Coloring vertices and faces of locally planar graphs

MICHAEL O. ALBERTSON

Department of Mathematics
Smith College
albertson@felix.smith.edu

Abstract. Let G be an embedded graph. A mapping $c : V(G) \cup F(G) \to \{1, 2, \ldots, r\}$ is called a *vertex-face r-coloring* if elements are assigned different colors whenever they are either adjacent or incident. Let $\chi_{V,F}(G)$ denote the minimum r such that G has a vertex-face r-coloring. Ringel conjectured and Borodin proved that if G is planar, then $\chi_{V,F}(G) \leq 6$. Let $w(G)$ denote the *width of G viz* the length of a shortest non-contractible cycle in G. If G is an embedded graph, let G_* denote the graph obtained from G by starring every non-triangle face. We will see that if $w(G_*)$ is large enough, then $\chi_{V,F}(G) \leq 9$.