

# The nonorientable genus of complete tripartite graphs: the basis cases

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**Abstract.** Finding the surfaces on which a particular graph can be embedded is a difficult problem, and exact answers are known only for some very special classes of graphs. Even for complete graphs, it took over 70 years to find the genus of an arbitrary complete graph  $K_n$ . The complete bipartite graphs  $K_{m,n}$  were somewhat easier, with the answers for orientable and nonorientable genus being given in the 1960's by Ringel. In 1976 Stahl and White conjectured that the nonorientable genus of the complete tripartite graph  $K_{l,m,n}$ , with  $l \geq m \geq n$ , is  $\left\lceil \frac{(l-2)(m+n-2)}{2} \right\rceil$ . Recently we, in joint work with Ellingham, Kawarabayashi, and Zha, have proved that this conjecture is true, with three exceptions:  $K_{4,4,1}$ ,  $K_{4,4,3}$ , and  $K_{3,3,3}$ . The proof is by induction on  $l$ . In this talk we discuss the basis cases for the induction, which are  $K_{m,m,n}$  and, when  $m$  is odd and  $n$  is even,  $K_{m+1,m,n}$ .