Modal auxiliaries and negation: A type-logical account

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Abstract. This paper proposes an analysis of modal auxiliaries in English in Type-Logical Grammar. The proposed analysis captures the scopal interactions between different types of modal auxiliaries and negation by incorporating the key analytic idea of Iatridou and Zeijlstra [6], who classify English modal auxiliaries into PPI and NPI types. In order to technically implement this analysis, we build on Kubota and Levine's [8, 10] treatment of modal auxiliaries as higher-order operators that take scope at the clausal level. The proposed extension of the Kubota/Levine analysis is shown to have several interesting consequences, including a formal derivability relation from the higher-order entry for auxiliaries to a lower-order VP/VP entry traditionally recognized in categorial grammar (CG) research. The systematic analysis of the scopal properties of auxiliaries and the somewhat more abstract meta-comparison between 'transformational' and 'non-transformational' analytic ideas that become possible in a type-logical setup highlight the value of taking a logical perspective on the syntax of natural language embodied in Type-Logical Grammar research.

Keywords: modal auxiliary · negation · scope · Type-Logical Grammar.

1 Introduction

Modal auxiliaries in English exhibit a somewhat puzzling patterns in terms of their scopal interactions with negation. So far as we are aware, this particular empirical domain has not been explored in detail in the literature of Type-Logical Grammar (TLG). In this paper, we show that by extending the analysis of auxiliary verbs as semantically higher-order operators proposed by Kubota and Levine [8, 10], a relatively simple analysis of the modal-negation scopal interaction becomes available.

The proposed analysis builds on the classification of English modal auxiliaries into two different types based on the polarity distinction proposed by Iatridou and Zeijlstra [6], and can be thought of as a precise logical formalization of the core ideas behind the reconstruction-based analysis by Iatridou and Zeijlstra in minimalist syntax. We show that our logical reconceptualization of Iatridou and Zeijlstra's configurational analysis has several interesting consequences. In particular, our type-logical account illuminates the relationship between the configurational analysis standard in the mainstream syntax and the lexicalist alternative familiar in the G/HPSG and CG literature more clearly than previous proposals in the respective traditions of generative grammar. We formulate our analysis in Hybrid Type-Logical Grammar (Hybrid TLG) [8, 9], but the main results of the present paper are largely neutral to the particular variant of TLG, and can be translated to other variants of CG.[3]

2 Modals and negation: the empirical landscape

It has long been noted that the scopal relationship between modals and negation is essentially unpredictable, though there are certain semantic aspects of modal operators which appear to be relevant.

(1) a. John should not criticize Mary. ($\Box\,\neg\mathbf{criticize}(\mathbf{m})(\mathbf{j})$)

b. John need not criticize Mary. ($\neg\Box\,\mathbf{criticize}(\mathbf{m})(\mathbf{j})$)

c. John may not criticize Mary. ($\Diamond\,\neg\mathbf{criticize}(\mathbf{m})(\mathbf{j})$, $\neg\Diamond\,\mathbf{criticize}(\mathbf{m})(\mathbf{j})$)

It is generally agreed that these variations in scope behavior do not admit of any purely semantic solution following from the meanings of the modals: both *should* and *need* denote (different flavors of) universal quantification over the relevant possible worlds, but have opposite scoping vis-à-vis negation. Similarly, *may* and *might* are both arguably variants of existential quantification over possible worlds, but the former can scope either way so far as negation is concerned, whereas the latter is necessarily wide-scoping. The following table lists the relevant patterns for the major familiar modal auxiliaries:

(2)

modal	scopal pattern
will	$\mathsf{F} > \neg$
would	$\mathfrak{W} > \neg$
shall	$\mathsf{F} > \neg$
should	$\Box > \neg$
ought	$\Box > \neg$
might	$\Diamond > \neg$
must	$\Box > \neg$
may	$\Diamond < > \neg$
can	$\Diamond < > \neg$
could	$\Diamond < > \neg$
need	$\neg > \Box$

[3] As a reviewer notes, transportability of the analysis depends significantly what is common between the two frameworks. Since the Displacement Calculus [14] is largely similar to Hybrid TLG, translation of the present analysis to the Displacement Calculus should for the most part be straightforward (see Morrill and Valentín [15] in this connection). Loweing to VP/VP is of course not available in Linear Categorial Grammar [12] and Abstract Categorial Grammar [4], but lowering to (NP $\multimap$ S) $\multimap$ (NP $\multimap$ S) should be possible.

The patterns reflected in this table can be summarized as follows. The great majority of modals outscope negation that they are syntactically associated with. The striking exception is the deontic necessity modal *need*, which invariably outscopes negation, and the three 'possibility' modals *can*, *could* and *may*, which appear to be neutral.

Iatridou and Zeijlstra [6] argue that a principled account of the patterns in (2) can be given directly in terms of the sensitivity displayed by the individual modals to the scope of negation. *Need* is a known negative polarity item (NPI; see Levine [11] for discussion of the somewhat unusual behavior of this NPI), and hence when it appears with a local negator, such as *not* or *never*, it always scopes under negation. Iatridou and Zeijlstra propose that the invariably wide scope of *must*, *should*, *ought*, etc., with respect to local negation reflects their status as *positive* polarity items (PPIs). On their account, the different scopal relations between different types of modals and negation is a consequence of the 'reconstruction' possibilities of modals depending on their polarity statuses—NPI, PPI or neutral modals—as summarized in the following table:[4]

(3)

	PPI modals	Neutral modals	NPI modals
Universal	*must*, *should*, *ought to*, *be to*	*have to*, *need to*	*need*
Existential	—	*can*, *may*	—

On Iatridou and Zeijlstra's account, the auxiliaries are raised to the head of TP, and hence above negP. In the case of a sentence such as *John need not worry*, *need* cannot be licensed unless it is reconstructed back under negP, due to its NPI status. By contrast, PPI modals such as *must*, *should* and *ought* are prohibited from reconstruction, again due to their lexical property as PPIs. Neutral modals

[4] One might wonder about the classification of *must* and *should* as PPIs, given that they can appear unproblematically in the scope of negation in sentences such as *I don't think that John should be even one little bit nice to anyone in that room*, where the NPIs *even*, *anyone* and *one little bit* appear with no hint of ill-formedness. But here it is crucial to bear in mind that polarity items as a broad class are known to be sensitive to not only semantic scope effects but syntactic contexts as well; see Richter and Soehn [19] for a survey of syntactic conditions on a range of NPIs in German. Iatridou and Zeijlstra argue that the same syntactic sensitivity holds for PPIs, and note that

> PPIs. . . are fine in the scope of negation or any other context that is known to ban PPIs if this context is clause-external (Szabolcsi 2004:24–27), as illustrated in (i)–(iv):
>
> (i) I don't think that John called someone. not > $[_{CP/IP}$ some
> (ii) No one thinks/says that John called someone. no one > $[_{CP/IP}$ some
> (iii) I regret that John called someone. regret > $[_{CP/IP}$ some
> (iv) Every boy who called someone got help. every $[_{CP/IP}$ some

What seems to hold for the PPI modals, then, is that they cannot be in the scope of negation that originates in syntactically *local* operators.

such as *can* and *may* optionally reconstruct to their original sites, giving rise to scope ambiguity with negation.

3 Higher order negation: the formal analysis

In this section, we present our type-logical analysis of modal-negation scope interaction. After reviewing Kubota and Levine's [8, 10] analysis of modal auxiliaries in Hybrid Type-Logical Grammar (Hybrid TLG) in section 3.1, we present our extended fragment that takes into account the scopal interactions with negation in section 3.2. Sections 3.3–3.5 discuss some consequences of our proposal that help clarify the relationship between the higher-order operator analysis we propose and alternative approaches in the literature.

3.1 Higher-order modals

Kubota and Levine [8, 10] posit the following type of lexical entries for modal auxiliaries in English (where $\mathsf{id}_{et} = \lambda P_{et}.P$ and VP_f, VP_b are abbreviations of $\mathrm{NP}\backslash \mathrm{S}_f$, $\mathrm{NP}\backslash \mathrm{S}_b$):

(4) $\lambda\sigma.\sigma(\mathsf{can't});\ \lambda\mathscr{F}.\neg\Diamond\mathscr{F}(\mathsf{id}_{et});\ \mathrm{S}_f{\upharpoonright}(\mathrm{S}_f{\upharpoonright}(\mathrm{VP}_f/\mathrm{VP}_b))$

The following derivation illustrates how the $\Box > \exists$ reading for *Someone must be present (at the meeting)* is captured in this analysis (see Appendix A for a formal fragment of Hybrid TLG):

(5)

$$
\dfrac{
\begin{array}{l}\lambda\sigma.\sigma(\mathsf{must});\\ \lambda\mathscr{F}.\Box\mathscr{F}(\mathsf{id}_{et});\\ \mathrm{S}_f{\upharpoonright}(\mathrm{S}_f{\upharpoonright}(\mathrm{VP}_f/\mathrm{VP}_b))\end{array}
\quad
\dfrac{\dfrac{
\begin{array}{l}\lambda\sigma.\sigma(\mathsf{someone});\\ \mathbf{\exists}_{\mathbf{person}};\\ \mathrm{S}_f{\upharpoonright}(\mathrm{S}_f{\upharpoonright}\mathrm{NP})\end{array}
\quad
\dfrac{\dfrac{\begin{bmatrix}\varphi_2;\\ x;\\ \mathrm{NP}\end{bmatrix}^2 \quad \dfrac{\begin{bmatrix}\varphi_1;\\ f;\ \mathrm{VP}_f/\mathrm{VP}_b\end{bmatrix}^1 \quad \begin{array}{l}\mathsf{be}\circ\mathsf{present};\\ \mathbf{present};\ \mathrm{VP}_b\end{array}}{\varphi_1\circ\mathsf{be}\circ\mathsf{present};\ f(\mathbf{present});\ \mathrm{VP}_f}{/}\mathrm{E}}{\varphi_2\circ\varphi_1\circ\mathsf{be}\circ\mathsf{present};\ f(\mathbf{present})(x);\ \mathrm{S}_f}\backslash\mathrm{E}}{\begin{array}{c}\lambda\varphi_2.\varphi_2\circ\varphi_1\circ\mathsf{be}\circ\mathsf{present};\\ \lambda x.f(\mathbf{present})(x);\mathrm{S}_f{\upharpoonright}\mathrm{NP}\end{array}}{\upharpoonright}\mathrm{I}^2
}{\begin{array}{c}\mathsf{someone}\circ\varphi_1\circ\mathsf{be}\circ\mathsf{present};\\ \mathbf{\exists}_{\mathbf{person}}(\lambda x.f(\mathbf{present})(x));\mathrm{S}_f\end{array}}{\upharpoonright}\mathrm{E}}{\begin{array}{c}\lambda\varphi_1.\mathsf{someone}\circ\varphi_1\circ\mathsf{be}\circ\mathsf{present};\\ \lambda f.\mathbf{\exists}_{\mathbf{person}}(\lambda x.f(\mathbf{present})(x));\mathrm{S}_f{\upharpoonright}(\mathrm{VP}_f/\mathrm{VP}_b)\end{array}}{\upharpoonright}\mathrm{I}^1
}{\mathsf{someone}\circ\mathsf{must}\circ\mathsf{be}\circ\mathsf{present};\ \Box\mathbf{\exists}_{\mathbf{person}}(\lambda x.\mathbf{present}(x));\ \mathrm{S}_f}{\upharpoonright}\mathrm{E}
$$

Here, the hypothetical reasoning for the NP hypothesis (indexed 2) is for the subject quantifier *someone*, which enters into the derivation once the whole clause is built (semantically scoping over it and 'lowering' its prosody in the gap position corresponding to the λ-bound prosodic variable φ_2). The derivation involves another set of steps of hypothetical reasoning, with the $\mathrm{VP}_f/\mathrm{VP}_b$ hypothesis (indexed 1). This lets the modal semantically take scope above the subject quantifier (with prosodic lowering similar to the case of quantifiers). We thus obtain the result in which the modal auxiliary and the subject quantifier appear in their respective surface positions but in which the modal outscopes the quantifier.

The key idea behind this analysis is that auxiliaries are treated like generalized quantifiers (which are of type $\mathrm{S}{\upharpoonright}(\mathrm{S}{\upharpoonright}\mathrm{NP})$ in Hybrid TLG) except that they 'quantify over' VP/VP type expressions rather than NPs. The meaning contribution of the modal is the propositional modal operator, so, on this analysis (unlike the VP/VP analysis more familiar in the CG literature), the semantic scope and the 'syntactic position' at which the modal is introduced in the derivation correspond to each other straightforwardly. The features f and b abbreviate the 'VFORM' features (in G/HPSG terms) *fin* and *bse* that mark finite and base forms of verbs respectively. This ensures that modals can only combine with base forms of verbs and after the modal is combined with the verb, the result is finite, and no other modal can stack on top of the resultant VP.

The main empirical motivation for this 'quantificational' analysis of modal auxiliaries comes from the famous scope anomaly in Gapping sentences noted by Siegel [20] and Oehrle [16], as in examples such as (6).

(6) John can't eat steak and Mary just (eat) pizza!
$\neg\Diamond\mathbf{eat}(\mathbf{steak})(\mathbf{j}) \wedge \mathbf{eat}(\mathbf{pizza})(\mathbf{m})$

We do not repeat the argument here, but refer the reader to Kubota and Levine [8, 10] for a detailed discussion. The key point is that the ordinary VP/VP analysis has difficulty in accounting for the wide scope interpretation of modals in examples like (6) in any straightforward manner (relatedly, assigning the semantic translation $\lambda\mathscr{K}.\mathscr{K}(\lambda g\lambda x.\Box g(x))$, which would correspond to the semantic translation of a syntactically type-raised entry of the lower-order VP/VP entry, would fail to capture the wide-scope reading in (6)).

Puthawala [18] has recently shown that the same type of scope anomaly is observed in Stripping as well, and that the Kubota/Levine analysis can be straightforwardly extended to the Stripping cases in (7) as well:

(7) a. John won't apply for the job, or Mary either.
$\neg(\mathsf{F}\,\mathbf{apply\text{-}for}(\iota(\mathbf{job}))(\mathbf{j}) \vee \mathsf{F}\,\mathbf{apply\text{-}for}(\iota(\mathbf{job}))(\mathbf{m}))$
b. Mary can't testify for the defense and John also!
$\neg\Diamond(\mathbf{testify\text{-}for}(\mathbf{defense})(\mathbf{m}) \wedge \mathbf{testify\text{-}for}(\mathbf{defense})(\mathbf{j}))$

As noted by Kubota and Levine [8, 10], an interesting consequence of the higher-order analysis of modal auxiliaries in TLG outlined above is that the more familiar VP/VP sign for the modal auxiliary standardly assumed in the CG literature is immediately derivable via hypothetical reasoning from the higher-order one posited in the lexicon. The proof goes as follows:

(8)
$$\dfrac{\dfrac{\dfrac{\begin{array}{c}\lambda\sigma.\sigma(\mathsf{can't});\\ \lambda\mathscr{F}.\neg\Diamond\mathscr{F}(\mathsf{id}_{et});\\ \mathrm{S}_f{\upharpoonright}(\mathrm{S}_f{\upharpoonright}(\mathrm{VP}_f/\mathrm{VP}_b))\end{array} \quad \dfrac{\dfrac{[\varphi_1; x; \mathrm{NP}]^1 \quad \dfrac{[\varphi_2; g; \mathrm{VP}_f/\mathrm{VP}_b]^2 \quad [\varphi_3; f; \mathrm{VP}_b]^3}{\varphi_2 \circ \varphi_3;\ g(f);\ \mathrm{VP}_f}/\mathrm{E}}{\varphi_1 \circ \varphi_2 \circ \varphi_3;\ g(f)(x);\ \mathrm{S}_f}\backslash\mathrm{E}}{\lambda\varphi_2.\varphi_1 \circ \varphi_2 \circ \varphi_3;\ \lambda g.G(f)(x);\ \mathrm{S}_f{\upharpoonright}(\mathrm{VP}_f/\mathrm{VP}_b)}{\upharpoonright}\mathrm{I}^2}{\varphi_1 \circ \mathsf{can't} \circ \varphi_3;\ \neg\Diamond f(x);\ \mathrm{S}_f}{\upharpoonright}\mathrm{E}}{\mathsf{can't} \circ \varphi_3;\ \lambda x.\neg\Diamond f(x);\ \mathrm{VP}_f}\backslash\mathrm{I}^1}{\mathsf{can't};\ \lambda f\lambda x.\neg\Diamond f(x);\ \mathrm{VP}_f/\mathrm{VP}_b}/\mathrm{I}^3$$

This is essentially a case of lowering in the sense of Hendriks [5] in a system that extends the Lambek calculus with a discontinuous connective (in our case, $\upharpoonright$). Ignoring directionality, it corresponds to the elemetary theorem $(((\phi \to \psi) \to \varrho) \to \varrho) \to \zeta \vdash (\phi \to \psi) \to \zeta$ in standard propositional logic.

We call the family of theorems of which (8) is an instance 'slanting'. In slanting derivations, the vertical slash $\upharpoonright$ is eliminated from the lexical specification of a scopal operator 'slanting'. In addition to clarifing the relationship between the higher-order and more familiar type assignments for scopal operators (see section 3.3), slanting is useful in ensuring the correct scoping relations between multiple operators in certain cases, as discussed in Kubota and Levine [9] with respect to the analysis of quantifier-coordination interaction and as we show below in connection to modal auxiliary scope (sections 3.4 and 3.5).

3.2 Capturing the modal/negation scope interaction

In order to capture the polarity sensitivity of different types of modal auxiliaries in English, we posit a syntactic feature *pol* for category S that takes one of the three values $+$, $-$ and $\varnothing$.[5] The treatment of polarity here follows the general approach to polarity marking in the CG literature by Dowty [3], Bernardi [2] and Steedman [21], but differs from them in some specific details. Intuitively, S_{pol+} and S_{pol-} are positively and negatively marked clauses respectively, and $\mathrm{S}_{pol\varnothing}$ is a 'smaller' clause that isn't yet assigned polarity marking. To avoid cluttering the notation, we suppress the feature name *pol* in what follows and write S_{pol+}, S_{pol-} and $\mathrm{S}_{pol\varnothing}$ simply as S_{+}, S_{-} and $\mathrm{S}_{\varnothing}$, respectively. Positive-polarity modals are then lexically specified to obligatorily take scope at the level of S_{+}. Negative-polarity modals on the other hand are lexically specified to take scope at the level of $\mathrm{S}_{\varnothing}$, before negation turns an 'unmarked' clause to a negatively marked clause. We assume further that complete sentences in English are marked either *pol*$+$ or *pol*$-$; thus, $\mathrm{S}_{\varnothing}$ does not count as a stand-alone sentence.

The analysis of PPI and NPI modals outlined above can be technically implemented by positing the following lexical entries for the modals and the negation morpheme (where $\alpha, \beta \in \{\varnothing, -\}$ and $\gamma \in \{bse, fin\}$):

(9) a. $\lambda\sigma.\sigma(\mathsf{should})$; $\lambda\mathscr{G}.\Box\mathscr{G}(\mathsf{id}_{et})$; $\mathrm{S}_{f,+}\upharpoonright(\mathrm{S}_{f,\beta}\upharpoonright(\mathrm{VP}_{f,\alpha}/\mathrm{VP}_{b,\alpha}))$

b. $\lambda\sigma.\sigma(\mathsf{need})$; $\lambda\mathscr{G}.\Box\mathscr{G}(\mathsf{id}_{et})$; $\mathrm{S}_{f,\varnothing}\upharpoonright(\mathrm{S}_{f,\varnothing}\upharpoonright(\mathrm{VP}_{f,\varnothing}/\mathrm{VP}_{b,\varnothing}))$

c. $\lambda\sigma.\sigma(\mathsf{not})$; $\lambda\mathscr{G}.\neg\mathscr{G}(\mathsf{id}_{et})$; $\mathrm{S}_{\gamma,-}\upharpoonright(\mathrm{S}_{\gamma,\varnothing}\upharpoonright(\mathrm{VP}_{b,\varnothing}/\mathrm{VP}_{b,\varnothing}))$

We assume that different modals are assigned the following syntactic categories, depending on their polarity sensitivity:

[5] We remain agnostic about the exact formal implementation of syntactic features in the present paper. This could be done, for example, via some mechanism of unification as in HPSG. Another approach would involve the use of dependent types, along lines suggested by Morrill [13] and worked out in some detail by Pogodalla and Pompigne [17]. So far as we can tell, the results of the current paper does not hinge on the specific choice on this matter.

(10)

PPI	**NPI**
$\mathbf{S}_{f,+}\restriction(\mathbf{S}_{f,\beta}\restriction(\mathbf{VP}_{f,\alpha}/\mathbf{VP}_{b,\alpha}))$	$\mathbf{S}_{f,\varnothing}\restriction(\mathbf{S}_{f,\varnothing}\restriction(\mathbf{VP}_{f,\varnothing}/\mathbf{VP}_{b,\varnothing}))$
should	*need*
must	*dare*
ought	
might	
can	*can*
could	*could*
may	*may*
will	*will*
would	*would*

We now illustrate the working of this fragment with the analyses for (11a) (which involves a PPI modal) and (11b) (which involves an NPI modal).

(11) a. John should not come.

b. John need not come.

The derivation for (11a) goes as follows:

(12) a.

$$\dfrac{\begin{array}{l}\mathsf{john};\\ \mathbf{j};\mathrm{NP}\end{array} \quad \dfrac{\left[\begin{array}{l}\varphi_4;\\ h;\mathrm{VP}_{f,\varnothing}/\mathrm{VP}_{b,\varnothing}\end{array}\right]^4 \quad \dfrac{[\varphi_1; f; \mathrm{VP}_{b,\varnothing}/\mathrm{VP}_{b,\varnothing}]^1 \quad \mathsf{come}; \mathbf{come}; \mathrm{VP}_{b,\varnothing}}{\varphi_1 \circ \mathsf{come}; f(\mathbf{come}); \mathrm{VP}_{b,\varnothing}}/\mathrm{E}}{\varphi_4\circ\varphi_1\circ\mathsf{come}; h(f(\mathbf{come})); \mathrm{VP}_{f,\varnothing}}/\mathrm{E}}{\mathsf{john}\circ\varphi_4\circ\varphi_1\circ\mathsf{come}; h(f(\mathbf{come}))(\mathbf{j}); \mathrm{S}_{f,\varnothing}}/\mathrm{E}$$

b.

$$\dfrac{\begin{array}{l}\lambda\sigma.\sigma(\mathsf{should});\\ \lambda\mathscr{G}.\Box\mathscr{G}(\mathsf{id}_{et});\\ \mathrm{S}_{f,+}\restriction(\mathrm{S}_{f,\beta}\restriction(\mathrm{VP}_{f,\alpha}/\mathrm{VP}_{b,\alpha}))\end{array} \quad \dfrac{\dfrac{\begin{array}{l}\lambda\sigma.\sigma(\mathsf{not});\\ \lambda\mathscr{G}.\neg\mathscr{G}(\mathsf{id}_{et});\\ \mathrm{S}_{\gamma,-}\restriction(\mathrm{S}_{\gamma,\varnothing}\restriction(\mathrm{VP}_{b,\varnothing}/\mathrm{VP}_{b,\varnothing}))\end{array} \quad \dfrac{\begin{array}{c}(12\mathrm{a})\\ \mathsf{john}\circ\varphi_4\circ\varphi_1\circ\mathsf{come};\\ h(f(\mathbf{come}))(\mathbf{j});\mathrm{S}_{f,\varnothing}\end{array}}{\begin{array}{l}\lambda\varphi_1.\mathsf{john}\circ\varphi_4\circ\varphi_1\circ\mathsf{come};\\ \lambda f.h(f(\mathbf{come}))(\mathbf{j});\\ \mathrm{S}_{f,\varnothing}\restriction(\mathrm{VP}_{b,\varnothing}/\mathrm{VP}_{b,\varnothing})\end{array}}\restriction\mathrm{I}^1}{\mathsf{john}\circ\varphi_4\circ\mathsf{not}\circ\mathsf{come}; \neg h(\mathbf{come})(\mathbf{j}); \mathrm{S}_{f,-}}\restriction\mathrm{E}}{\begin{array}{c}\lambda\varphi_4.\mathsf{john}\circ\varphi_4\circ\mathsf{not}\circ\mathsf{come};\\ \lambda h.\neg h(\mathbf{come})(\mathbf{j});\mathrm{S}_{f,-}\restriction(\mathrm{VP}_{f,\varnothing}/\mathrm{VP}_{b,\varnothing})\end{array}}\restriction\mathrm{I}^4}{\mathsf{john}\circ\mathsf{should}\circ\mathsf{not}\circ\mathsf{come}; \Box\neg\mathbf{come}(\mathbf{j}); \mathrm{S}_{f,+}}\restriction\mathrm{E}$$

The key point here is that although both *should* and *not* are lexically specified to take scope at the clausal level, their scopal relation is fixed. Specifically, once *should* takes scope, the resultant clause is S_+, which is incompatible with the specification on the argument category for *not*. This means that negation is forced to take scope before the PPI modal does.

Exactly the opposite relation holds between the NPI modal *need* and negation. Here, after negation takes scope, we have S_-, but this specification is incompatible with the argument category for the NPI modal, which requires the clause it scopes over to be $\mathrm{S}_\varnothing$. Thus, as in (13), the only possibility is to have *need* take scope before the negation does, which gives us the $\neg > \Box$ Scopal relation.[6]

[6] Extending the present analysis to cases involving negative quantifiers (e.g. *Nothing need be said about this*) is a task that we leave for future work.

(13) a.

$$\dfrac{\begin{array}{l}\mathsf{john};\\ \mathbf{j}; \mathrm{NP}\end{array} \quad \dfrac{\begin{bmatrix}\varphi_4;\\ h; \mathrm{VP}_{f,\varnothing}/\mathrm{VP}_{b,\varnothing}\end{bmatrix}^4 \quad \dfrac{[\varphi_1; f; \mathrm{VP}_{b,\varnothing}/\mathrm{VP}_{b,\varnothing}]^1 \quad \mathsf{come}; \mathbf{come}; \mathrm{VP}_{b,\varnothing}}{\varphi_1 \circ \mathsf{come}; f(\mathbf{come}); \mathrm{VP}_{b,\varnothing}}/\mathrm{E}}{\varphi_4 \circ \varphi_1 \circ \mathsf{come}; h(f(\mathbf{come})); \mathrm{VP}_{f,\varnothing}}/\mathrm{E}}{\mathsf{john} \circ \varphi_4 \circ \varphi_1 \circ \mathsf{come}; h(f(\mathbf{come}))(\mathbf{j}); \mathrm{S}_{f,\varnothing}}/\mathrm{E}$$

b.

$$\dfrac{\begin{array}{l}\lambda\sigma.\sigma(\mathsf{not});\\ \lambda\mathscr{G}.\neg\mathscr{G}(\mathsf{id}_{et});\\ \mathrm{S}_{\gamma,-}{\upharpoonright}(\mathrm{S}_{\gamma,\varnothing}{\upharpoonright}(\mathrm{VP}_{b,\varnothing}/\mathrm{VP}_{b,\varnothing}))\end{array} \quad \dfrac{\dfrac{\begin{array}{l}\lambda\sigma.\sigma(\mathsf{need});\\ \lambda\mathscr{G}.\Box\mathscr{G}(\mathsf{id}_{et});\\ \mathrm{S}_{f,\varnothing}{\upharpoonright}(\mathrm{S}_{f,\varnothing}{\upharpoonright}(\mathrm{VP}_{f,\varnothing}/\mathrm{VP}_{b,\varnothing}))\end{array} \quad \dfrac{\begin{array}{c}(13a)\\ \mathsf{john} \circ \varphi_4 \circ \varphi_1 \circ \mathsf{come};\\ h(f(\mathbf{come}))(\mathbf{j}); \mathrm{S}_{f,\varnothing}\end{array}}{\begin{array}{l}\lambda\varphi_4.\mathsf{john} \circ \varphi_4 \circ \varphi_1 \circ \mathsf{come};\\ \lambda h.h(f(\mathbf{come}))(\mathbf{j});\\ \mathrm{S}_{f,\varnothing}{\upharpoonright}(\mathrm{VP}_{f,\varnothing}/\mathrm{VP}_{b,\varnothing})\end{array}}{\upharpoonright}\mathrm{I}^4}{\mathsf{john} \circ \mathsf{need} \circ \varphi_1 \circ \mathsf{come}; \Box f(\mathbf{come})(\mathbf{j}); \mathrm{S}_{f,\varnothing}}{\upharpoonright}\mathrm{E}}{\begin{array}{l}\lambda\varphi_1.\mathsf{john} \circ \mathsf{need} \circ \varphi_1 \circ \mathsf{come};\\ \lambda f.\Box f(\mathbf{come})(\mathbf{j}); \mathrm{S}_{f,\varnothing}{\upharpoonright}(\mathrm{VP}_{b,\varnothing}/\mathrm{VP}_{b,\varnothing})\end{array}}{\upharpoonright}\mathrm{I}^1}{\mathsf{john} \circ \mathsf{need} \circ \mathsf{not} \circ \mathsf{come}; \neg\Box\mathbf{come}(\mathbf{j}); \mathrm{S}_{f,-}}{\upharpoonright}\mathrm{E}$$

We assume that modals that give rise to scope ambiguity with negation are simply ambiguous between PPI and NPI variants, as in (10). This accounts for the scope ambiguity of examples such as (1c).[7]

3.3 Slanting and the VP/VP analysis of auxiliaries

The analysis of modal scope presented above can, in a sense, be thought of as a logical reconceptualization of the configurational account proposed by Iatridou and Zeijlstra. Instead of relying on reconstruction and movement, our analysis simply regulates the relative scope relations between the auxiliary and negation via the three-way distinction of the polarity-marking feature *pol*, but aside

[7] Though we have chosen to posit two distinction lexical entries for the 'neutral' modals (*can*, *could* and *may*) for high and low scoping possibilities with respect to negation, corresponding respectively to the scoping properties of the unambiguous modals, it is easy to collapse these two entries for these modals by making the polarity features for the two S's and two VPs in the complex higher-order category for the modal totally underspecified and unconstrained (except for one constraint $\langle \alpha, \beta \rangle \neq \langle \varnothing, - \rangle$, to exclude the possibility of double negation marking **can not not*), along the following lines:

(i) $\lambda\sigma.\sigma(\mathsf{can}); \lambda\mathscr{G}.\Diamond\mathscr{G}(\mathsf{id}_{et}); \mathrm{S}_{f,\alpha}{\upharpoonright}(\mathrm{S}_{f,\beta}{\upharpoonright}(\mathrm{VP}_{f,\delta}/\mathrm{VP}_{b,\zeta}))$

By (partially) resolving underspecification, we can derive both the 'PPI' and 'NPI' variants of the modal lexical entry in (10) from (i), thus capturing scope ambiguity via a single lexical entry. (i) allows for other instantiations of feature specification, but these are either redundant (yielding either high or low scope that are already derivable with the PPI and NPI instantiations in (10)), or useless (i.e. cannot be used in any well-formed syntactic derivation), and hence harmless. Thus, if desired, the lexical ambiguity we have tentatively assumed in the main text can be eliminated by adopting the more general lexical entry along the lines of (i) without the danger of overgeneration.

from this technical difference, the essential analytic idea is the same: the semantic scope of the modal and negation operators transparently reflects the form of the abstract combinatoric structure that is not directly visible from surface constituency, be it a level of syntactic representation (i.e. LF, as in Iatridou and Zeijlstra's account), or the structure of the proof that yields the pairing of surface string semantic translation (as in our approach, and more generally, in CG-based theories of natural language syntax/semantics).

One might then wonder whether the two analyses are mere notational variants or if there is any advantage gained by recasting the LF-based analysis in a type-logical setup. We do think that our approach has the advantage of being fully explicit, without relying on the notions of reconstruction and movement whose exact details remain somewhat elusive. However, rather than dwelling on this point, we would like to point out an interesting consequence that immediately follows from our account and which illuminates the relationship between the 'transformational' analysis of auxiliaries (of the sort embodied in our analysis of modal auxiliaries as 'VP-modifer quantifiers') and the lexicalist alternatives in the tradition of non-transformational syntax (such as G/HPSG and CG).

To see the relevant point, note first that PPI modals such as *should* can be derived in the lower-order category $\mathrm{VP}_{f,+}/\mathrm{VP}_{b,\delta}$ as follows (here, $\alpha, \beta, \delta \in \{\varnothing, -\}$):

(14)

$$\dfrac{\dfrac{\begin{array}{l}\lambda\sigma.\sigma(\mathsf{should});\\ \lambda\mathscr{G}.\Box\mathscr{G}(\mathsf{id}_{et});\\ \mathrm{S}_{f,+}\restriction(\mathrm{S}_{f,\beta}\restriction(\mathrm{VP}_{f,\alpha}/\mathrm{VP}_{b,\alpha}))\end{array} \quad \dfrac{\dfrac{\begin{bmatrix}\varphi_3;\\ x;\mathrm{NP}\end{bmatrix}^3 \quad \dfrac{\begin{bmatrix}\varphi_1;\\ f;\mathrm{VP}_{f,\delta}/\mathrm{VP}_{b,\delta}\end{bmatrix}^1 \quad \begin{bmatrix}\varphi_2;\\ g;\mathrm{VP}_{b,\delta}\end{bmatrix}^2}{\varphi_1\circ\varphi_2;\ f(g);\ \mathrm{VP}_{f,\delta}}/\mathrm{E}}{\varphi_3\circ\varphi_1\circ\varphi_2;\ f(g)(x);\ \mathrm{S}_{f,\delta}}\backslash\mathrm{E}}{\lambda\varphi_1.\varphi_3\circ\varphi_1\circ\varphi_2;\ \lambda f.f(g)(x);\ \mathrm{S}_{f,\delta}\restriction(\mathrm{VP}_{f,\delta}/\mathrm{VP}_{b,\delta})}\restriction\mathrm{I}^1}{\dfrac{\varphi_3\circ\mathsf{should}\circ\varphi_2;\ \Box g(x);\ \mathrm{S}_{f,+}}{\dfrac{\mathsf{should}\circ\varphi_2;\ \lambda x.\Box g(x);\ \mathrm{VP}_{f,+}}{\mathsf{should};\ \lambda g\lambda x.\Box g(x);\ \mathrm{VP}_{f,+}/\mathrm{VP}_{b,\delta}}/\mathrm{I}^2}\backslash\mathrm{I}^3}\restriction\mathrm{E}$$

Similarly, the negation morpheme *not* can be slanted to the $\mathrm{VP}_{b,-}/\mathrm{VP}_{b,\varnothing}$ category:

(15)

$$\dfrac{\dfrac{\begin{array}{l}\lambda\sigma.\sigma(\mathsf{not});\\ \lambda\mathscr{G}.\neg\mathscr{G}(\mathsf{id}_{et});\\ \mathrm{S}_{\gamma,-}\restriction(\mathrm{S}_{\gamma,\varnothing}\restriction(\mathrm{VP}_{b,\varnothing}/\mathrm{VP}_{b,\varnothing}))\end{array} \quad \dfrac{\dfrac{\begin{bmatrix}\varphi_3;\\ x;\mathrm{NP}\end{bmatrix}^3 \quad \dfrac{\begin{bmatrix}\varphi_1;\\ f;\mathrm{VP}_{b,\varnothing}/\mathrm{VP}_{b,\varnothing}\end{bmatrix}^1 \quad \begin{bmatrix}\varphi_2;\\ g;\mathrm{VP}_{b,\varnothing}\end{bmatrix}^2}{\varphi_1\circ\varphi_2;\ f(g);\ \mathrm{VP}_{b,\varnothing}}/\mathrm{E}}{\varphi_3\circ\varphi_1\circ\varphi_2;\ f(g)(x);\ \mathrm{S}_{b,\varnothing}}\backslash\mathrm{E}}{\lambda\varphi_1.\varphi_3\circ\varphi_1\circ\varphi_2;\ \lambda f.f(g)(x);\ \mathrm{S}_{b,\varnothing}\restriction(\mathrm{VP}_{b,\varnothing}/\mathrm{VP}_{b,\varnothing})}\restriction\mathrm{I}^1}{\dfrac{\varphi_3\circ\mathsf{not}\circ\varphi_2;\ \neg g(x);\ \mathrm{S}_{b,-}}{\dfrac{\mathsf{not}\circ\varphi_2;\ \lambda x.\neg g(x);\ \mathrm{VP}_{b,-}}{\mathsf{not};\ \lambda g\lambda x.\neg g(x);\ \mathrm{VP}_{b,-}/\mathrm{VP}_{b,\varnothing}}/\mathrm{I}^2}\backslash\mathrm{I}^3}\restriction\mathrm{E}$$

These two lowered categories can be combined to produce the following sign:

(16)

$$\dfrac{\dfrac{\textsf{should};\ \lambda g\lambda x.\Box g(x);\ \mathrm{VP}_{f,+}/\mathrm{VP}_{b,\delta} \qquad \dfrac{\textsf{not};\ \lambda g\lambda x.\neg g(x);\ \mathrm{VP}_{b,-}/\mathrm{VP}_{b,\varnothing} \quad [\varphi_1; g; \mathrm{VP}_{b,\varnothing}]^1}{\textsf{not}\circ\varphi_1;\ \lambda x.\neg g(x);\ \mathrm{VP}_{b,-}}\,/\mathrm{E}}{\textsf{should}\circ\textsf{not}\circ\varphi_1;\ \lambda x.\Box\neg g(x);\ \mathrm{VP}_{f,+}}\,/\mathrm{E}}{\textsf{should}\circ\textsf{not};\ \lambda g\lambda x.\Box\neg g(x);\ \mathrm{VP}_{f,+}/\mathrm{VP}_{b,\varnothing}}\,/\mathrm{I}^1$$

Slanting the NPI modal *need*, on the other hand, yields the following result:

(17)

$$\dfrac{\dfrac{\dfrac{\begin{array}{l}\lambda\sigma.\sigma(\textsf{need});\\ \lambda\mathscr{G}.\Box\mathscr{G}(\textsf{id}_{et});\\ \mathrm{S}_{f,\varnothing}\upharpoonright(\mathrm{S}_{f,\varnothing}\upharpoonright(\mathrm{VP}_{f,\varnothing}/\mathrm{VP}_{b,\varnothing}))\end{array} \quad \dfrac{\dfrac{\begin{bmatrix}\varphi_3;\\ x; \mathrm{NP}\end{bmatrix}^3 \quad \dfrac{\begin{bmatrix}\varphi_1;\\ f; \mathrm{VP}_{f,\varnothing}/\mathrm{VP}_{b,\varnothing}\end{bmatrix}^1 \quad \begin{bmatrix}\varphi_2;\\ g; \mathrm{VP}_{b,\varnothing}\end{bmatrix}^2}{\varphi_1\circ\varphi_2;\ f(g);\ \mathrm{VP}_{f,\varnothing}}\,/\mathrm{E}}{\varphi_3\circ\varphi_1\circ\varphi_2;\ f(g)(x);\ \mathrm{S}_{f,\varnothing}}\,\backslash\mathrm{E}}{\lambda\varphi_1.\varphi_3\circ\varphi_1\circ\varphi_2;\ \lambda f.f(g)(x);\ \mathrm{S}_{f,\varnothing}\upharpoonright(\mathrm{VP}_{f,\varnothing}/\mathrm{VP}_{b,\varnothing})}\,\upharpoonright\mathrm{I}^1}{\varphi_3\circ\textsf{need}\circ\varphi_2;\ \Box g(x);\ \mathrm{S}_{f,\varnothing}}\,\upharpoonright\mathrm{E}}{\textsf{need}\circ\varphi_2;\ \lambda x.\Box g(x);\ \mathrm{VP}_{f,\varnothing}}\,\backslash\mathrm{I}^3}{\textsf{need};\ \lambda g\lambda x.\Box g(x);\ \mathrm{VP}_{f,\varnothing}/\mathrm{VP}_{b,\varnothing}}\,/\mathrm{I}^2$$

Note that this resultant category cannot be combined with the lowered negation category in (15) due to feature mismatch (*need* requires its argument to be $\mathrm{VP}_{b,\varnothing}$, but *not* marks the VP as $\mathrm{VP}_{b,-}$). Thus, the lowered *need* is correctly prevented from outscoping negation.

It is however possible to derive *need not* as a complex auxiliary with the correct negation-outscoping semantics:

(18) a.

$$\dfrac{\begin{bmatrix}\varphi_3;\\ x; \mathrm{NP}\end{bmatrix}^3 \quad \dfrac{\begin{bmatrix}\varphi_4;\\ h; \mathrm{VP}_{f,\varnothing}/\mathrm{VP}_{b,\varnothing}\end{bmatrix}^4 \quad \dfrac{\begin{bmatrix}\varphi_1;\\ f; \mathrm{VP}_{b,\varnothing}/\mathrm{VP}_{b,\varnothing}\end{bmatrix}^1 \quad \begin{bmatrix}\varphi_2;\\ g; \mathrm{VP}_{b,\varnothing}\end{bmatrix}^2}{\varphi_1\circ\varphi_2;\ f(g);\ \mathrm{VP}_{b,\varnothing}}\,/\mathrm{E}}{\varphi_4\circ\varphi_1\circ\varphi_2;\ h(f(g));\ \mathrm{VP}_{f,\varnothing}}\,/\mathrm{E}}{\varphi_3\circ\varphi_4\circ\varphi_1\circ\varphi_2;\ h(f(g))(x);\ \mathrm{S}_{f,\varnothing}}\,\backslash\mathrm{E}$$

b.

$$\dfrac{\dfrac{\dfrac{\begin{array}{l}\lambda\sigma.\sigma(\textsf{not});\\ \lambda\mathscr{G}.\neg\mathscr{G}(\textsf{id}_{et});\\ \mathrm{S}_{\gamma,-}\upharpoonright(\mathrm{S}_{\gamma,\varnothing}\upharpoonright(\mathrm{VP}_{b,\varnothing}/\mathrm{VP}_{b,\varnothing}))\end{array} \quad \dfrac{\dfrac{\begin{array}{l}\lambda\sigma.\sigma(\textsf{need});\\ \lambda\mathscr{G}.\Box\mathscr{G}(\textsf{id}_{et});\\ \mathrm{S}_{f,\varnothing}\upharpoonright(\mathrm{S}_{f,\varnothing}\upharpoonright(\mathrm{VP}_{f,\varnothing}/\mathrm{VP}_{b,\varnothing}))\end{array} \quad \dfrac{\begin{array}{c}\text{(18a)}\\ \varphi_3\circ\varphi_4\circ\varphi_1\circ\varphi_2;\\ h(f(g))(x); \mathrm{S}_{f,\varnothing}\end{array}}{\begin{array}{l}\lambda\varphi_4.\varphi_3\circ\varphi_4\circ\varphi_1\circ\varphi_2;\\ \lambda h.h(f(g))(x);\\ \mathrm{S}_{f,\varnothing}\upharpoonright(\mathrm{VP}_{f,\varnothing}/\mathrm{VP}_{b,\varnothing})\end{array}}\,\upharpoonright\mathrm{I}^4}{\varphi_3\circ\textsf{need}\circ\varphi_1\circ\varphi_2;\ \Box f(g)(x);\ \mathrm{S}_{f,\varnothing}}\,\upharpoonright\mathrm{E}}{\begin{array}{l}\lambda\varphi_1.\varphi_3\circ\textsf{need}\circ\varphi_1\circ\varphi_2;\\ \lambda f.\Box f(g)(x); \mathrm{S}_{f,\varnothing}\upharpoonright(\mathrm{VP}_{b,\varnothing}/\mathrm{VP}_{b,\varnothing})\end{array}}\,\upharpoonright\mathrm{I}^1}{\varphi_3\circ\textsf{need}\circ\textsf{not}\circ\varphi_2;\ \neg\Box g(x);\ \mathrm{S}_{f,-}}\,\upharpoonright\mathrm{E}}{\textsf{need}\circ\textsf{not}\circ\varphi_2;\ \lambda x.\neg\Box g(x);\ \mathrm{VP}_{f,-}}\,\backslash\mathrm{I}^3}{\textsf{need}\circ\textsf{not};\ \lambda g\lambda x.\neg\Box g(x);\ \mathrm{VP}_{f,-}/\mathrm{VP}_{b,\varnothing}}\,/\mathrm{I}^2$$

Note also that we can derive string-level signs for modals that mimic the higher order version in their ability to outscope generalized quantifiers:

(19)

$$
\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{[\varphi_3; \mathscr{P}; \mathrm{S}_{f,\alpha}/\mathrm{VP}_{f,\alpha}]^3 \quad \dfrac{[\varphi_2; f; \mathrm{VP}_{f,\alpha}/\mathrm{VP}_{b,\alpha}]^2 \quad [\varphi_1; P; \mathrm{VP}_{b,\alpha}]^1}{\varphi_2 \circ \varphi_1;\ f(P);\ \mathrm{VP}_{f,\alpha}}}{\varphi_3 \circ \varphi_2 \circ \varphi_1;\ \mathscr{P}(f(P));\ \mathrm{S}_{f,\alpha}}}{\lambda\varphi_2.\varphi_3 \circ \varphi_2 \circ \varphi_1;\ \lambda f.\mathscr{P}(P);\ \mathrm{S}_{f,\alpha}{\upharpoonright}(\mathrm{VP}_{f,\alpha}/\mathrm{VP}_{b,\alpha})}{\upharpoonright}\mathrm{I}^2 \quad \begin{array}{l}\lambda\sigma.\sigma(\mathsf{can});\\ \lambda\mathscr{F}.\Diamond\mathscr{F}(\mathsf{id}_{et});\\ \mathrm{S}_{f,+}{\upharpoonright}(\mathrm{S}_{f,\beta}{\upharpoonright}(\mathrm{VP}_{f,\alpha}/\mathrm{VP}_{b,\alpha}))\end{array}}{\varphi_3 \circ \mathsf{can} \circ \varphi_1;\ \Diamond\mathscr{P}(P);\ \mathrm{S}_{f,+}}}{\mathsf{can} \circ \varphi_1;\ \lambda\mathscr{P}.\Diamond\mathscr{P}(P);\ (\mathrm{S}_{f,\alpha}/\mathrm{VP}_{f,\alpha})\backslash\mathrm{S}_{f,+}}{\upharpoonright}\mathrm{I}^3}{\mathsf{can};\ \lambda P\lambda\mathscr{P}.\Diamond\mathscr{P}(P);\ ((\mathrm{S}_{f,\alpha}/\mathrm{VP}_{f,\alpha})\backslash\mathrm{S}_{f,+})/\mathrm{VP}_{b,\alpha}}{\upharpoonright}\mathrm{I}^1
$$

In short, in our type-logical setup, alternative lexical signs that correspond to the lexical entries for the relevant expressions that are directly specified in the lexicon in lexicalist theories of syntax are all derivable as theorems from the more abstract, higher-order entries we have posited above. This is essentially the consequence of the slanting lemma (whose basic form is shown in (8) in Appendix B) in the revised system augmented with the polarity markings. Significantly, the polarity markings ensure that slanting of the higher-order modals and negation preserves the correct scope relations between these operators.

The formal derivability of the lower-order entry from the higher-order entry is an interesting and useful result, as it potentially illuminates the deeper relationship between the 'transformational' and 'lexicalist' analyses of auxiliaries in the different traditions of the generative grammar literature. The two approaches have tended to be seen as reflecting fundamentally incompatible assumptions about the basic architecture of grammar, but if a formal connection can be established between the two at an abstract level by making certain (not totally implausible) assumptions, then the two may not be as different from each other as they have appeared to be throughout the whole history of the controversy between the transformational and non-transformational approaches to syntax. In any event, we take our result above to indicate that the logic-based setup of Type-Logical Grammar can be fruitfully employed for the purpose of meta-comparison of different approaches to grammatical phenomena in the syntactic literature.

3.4 Slanting and coordination

The slanting lemma moreover plays a crucial role in deriving the correct scope relations in certain examples involving coordination of higher-order operators. For example, consider the conjunction of modals in (20).

(20) Every physicist can and should learn how to teach quantum mechanics to the undergratuate literature majors.

There is a reading for this sentence in which the two modals outscope the subject universal quantifier in each conjunct ('it is possible that every physicist learns . . . and it is deontically necessary that every physicist learns. . . ').

Assuming that *and* is of type $(\mathrm{X}\backslash\mathrm{X})/\mathrm{X}$, combining only expressions whose prosodies are strings, it may appear impossible to derive (20) on the relevant reading, since the modals in (20) must be higher-order to outscope the subject quantifier, and therefore must have functional prosodies. In fact, however, a straightfoward derivation is available with no additional asssumptions or machinery. Note first that the modal auxiliary can be derived in the ((S/VP)\S)/VP Type (see the discussion in section 3.3; the complete derivation is given in (19) in Appendix B):

(21) $\textsf{can}; \lambda P \lambda \mathscr{P}.\Diamond\, \mathscr{P}(P); ((\mathrm{S}_{f,\alpha}/\mathrm{VP}_{f,\alpha})\backslash \mathrm{S}_{f,+})/\mathrm{VP}_{b,\alpha}$

By conjoining two such modals via generalized conjunction, we obtain:

(22) $\textsf{can} \circ \textsf{and} \circ \textsf{should}; \lambda R \lambda \mathscr{R}.\Diamond \mathscr{R}(R) \wedge \Box \mathscr{R}(R); ((\mathrm{S}_{f,\alpha}/\mathrm{VP}_{f,\alpha})\backslash \mathrm{S}_{f,+})/\mathrm{VP}_{b,\alpha}$

We apply this functor first to the sign with VP type derived for *learn how to teach quantum mechanics to the undergratuate literature majors*, and finally to the slanted version of the quantified subject *every physicist*, derivable as in (23):

(23)
$$\dfrac{\dfrac{\dfrac{\dfrac{[\varphi_1; y; \mathrm{NP}]^1 \quad [\varphi_2; P; \mathrm{VP}_{f,\alpha}]^2}{\varphi_1 \circ \varphi_2;\ P(y);\ \mathrm{S}_{f,\alpha}}}{\lambda \varphi_1.\varphi_1 \circ \varphi_2;\ \lambda y.P(y);\ \mathrm{S}_{f,\alpha}{\upharpoonright}\mathrm{NP}}{\upharpoonright}\mathrm{I}^1 \quad \begin{array}{l}\lambda \sigma_1.\sigma_1(\textsf{every} \circ \textsf{physicist});\\ \mathbf{V}_{\mathbf{phys}}; \mathrm{S}_{f,\alpha}{\upharpoonright}(\mathrm{S}_{f,\alpha}{\upharpoonright}\mathrm{NP})\end{array}}{\textsf{every} \circ \textsf{physicist} \circ \varphi_2;\ \mathbf{V}_{\mathbf{phys}}(\lambda y.P(y));\ \mathrm{S}_{f,\alpha}}}{\textsf{every} \circ \textsf{physicist};\ \lambda P.\mathbf{V}_{\mathbf{phys}}(\lambda y.P(y));\ \mathrm{S}_{f,\alpha}/\mathrm{VP}_{f,\alpha}}{\upharpoonright}\mathrm{I}^2$$

This yields the following result, with the correct semantic translation for (20):

(24)
$$\dfrac{\begin{array}{l}(23)\\ \textsf{every} \circ \textsf{physicist};\\ \mathbf{V}_{\mathbf{phys}}; \mathrm{S}_{f,\alpha}/\mathrm{VP}_{f,\alpha}\end{array} \quad \dfrac{\begin{array}{l}(22)\\ \textsf{can} \circ \textsf{and} \circ \textsf{should};\\ \lambda R \lambda \mathscr{R}.\Diamond \mathscr{R}(R) \wedge \Box \mathscr{R}(R);\\ ((\mathrm{S}_{f,\alpha}/\mathrm{VP}_{f,\alpha})\backslash \mathrm{S}_{f,+})/\mathrm{VP}_{b,\alpha}\end{array} \quad \begin{array}{c}\vdots\\ \textsf{learn} \ldots; \mathbf{LHT}; \mathrm{VP}_{b,\alpha}\end{array}}{\begin{array}{l}\textsf{can} \circ \textsf{and} \circ \textsf{should} \circ \textsf{learn} \circ \ldots;\\ \lambda \mathscr{R}.\Diamond \mathscr{R}(\mathbf{LHT}) \wedge \Box \mathscr{R}(\mathbf{LHT}); (\mathrm{S}_{f,\alpha}/\mathrm{VP}_{f,\alpha})\backslash \mathrm{S}_{f,+}\end{array}}}{\begin{array}{l}\textsf{every} \circ \textsf{physicist} \circ \textsf{can} \circ \textsf{and} \circ \textsf{should} \circ \textsf{learn} \circ \ldots;\\ \Diamond \mathbf{V}_{\mathbf{phys}}(\mathbf{LHT}) \wedge \Box \mathbf{V}_{\mathbf{phys}}(\mathbf{LHT}); \mathrm{S}_{f,+}\end{array}}$$

3.5 VP fronting

Work in phrase-structure-theoretic approaches to the syntax/semantics interface has tended to follow the treatment of negation in Kim and Sag [7], which distinguishes *not* (and possibly *never*) as complements of auxiliaries from *not* as adjuncts to the auxiliaries' VP complements. This approach is supposedly motivated by the ambiguity of sentences with *could not/never* sequences, where both $\neg > \Diamond$ and $\Diamond > \neg$ readings are available.

There is, in fact, a very sparse empirical base in English for this phrase structure-based analysis of modal/negation scoping relations, a fact that Kim and Sag [7] themselves tacitly acknowledge. One of the few lines of argument that Kim and Sag [7] appeal to is the fact that fronted VPs containing *not* adjuncts are always interpreted with narrowly scoping negation, as illustrated in (25):

(25) ... and NOT vote, you certainly can __, if the nominees are all second-rate.

Data of this sort are intended to provide empirical support for the putative correlation of phrase structural position with the scope of negation, and the particular empirical fact about fronted VP with negation exemplified by (25) needs to be accounted for in any approach to modal/negation interaction in any theoretical framework. But there seems no strong reason to prefer the phrase structural account to any of a number of alternatives.

Indeed, we can readily capture the pattern in (25) in our approach by requiring that topicalization clauses are subject to polarity requirements which entail narrow scope for the negation within the fronted VP. We start by presenting the topicalization operator in (26a) (with the polymorphic syntactic type X), illustrating its ordinary operation to produce (26b) (where the semantics is simply an identity function, since we ignore the pragmatic effects of topicalization):[8]

(26) a. $\lambda\varphi\lambda\sigma.\varphi \circ \sigma(\boldsymbol{\epsilon})$; $\lambda P\lambda\mathscr{C}.\mathscr{C}(P)$; $(\mathrm{S}_{f,\beta}{\upharpoonleft}(\mathrm{S}_{f,\beta}{\upharpoonleft}\mathrm{X})){\upharpoonleft}\mathrm{X}$ where $\beta \in \{+,-\}$

b. ... and vote, John can __.

c. #... and not vote, John can __. ($\neg > \Diamond$)

The derivation for (26b) is given in (27).

(27)
$$
\begin{prooftree}
\AxiomC{$\begin{array}{l}\mathsf{can};\\ \lambda P\lambda y.\Diamond P(y); \mathrm{VP}_{f,+}/\mathrm{VP}_{b,\alpha}\end{array}$}
\AxiomC{$\begin{bmatrix}\varphi_1;\\ Q; \mathrm{VP}_{b,\alpha}\end{bmatrix}^1$}
\BinaryInfC{$\mathsf{can} \circ \varphi_1;\ \lambda y.\Diamond Q(y);\ \mathrm{VP}_{f,+}$}
\AxiomC{$\begin{array}{l}\mathsf{john};\\ \mathbf{j}; \mathrm{NP}\end{array}$}
\BinaryInfC{$\mathsf{john} \circ \mathsf{can} \circ \varphi_1;\ \Diamond Q(\mathbf{j});\ \mathrm{S}_{f,+}$}
\RightLabel{${\upharpoonleft}\mathrm{I}^1$}
\UnaryInfC{$\lambda\varphi_1.\mathsf{john} \circ \mathsf{can} \circ \varphi_1;\ \lambda Q.\Diamond Q(\mathbf{j});\ \mathrm{S}_{f,+}{\upharpoonleft}\mathrm{VP}_{b,\alpha}$}
\AxiomC{$\begin{array}{l}\lambda\varphi\lambda\sigma.\varphi \circ \sigma(\boldsymbol{\epsilon});\\ \lambda P\lambda\mathscr{C}.\mathscr{C}(P);\\ (\mathrm{S}_{f,\beta}{\upharpoonleft}(\mathrm{S}_{f,\beta}{\upharpoonleft}\mathrm{X})){\upharpoonleft}\mathrm{X}\end{array}$}
\AxiomC{$\begin{array}{l}\mathsf{vote};\\ \mathbf{vote};\\ \mathrm{VP}_{b,\alpha}\end{array}$}
\BinaryInfC{$\begin{array}{l}\lambda\sigma.\mathsf{vote} \circ \sigma(\boldsymbol{\epsilon});\\ \lambda\mathscr{C}.\mathscr{C}(\mathbf{vote});\\ \mathrm{S}_{f,\beta}{\upharpoonleft}(\mathrm{S}_{f,\beta}{\upharpoonleft}\mathrm{VP}_{b,\alpha})\end{array}$}
\BinaryInfC{$\mathsf{vote} \circ \mathsf{john} \circ \mathsf{can} \circ \boldsymbol{\epsilon};\ \Diamond\mathbf{vote}(\mathbf{j});\ \mathrm{S}_{f,+}.$}
\end{prooftree}
$$

The requirement on the topicalization operator in (26a) effectively means that $\mathrm{S}_{\varnothing}$ is 'too small' to host a topicalized phrase. That is, in order to license topicalization, the clause needs to have already 'fixed' the polarity value to either $+$ or $-$. This condition turns out to have the immediate effect or enforcing narrow scope on negation in fronted VPs.

To see how this condition works, let's suppose it did not hold; that is, suppose that β could take any of the three polarity values. Then the following would be one way in which *not* inside a topicalized phrase would outscope the modal.

(28) a.
$$
\begin{prooftree}
\AxiomC{$\begin{bmatrix}\varphi_4;\\ Q; \mathrm{VP}_{b,\varnothing}\end{bmatrix}^1$}
\AxiomC{$\begin{bmatrix}\varphi_5;\\ g; \mathrm{VP}_{b,\varnothing}/\mathrm{VP}_{b,\varnothing}\end{bmatrix}^2$}
\BinaryInfC{$\varphi_5 \circ \varphi_4;\ g(Q);\ \mathrm{VP}_{b,\varnothing}$}
\AxiomC{$\begin{array}{l}\mathsf{john};\\ \mathbf{j};\\ \mathrm{NP}\end{array}$}
\BinaryInfC{$\mathsf{john} \circ \varphi_5 \circ \varphi_4;\ g(Q)(\mathbf{j});\ \mathrm{S}_{b,\varnothing}$}
\RightLabel{${\upharpoonleft}\mathrm{I}^2$}
\UnaryInfC{$\begin{array}{l}\lambda\varphi_5.\mathsf{john} \circ \varphi_5 \circ \varphi_4;\\ \lambda g.g(Q)(\mathbf{j}); \mathrm{S}_{b,\varnothing}{\upharpoonleft}(\mathrm{VP}_{b,\varnothing}/\mathrm{VP}_{b,\varnothing})\end{array}$}
\AxiomC{$\begin{array}{l}\lambda\sigma_0.\sigma_0(\mathsf{can});\\ \lambda\mathscr{F}.\Diamond\mathscr{F}(\mathsf{id}_{et});\\ \mathrm{S}_{f,\varnothing}{\upharpoonleft}(\mathrm{S}_{b,\varnothing}{\upharpoonleft}(\mathrm{VP}_{b,\varnothing}/\mathrm{VP}_{b,\varnothing}))\end{array}$}
\BinaryInfC{$\mathsf{john} \circ \mathsf{can} \circ \varphi_4;\ \Diamond Q(\mathbf{j});\ \mathrm{S}_{f,\varnothing}$}
\RightLabel{${\upharpoonleft}\mathrm{I}^1$}
\UnaryInfC{$\lambda\varphi_4.\mathsf{john} \circ \mathsf{can} \circ \varphi_4;\ \Diamond Q(\mathbf{j});\ \mathrm{S}_{f,\varnothing}{\upharpoonleft}\mathrm{VP}_{b,\varnothing}$}
\end{prooftree}
$$

[8] Here and below, $\boldsymbol{\epsilon}$ denotes the null string.

b.

$$
\dfrac{
  \dfrac{
    \begin{array}{l}\text{(28a)}\\ \boxed{\begin{array}{l}\lambda\varphi_4.\mathsf{john}\circ\\ \quad\mathsf{can}\circ\varphi_4;\\ \Diamond Q(\mathbf{j});\\ \mathrm{S}_{f,\varnothing}{\upharpoonright}\mathrm{VP}_{b,\varnothing}\end{array}}\end{array}
    \quad
    \dfrac{
      \dfrac{\begin{array}{l}\mathsf{vote};\\ \mathbf{vote};\\ \mathrm{VP}_{b,\varnothing}\end{array}\quad \begin{bmatrix}\varphi_1;\\ f;\\ \mathrm{VP}_{b,\varnothing}/\mathrm{VP}_{b,\varnothing}\end{bmatrix}^3}{\begin{array}{l}\varphi_1\circ\mathsf{vote};\\ f(\mathbf{vote});\mathrm{VP}_{b,\varnothing}\end{array}}
      \quad
      \begin{array}{l}\lambda\varphi_2\lambda\sigma_1.\\ \quad\varphi_2\circ\sigma_1(\boldsymbol{\epsilon});\\ \lambda\alpha\lambda\mathscr{C}.\mathscr{C}(\alpha);\\ (\mathrm{S}_{f,\beta}{\upharpoonright}(\mathrm{S}_{f,\beta}{\upharpoonright}\mathrm{X})){\upharpoonright}\mathrm{X}\end{array}
    }{
      \boxed{\begin{array}{l}\lambda\sigma_1.\varphi_1\circ\mathsf{vote}\circ\sigma_1(\boldsymbol{\epsilon});\\ \lambda\mathscr{C}.\mathscr{C}(f(\mathbf{vote}));\mathrm{S}_{f,\varnothing}{\upharpoonright}(\mathrm{S}_{f,\varnothing}{\upharpoonright}\mathrm{VP}_{b,\varnothing})\end{array}}
    }
  }{
    \dfrac{\varphi_1\circ\mathsf{vote}\circ\mathsf{john}\circ\mathsf{can}\circ\boldsymbol{\epsilon};\ \Diamond f(\mathbf{vote})(\mathbf{j});\ \mathrm{S}_{f,\varnothing}}{\begin{array}{l}\lambda\varphi_1.\varphi_1\circ\mathsf{vote}\circ\mathsf{john}\circ\mathsf{can}\circ\boldsymbol{\epsilon};\\ \lambda f.\Diamond f(\mathbf{vote})(\mathbf{j});\mathrm{S}_{f,\varnothing}{\upharpoonright}(\mathrm{VP}_{b,\varnothing}/\mathrm{VP}_{b,\varnothing})\end{array}}{\upharpoonright}\mathrm{I}^3
  }
  \quad
  \begin{array}{l}\lambda\sigma.\sigma(\mathsf{not});\\ \lambda\mathscr{G}.\neg\mathscr{G}(\mathsf{id}_{et});\\ \mathrm{S}_{f,-}{\upharpoonright}(\mathrm{S}_{f,\varnothing}{\upharpoonright}(\mathrm{VP}_{b,\varnothing}/\mathrm{VP}_{b,\varnothing}))\end{array}
}{
  \mathsf{not}\circ\mathsf{vote}\circ\mathsf{john}\circ\mathsf{can};\ \neg\Diamond\mathbf{vote}(\mathbf{j});\ \mathrm{S}_{f,-}
}
$$

Here, the derivation uses the NPI version of *can*, in order to license the negation wide scope reading. Since the negation is inside the topicalized phrase rather than the main clause, topicalization needs to be hosted by a clause to which negation hasn't yet combined. But this is precisely the possibility that the restriction $\beta \in \{+,-\}$ excludes (note the conflict in the greyed-in expressions). Using the other version of *can* will only produce the other scopal relation (one in which the modal outscopes negation), so, this option is not available for licensing the reading in question. Thus neither version of *can* admits a derivation resulting in wide scope for topicalized negation, and the same result holds for all NPI (i.e. narrow-scoping) modals.

There is in contrast no difficulty in obtaining the narrow scope interpretation of negation, as shown in (26c), with α and $\delta = -$, and $\beta = +$.

(29) a.

$$
\dfrac{
  \dfrac{\begin{array}{l}\mathsf{vote};\\ \mathbf{vote};\\ \mathrm{VP}_{b,\varnothing}\end{array}\quad \begin{array}{c}(15)\\ \mathsf{not};\ \lambda Q\lambda y.\neg Q(y);\ \mathrm{VP}_{b,-}/\mathrm{VP}_{b,\varnothing}\end{array}}{\mathsf{not}\circ\mathsf{vote};\ \lambda y.\neg\mathbf{vote}(y);\ \mathrm{VP}_{b,-}}
  \quad
  \begin{array}{l}\lambda\varphi_2\lambda\sigma_1.\varphi_2\circ\sigma_1(\boldsymbol{\epsilon});\\ \lambda\alpha\lambda\mathscr{C}.\mathscr{C}(\alpha);(\mathrm{S}_{f,\beta}{\upharpoonright}(\mathrm{S}_{f,\beta}{\upharpoonright}\mathrm{X})){\upharpoonright}\mathrm{X}\end{array}
}{
  \lambda\sigma_1.\mathsf{not}\circ\mathsf{vote}\circ\sigma_1(\boldsymbol{\epsilon});\ \lambda\mathscr{C}.\mathscr{C}(\lambda y.\neg\mathbf{vote}(y));\ \mathrm{S}_{f,\beta}{\upharpoonright}(\mathrm{S}_{f,\beta}{\upharpoonright}\mathrm{VP}_{b,-})
}
$$

b.

$$
\dfrac{
  \begin{array}{l}\text{(29a)}\\ \lambda\sigma_1.\mathsf{not}\circ\mathsf{vote}\circ\sigma_1(\boldsymbol{\epsilon});\\ \lambda\mathscr{C}.\mathscr{C}(\lambda y.\neg\mathbf{vote}(y));\\ \mathrm{S}_{f,\beta}{\upharpoonright}(\mathrm{S}_{f,\beta}{\upharpoonright}\mathrm{VP}_{b,-})\end{array}
  \quad
  \dfrac{
    \dfrac{
      \begin{array}{l}\lambda\sigma_0.\sigma_0(\mathsf{can});\\ \lambda\mathscr{F}.\Diamond\mathscr{F}(\mathsf{id}_{et});\\ \mathrm{S}_{f,+}{\upharpoonright}(\mathrm{S}_{b,-}{\upharpoonright}(\mathrm{VP}_{b,\alpha}/\mathrm{VP}_{b,\alpha}))\end{array}
      \quad
      \dfrac{
        \dfrac{
          \dfrac{\begin{bmatrix}\varphi_1;\\ P;\\ \mathrm{VP}_{b,-}\end{bmatrix}^1\quad\begin{bmatrix}\varphi_3;\\ f;\\ \mathrm{VP}_{b,-}/\mathrm{VP}_{b,-}\end{bmatrix}^3}{\varphi_3\circ\varphi_1;\ f(P);\ \mathrm{VP}_{b,-}}
          \quad
          \begin{array}{l}\mathsf{john};\\ \mathbf{j};\\ \mathrm{NP}\end{array}
        }{\begin{array}{c}\lambda\varphi_3.\mathsf{john}\circ\varphi_3\circ\varphi_1;\\ \lambda P.f(P)(\mathbf{j});\mathrm{S}_{f,+}\end{array}}
      }{\begin{array}{l}\lambda\varphi_3.\mathsf{john}\circ\varphi_3\circ\varphi_1;\\ \lambda P.f(P)(\mathbf{j});\mathrm{S}_{f,+}{\upharpoonright}(\mathrm{VP}_{b,-}/\mathrm{VP}_{b,-})\end{array}}{\upharpoonright}\mathrm{I}^3
    }{
      \mathsf{john}\circ\mathsf{can}\circ\varphi_1;\ \Diamond P(\mathbf{j});\ \mathrm{S}_{f,+}
    }
  }{
    \lambda\varphi_1.\mathsf{john}\circ\mathsf{can}\circ\varphi_1;\ \lambda P.\Diamond P(\mathbf{j});\ \mathrm{S}_{f,+}{\upharpoonright}\mathrm{VP}_{b,-}
  }{\upharpoonright}\mathrm{I}^1
}{
  \mathsf{not}\circ\mathsf{vote}\circ\mathsf{john}\circ\mathsf{can}\circ\boldsymbol{\epsilon};\ \Diamond\neg\mathbf{vote}(\mathbf{j});\ \mathrm{S}_{f,+}
}
$$

The slanted version of *not* combines freely with its VP argument to yield a topicalized VP_{-}, but the type of the mother—in particular, its polarity specification—is determined by the highest scoping operator, *can*, which yields a positive polarity clause.

4 Conclusion

In this paper, we proposed an explicit analysis of scope interactions between modal auxiliaries and negation in English in Type-Logical Grammar. The proposed analysis builds on two previous works in somewhat different research traditions: (i) Iatridou and Zeijlstra's [6] configurational analysis of modal auxiliaries that captures their scopal properties in terms of the distinction between PPI and NPI modals; (ii) Kubota and Levine's [8, 10] analysis of modal auxiliaries in Type-Logical Grammar as higher-order operators that take clausal scope (unlike the more traditional VP/VP analysis in lexicalist theories such as CG and G/HPSG). Our analysis captures the different scoping patterns of different types of modals via the polarity-marking distinction, whose core analytic idea is due to Iatridou and Zeijlstra, but it does so without making recourse to the notion of reconstruction, which is a type of lowering movement whose exact formal implementation in minimalist syntax is somewhat unclear. Our analysis moreover clarifies the relationship between configurational (or transformational) and non-transformational analyses of modal auxiliaries by showing precisely how the latter type of analysis can be thought of as a derivative of the former type of analysis when both are recast within a logical calculus that allows one to *derive* (in the literal sense of 'derive' in formal logic) certain types of lexical descriptions from more abstract and seemingly unrelated lexical descriptions. We take this result to be highly illuminating, as it helps clarify a deeper connection between different stripes of syntactic research that is in no sense obvious unless one takes a logical perspective on grammatical composition.

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A Hybrid Type-Logical Grammar

A.1 Syntactic types

(30) $\mathcal{A} := \{\ \mathrm{S}, \mathrm{NP}, \mathrm{N}, \ldots\}$ (atomic type)
$\mathcal{D} := \mathcal{A} \mid \mathcal{D}\backslash\mathcal{D} \mid \mathcal{D}/\mathcal{D}$ (directional type)
$\mathcal{T} := \mathcal{D} \mid \mathcal{T}\upharpoonright\mathcal{T}$ (type)

Note: The algebra of syntactic types is *not* a free algebra generated over the set of atomic types with the three binary connectives /, \, and ↾. Specifically, given the definitions in (30), in Hybrid TLG, a vertical slash cannot occur 'under' a directional slash. Thus, $\mathrm{S}/(\mathrm{S}\upharpoonright\mathrm{NP})$ is not a well-formed syntactic type. This is a deliberate design, and Hybrid TLCG differs from closely related variants of TLG (such as the Displacement Calculus Morrill [14] and NL_λ Barker and Shan [1]) in this respect.

A.2 Mapping from syntactic types to semantic types

(31) a. $\mathsf{Sem}(\mathrm{NP}) = e$
b. $\mathsf{Sem}(\mathrm{S}) = t$
c. $\mathsf{Sem}(\mathrm{N}) = e \to t$

(32) For any complex syntactic category of the form α/β (or $\alpha\backslash\beta$, $\alpha\upharpoonright\beta$),
$\mathsf{Sem}(\alpha/\beta)$ (= $\mathsf{Sem}(\alpha\backslash\beta) = \mathsf{Sem}(\alpha\upharpoonright\beta)$) = $\mathsf{Sem}(\beta) \to \mathsf{Sem}(\alpha)$

A.3 Mapping from syntactic types to prosodic types

(33) For any directional type $\mathcal{D}$, $\mathsf{Pros}(\mathcal{D}) = \textbf{st}$ (with $\textbf{st}$ for 'strings').

(34) For any complex syntactic type $A{\upharpoonright}B$ involving the vertical slash ${\upharpoonright}$, $\mathsf{Pros}(A{\upharpoonright}B) = \mathsf{Pros}(B) \to \mathsf{Pros}(A)$.

A.4 Deductive rules

(35)

Connective	Introduction	Elimination
$/$	$\dfrac{\dfrac{\begin{array}{ccc} \vdots & \dfrac{[\varphi;\ x;\ A]^n}{\vdots} & \vdots \end{array}}{b \circ \varphi;\ \boldsymbol{\mathcal{F}};\ B}}{b;\ \lambda x.\boldsymbol{\mathcal{F}};\ B/A}\ /\mathrm{I}^n$	$\dfrac{a;\ \boldsymbol{\mathcal{F}};\ A/B \quad b;\ \boldsymbol{\mathcal{G}};\ B}{a \circ b;\ \boldsymbol{\mathcal{F}}(\boldsymbol{\mathcal{G}});\ A}\ /\mathrm{E}$
$\backslash$	$\dfrac{\dfrac{\begin{array}{ccc} \vdots & \dfrac{[\varphi;\ x;\ A]^n}{\vdots} & \vdots \end{array}}{\varphi \circ b;\ \boldsymbol{\mathcal{F}};\ B}}{b;\ \lambda x.\boldsymbol{\mathcal{F}};\ A\backslash B}\ \backslash\mathrm{I}^n$	$\dfrac{b;\ \boldsymbol{\mathcal{G}};\ B \quad a;\ \boldsymbol{\mathcal{F}};\ B\backslash A}{b \circ a;\ \boldsymbol{\mathcal{F}}(\boldsymbol{\mathcal{G}});\ A}\ \backslash\mathrm{E}$
${\upharpoonright}$	$\dfrac{\dfrac{\begin{array}{ccc} \vdots & \dfrac{[\varphi;\ x;\ A]^n}{\vdots} & \vdots \end{array}}{b;\ \boldsymbol{\mathcal{F}};\ B}}{\lambda\varphi.b;\ \lambda x.\boldsymbol{\mathcal{F}};\ B{\upharpoonright}A}\ {\upharpoonright}\mathrm{I}^n$	$\dfrac{a;\ \boldsymbol{\mathcal{F}};\ A{\upharpoonright}B \quad b;\ \boldsymbol{\mathcal{G}};\ B}{a(b);\ \boldsymbol{\mathcal{F}}(\boldsymbol{\mathcal{G}});\ A}\ {\upharpoonright}\mathrm{E}$

Notes: Corresponding to the asymmetry in the status of the directional slashes ($/$, $\backslash$) and the vertical slash (${\upharpoonright}$) in the definitions of syntactic types, there is an asymmetry in the definitions of the deductive rules for the two types of slashes.

Note in particular that in the Introduction rules for $/$ ($\backslash$), instead of lambda binding, the prosodic variable of the hypothesis that is withdrawn is *removed* from the prosodic term on the condition that it appears on the right (left) edge of the prosody of the expression that feeds into the rule. (One way to make sense of this is to take the $/,\backslash$ Introduction rules as abbreviations of theorems in which the variable is first bound by left and right lambda abstraction as usual [23], immediately followed by a step of feeding an empty string to the prosodic function thus obtained.)

So far as we can tell, fixing the prosodic type to be $\textbf{st}$ for directional (i.e. Lambek) syntactic types is crucial for ensuring the particular way in which the directional and vertical slashes interact with one another in the various Slanting lemma and related results (which play important roles in the linguistic analyses we have presented above).