

**Recent Progress on Turán's Brickyard Problem:  
Improved Lower Bounds for the Crossing Numbers of  $K_{m,n}$  and  $K_n$**

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**Abstract.** In the earliest instance of a crossing number problem, Turán conjectured in 1945 that the crossing number  $cr(K_{m,n})$  of  $K_{m,n}$  is  $\lfloor (m-1)/2 \rfloor \lfloor m/2 \rfloor \lfloor (n-1)/2 \rfloor \lfloor n/2 \rfloor$ .

By using some elementary topological arguments, we set up a quadratic optimization problem whose minimum yields a lower bound for  $cr(K_{m,n})$ . Although the quadratic problem is intractable because of its size, by using some very recent relaxation techniques for quadratic programming we were able to show that  $cr(K_{m,n})$  is at least 0.83 of its conjectured value, for each fixed  $m$  and sufficiently large  $n$ . This also implies that the crossing number of the complete graph  $K_n$  is asymptotically at least 0.83 of its long-conjectured value. This is joint work with Etienne DeKlerk, John Maharry, Dima Pasechnik, and Bruce Richter.