits (25), comprising the phonology and semantics in (26).

(25) $\boldsymbol{bake}^*(\langle \boldsymbol{john}, \boldsymbol{bread}\rangle \wedge \langle \boldsymbol{mary}, \boldsymbol{cake}\rangle)$
$= \langle \boldsymbol{bake}^*_\pi(\langle \textsf{john}, \textsf{bread}\rangle \wedge \langle \textsf{mary}, \textsf{cake}\rangle), \boldsymbol{bake}^*_\sigma(\langle \mathbf{j}, \mathbf{b}\rangle \wedge \langle \mathbf{m}, \mathbf{c}\rangle)\rangle$

(26) a. $\boldsymbol{bake}^*_\pi(\langle \textsf{john}, \textsf{bread}\rangle \wedge \langle \textsf{mary}, \textsf{cake}\rangle)$
$= \boldsymbol{bake}_\pi(\langle \textsf{john}, \textsf{bread}\rangle) \circ \textsf{and} \circ \textsf{mary} \circ \textsf{cake}$
$= \textsf{john} \circ \textsf{bakes} \circ \textsf{bread} \circ \textsf{and} \circ \textsf{mary} \circ \textsf{cake}$
b. $\boldsymbol{bake}^*_\sigma(\langle \mathbf{j}, \mathbf{b}\rangle \wedge \langle \mathbf{m}, \mathbf{c}\rangle)$
$= \boldsymbol{bake}_\sigma(\langle \mathbf{j}, \mathbf{b}\rangle) \wedge \boldsymbol{bake}_\sigma(\langle \mathbf{m}, \mathbf{c}\rangle) = \mathbf{bake}(\mathbf{j},\mathbf{b}) \wedge \mathbf{bake}(\mathbf{m},\mathbf{c})$

The key insight that enables Oehrle to account for the scope ambiguity of negative and modal auxiliaries is that when propositional operators like negation interact with verb meaning, there are two maps from $\mathbf{L}$[NP×NP] to $\mathbf{2}$, with two different semantic results. The first option is to compose the negation operator **neg** with the lexical verb $\boldsymbol{v}$ and then extend it with the * operator to obtain $(\mathbf{neg} \circ \boldsymbol{v})^*$, which produces a function that takes arguments in the domain of conjoined pairs of verbal arguments. This yields the conjunction wide-scope interpretation since * extends verb meanings that are already negated and which take unconjoined pairs of arguments to the domain of conjoined pairs of arguments. The other option is to compose the negation operator **neg** with the result of the application of *, which gives us $\mathbf{neg} \circ \boldsymbol{v}^*$. Since $\boldsymbol{v}^*$ is the closure of $\boldsymbol{v}$ under meet and join which semantically correspond to conjunction and disjunction, this yields a function that takes conjoined pairs of arguments as input, and then first form unnegated conjunction or disjunction of two propositions obtained by applying the verb meaning to the each of the conjoined pair of arguments, which is then passed on to the negation operator as an argument to produce a negated proposition, which corresponds to the negation wide-scope interpretation.

Thus, in Oehrle's analysis, the assumption that argument pairs of verbs can be treated as conjoinable constituents and that the * operator that maps verb meanings from NP×NP to **L**[NP×NP] which contains such conjoined argument pairs plays a crucial role in deriving the scopal interactions between conjunction and operators such as negation and modals. While the elegance and systematicity by which the auxiliary wide-scope readings are derived is remarkable, Oehrle's analysis relies on several nonstandard assumptions about both the basic clause structure of English and the syntax of Gapping. In particular, since the analysis crucially hinges on the assumption that the remnants that appear in the right conjunct are pairs of *arguments* of a verb, it is not clear how the analysis might be extended to cases involving adjuncts in the remnant, such as (1c) from section 1. Since adverbs are adjuncts which are functions that take verbs as arguments rather than themselves being arguments of the verb, it is not clear how examples like (1c) can be licensed in Oehrle's setup. Note furthermore that such argument-adjunct pairs in Gapping can also induce the same kind of scopal interaction with auxiliaries as the argument-pair examples examined above:

(27) Terry can't go there with me and Pat with you—one and the same person has to accompany them both.

This suggests that generating surface strings like (1c) isn't enough and that the mechanism for licensing the two scoping possibilities for argument pairs has to be extended to cases involving adjuncts too. However, given the nonstandard assumptions about syntax that Oehrle's analysis builds on, it is not clear whether such an extension can be worked out straightforwardly.

4 Conclusion

We have proposed a system of TLCG that models phonologies of linguistic signs by λ-terms, allowing for higher-order abstraction over string-type entities. The flexible treatment of linguistic expressions manifesting discontinuous constituency that the present system allows for enables a straightforward treatment of Gapping which subsumes this construction—despite its appearance—under the law of coordination of likes. Furthermore, this analysis provides an immediate solution for a seemingly separate puzzle of apparently anomalous wide-scoping auxiliaries in Gapping, for which no explicit analysis exists except for Oehrle (1987) (which itself suffers from a different kind of problem).

The proposed calculus is unique among contemporary alternatives of CG-based syntactic frameworks in that it recognizes *both* directionality-sensitive modes of implication traditionally assumed in TLCG and the directionality-insensitive mode of implication from the more recent variants of CG such as λ-Grammar and ACG that deal with word order by enriching operations available in the (morpho-)phonological component (in particular, by having a full-fledged λ-calculus for it). This novel architecture of the present theory raises two related larger questions. First, one might wonder whether the relatively elaborate theoretical setup of the present system is justified. Second, the formal properties of the proposed system is as yet unexplored.

For the first, more empirical question, note that what enables subsuming Gapping under the case of like-categorial coordination in the present approach is its ability to analyze any *substring* containing a verb within a sentence as a constituent that can be abstracted over. This requires an interaction of the directional and non-directional slashes precisely of the kind that the present approach provides. In a system with only one mode of implication in the syntactic component (corresponding to our $|$), significant complications will arise, since in such an approach, verb phonologies in the lexicon are not simply strings but rather are n-place functions over strings (e.g., for transitive verbs, of the form $\lambda\pi_1\lambda\pi_2.\pi_2 \circ \mathsf{bought} \circ \pi_1$, of type $st \rightarrow st \rightarrow st$) that specify the relative positions of their arguments purely in the phonological representation. Abstracting over such a sign creates a higher-order phonological entity. To simulate the results of our analysis of Gapping in such a framework, one would then need to define a polymorphic entry for *and* which would yield the correct surface string for the right conjunct from such higher-order functions for each case in which a different type of functional phonology is abstracted over. But defining the appropriate entry for the conjunction word that would extend to cases involving auxiliaries—which have still more complex phonological types—is a non-trivial task, to put it mildly. It thus seems reasonable to conclude that, however one implements it, the kind of interaction between (tectogrammatical) syntax and surface linearization that the present system enables (via the interactions between $/,\backslash$ and $|$) needs to be part of the formal calculus for dealing with natural language syntax.

And this brings up the second question: if such a mixed system is empirically motivated, what are its exact formal underpinnings? Although previous proposals exist that propose calculi that recognize both directional and non-directional modes of implication within a single system (cf. de Groote (1996), Polakow and Pfenning (1999)), our system differs from these formal systems in that it allows for the two kinds of reasoning to freely feed into one another. In fact, this is precisely the source of the flexibility exploited in our analysis of Gapping, and, so far as we are aware, such a system is unprecedented and its mathematical properties are unknown. Given the linguistic motivation that we have demonstrated in this paper, the mathematical properties of the proposed system should be studied closely. We acknowledge this as an important issue to be investigated in future work.

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