Biased Expansion Graphs, Dowling Lattices, and Neil's Advisor's 3-decomposition Theorem

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Abstract. A biased expansion Omega of a graph D is a kind of branched covering graph of D. A simple kind of biased expansion, built from a group, is called a group expansion. Another kind is a multary quasigroup expansion. (A multary, or n-ary, quasigroup is a set with a multary operation that is like a group operation but with n arguments and without an analog of associativity.)

If D is 3-connected, then Omega is a group expansion. If D is only 2-connected, then Omega is built from group expansions and multary quasigroup expansions in analogy to Tutte's 3-decomposition of graphs.

Biased expansions have associated matroids; those of group expansions of complete graphs are Dowling's lattices. If D is 2-connected and not extendible on the same vertex set, then it is built out of Dowling lattices and irreducible multary quasigroup expansions.

The 3-connectedness theorem implies that a multary quasigroup with enough factorizations must be a group in disguise. If it does not have enough factorizations, it is obtained by composition of multary operations from disguised groups and irreducible multary quasigroups.