Physics 5300, Theoretical Mechanics Spring 2015

Quiz 1 Solutions

Given: Friday Jan 16

Problem 1 A particle moves in a circle (center O and radius R) with constant angular velocity ω counterclockwiswe:

$$\vec{r}(t) = R\cos(\omega t)\hat{x} + R\sin(\omega t)\hat{y}$$

Find

(i) The velocity at time t. (2 points)

$$\vec{v}(t) = -R\omega\sin(\omega t)\hat{x} + R\omega\cos(\omega t)\hat{y}$$

(ii) The acceleration at time t. (2 points)

$$\vec{a}(t) = -R\omega^2\cos(\omega t)\hat{x} - R\omega^2 sin(\omega t)\hat{y}$$

(iii) What is the magnitude and direction of the acceleration? (describe in any way you like, but the result should be clear). (2 points)

$$|\vec{a}|^2 = R\omega^2$$

We see that

$$\vec{a} = -R\omega^2\vec{x}$$

Thus the acceleration points radially inwards.

Problem 2 The unit vector $\hat{r}$ in 2-d polar coordinates is equal to

$$\hat{r} = \cos\phi\,\hat{x} + \sin\phi\,\hat{y}$$

Find the corresponding expresson for the unit vector $\hat{\phi}$. (4 points)

$\hat{\phi}$ is perpendicular to $\hat{r}$, and at $\phi = 0$, it points in the positive y direction. Thus writing

$$\hat{\phi} = a\,\hat{x} + b\,\hat{y}$$

We see that

$$a\cos\phi + b\sin\phi = 0$$

Thus

$$\frac{a}{b} = -\frac{\sin\phi}{\cos\phi}$$

so we can write

$$a = -k\sin\phi, \quad b = k\cos\phi$$

for some constant k. We also have, since $\hat{\phi}$ is a unit vector

$$a^2 + b^2 = 1$$

This gives

$$k = \pm 1$$

so we have the two possibilties

$$\hat{\phi} = -\sin\phi\,\hat{x} + \cos\phi\,\hat{y}$$

and

$$\hat{\phi} = \sin\phi\,\hat{x} - \cos\phi\,\hat{y}$$

Using the last geometric consition that $\hat{\phi}$ point in the counterclockwise direction along the circle, we get

$$\hat{\phi} = -\sin\phi\,\hat{x} + \cos\phi\,\hat{y}$$