Various kinds of tight designs and their existence problems

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Abstract. This talk has two purposes. We first review the concepts of t-designs and tight t-designs in various spaces, such as spheres or association schemes. In the second part, which is a joint work with Etsuko Bannai, we discuss the classification problem of tight Euclidean 4-designs. More details of the second part is as follows.
A finite set X on the unit sphere S^{n-1} in Euclidean space $\mathbf{R}^n$ is called a spherical t-design if

$$\frac{1}{|S^{n-1}|}\int_{S^{n-1}} f(x)d\sigma(x) = \frac{1}{|X|}\sum_{x\in X} f(x)$$

holds for any polynomials in n variables of degree at most t (Delsarte-Goethals-Seidel(1977)). Neumaier and Seidel generalized this concept and gave a definition of Euclidean designs (1988). That is, a finite set X in $\mathbf{R}^n$ is called a Euclidean t-design if

$$\sum_{i=1}^{p}\frac{\omega(X_i)}{|S_i|}\int_{S_i} f(x)d\sigma_i(x) = \sum_{x\in X}\omega(x)f(x)$$

holds for any polynomials in n variables of degree at most t, where $\{S_i \mid i = 1, 2, \ldots, p\}$ is the set of concentric spheres centered the origin and $S_i \cap X \neq \emptyset$, $\omega : X \longrightarrow \mathbf{R}_{>0}$ is a weight function on X. Neumaier-Seidel(1988) and Delsarte-Seidel(1989) proved that if a Euclidean $2e$-design X intersects with at least $[\frac{e}{2}]+1$ if $O \notin X$ (or $[\frac{e+1}{2}]+1$ if $O \in X$) concentric spheres centered the origin, then $|X| \geq \binom{n+e}{e}$. We call a $2e$-design X is tight if $|X| = \binom{n+e}{e}$ and X intersects with at least $[\frac{e}{2}]+1$ if $O \notin X$ (or $[\frac{e+1}{2}]+1$ if $O \in X$) concentric spheres. We give the classification of Euclidean tight 4-designs with constant weight. We also talk about some special cases of Euclidean tight 4-designs with non-constant weight.