Physics 5300, Theoretical Mechanics   Spring 2015

Assignment 8 solutions

The problems numbers below are from Classical Mechanics, John R. Taylor, University Science Books (2005).

Problem 1   Taylor 16.2

Solution: Each mass is m, and the spacing is b. The force on the mass is

$$T\sin\theta_1 - T\sin\theta_2 \approx T(\tan\theta_1 - \tan\theta_2) = T[\frac{y_{n+1}-y_n}{b} - \frac{y_n - y_{n-1}}{b}] \approx Tb\frac{d^2y}{dx^2} \qquad (1)$$

The motion is given by

$$m\ddot{y} \approx Tb\frac{d^2y}{dx^2} \qquad (2)$$

$$\ddot{y} \approx \frac{Tb}{m}\frac{d^2y}{dx^2} \qquad (3)$$

Thus taking a limit where $\frac{Tb}{m} = 1$, we get the wave equation

$$\ddot{y} = y'' \qquad (4)$$

Problem 2   Taylor 16.5

Solution:

$$\frac{d}{dx}f(x-ct) = f'(x-ct), \quad \frac{d^2}{dx^2}f(x-ct) = f''(x-ct) \qquad (5)$$

$$\frac{d}{dt}f(x-ct) = -cf'(x-ct), \quad \frac{d^2}{dx^2}f(x-ct) = (-c)^2f''(x-ct) \qquad (6)$$

Thus

$$\frac{d^2f}{dx^2} = c^2\frac{d^2f}{dt^2} \qquad (7)$$

Problem 3   Taylor 14.2

Solution: (a)The radius is $r = 5 \times 10^{-15}m$, so the area is $\pi r^2 = 25\pi 10^{-30}m^2 \approx 8 \times 10^{-29}m^2$. We thus have $\sigma = .8\, barn$.

For an atom, $r = 10^{-9}m$, so $\sigma = \pi \times 10^{-18}\, m^2$. This is $\sigma = 3 \times 10^{10}\, barns$.

Problem 4 Taylor 14.3

Solution: The mass of each atom is m. The total mass is M. Thus the number of atoms is M/m. The cross section area we take to be A. The depth is L. Thus the volume is $V = AL$. The density is ρ. Thus the mass is $M = \rho V = \rho AL$. Thus

$$\frac{N}{A} = \frac{M}{mA} = \frac{\rho AL}{mA} = \frac{\rho L}{m} = \frac{.07 \times 50}{1.67 \times 10^{-24}} = 2 \times 10^{24} \tag{8}$$

Problem 5 Taylor 14.14

Solution: (a) We have

$$d\sigma = 2db \tag{9}$$

$$d\Omega = 2d\theta \tag{10}$$

Thus

$$\frac{d\sigma}{d\Omega} = \frac{|db}{d\theta}| \tag{11}$$

(b) We have

$$b = R\cos\frac{\theta}{2} \tag{12}$$

$$\frac{db}{d\theta} = -\frac{R}{2}\sin\frac{\theta}{2} \tag{13}$$

$$\frac{d\sigma}{d\Omega} = |\frac{db}{d\theta}| = \frac{R}{2}\sin\frac{\theta}{2} \tag{14}$$

(c)

$$\sigma = \int_{-\pi}^{\pi} d\theta \frac{R}{2}|\sin\frac{\theta}{2}| = 2R \tag{15}$$

Problem 6 Taylor 14.24

Solution: (a) We have

$$\theta_{cm} = 2\theta_{lab}, \quad \cos\theta_{cm} = \cos(2\theta_{lab}) = 2cos^2\theta_{lab} - 1 \tag{16}$$

$$\frac{d\cos\theta_{cm}}{d\cos(\theta_{lab}} = -2\cos\theta_{lab} \tag{17}$$

Thus

$$|\frac{d\cos\theta_{cm}}{d\cos(\theta_{lab}}| = 42\cos\theta_{lab} \tag{18}$$

and the result follows from eq 14.45:

$$(\frac{d\sigma}{d\Omega})_{lab} = |\frac{d\cos\theta_{cm}}{d\cos(\theta_{lab}}|(\frac{d\sigma}{d\Omega})_{cm} \tag{19}$$

(b)

$$(\frac{d\sigma}{d\Omega})_{cm} = \frac{R^2}{4} \tag{20}$$

Thus

$$(\frac{d\sigma}{d\Omega})_{lab} = 4\cos\theta_{lab}(\frac{d\sigma}{d\Omega})_{cm} = 4\cos\theta_{lab}\frac{R^2}{4} = R^2|\cos\theta_{lab}| \tag{21}$$

Note that θ_{cm} ranges between 0 and π, so θ_{lab} ranges between 0 and $\frac{\pi}{2}$. Thus

$$\int d\Omega R^2\cos\theta_{lab} = 2\pi\int_0^1 d\cos\theta_{lab} R^2|\cos\theta_{lab}| = \pi R^2 \tag{22}$$